International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 6 (2018) pp. 3361-3368


# Coriolis Force and Wall Velocity Effects for MHD Rotating Fluid Past a Semi-Infinite Vertical Moving Plate

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**Abstract**

This study deals with the problem of the thermal and mass transfer of an incompressible rotating and moving fluid in the presence of a magnetic field. The considered plate is vertical and moving also and the Hall effect is taken into account. The similarity equations are solved numerically by a finite difference scheme via the technique of Lobatto III. Velocity, temperature and concentration profiles are presented graphically and the results are discussed and focused on the main effects the wall velocity $\lambda$ and the Coriolis force parameter $\xi$. The heat and mass transfer rates are entered for various and reasonable values of these specific parameters.

All the velocities decrease with the increase of these parameters, whereas the temperature and the concentration profiles increase. An opposite effect is found on $Nu_x$ and $Sh_x$ when $\xi$ and $\lambda$ increase.

**Keywords:** Magneto-hydrodynamic, Hall effect, wall velocity, Coriolis force, moving plate, moving –rotating fluid .

## NOMENCLATURE

### *Characters*

| | |
|---|---|
| $B_0$ | imposed magnetic field (T) |
| $C$ | concentration (Kg m$^{-3}$) |
| $c_p$ | specific heat at constant pressure (J Kg $^{-1}$ K$^{-1}$) |
| $D$ | diffusion coefficient (m$^2$ s$^{-1}$) |
| $E$ | electric charge (coulomb m$^{-3}$) |
| $f(\eta)$, $h(\eta)$, $\theta(\eta)$, $\phi(\eta)$ | similarity functions for ($u$, $w$, $T$, $C$) |
| $g$ | |
| $Gc$ | acceleration due to gravity (m s$^{-2}$) |
| | local Grashof number due to |
| $Gr$ | concentration differences |
| | local Grashof number due to |
| $h_m$ | temperature differences |
| $h_w$ | mass transfer coefficient (m s$^{-1}$) |
| $J$ | heat transfer coefficient (W m$^{-2}$ K$^{-1}$) |
| $k$ | electric current density (A m$^{-2}$) |
| $M$ | thermal conductivity (W m$^{-1}$ K$^{-1}$) |
| $m$ | magnetic parameter |
| $m_e$ | Hall parameter |
| $m_w$ | mass of the electron (Kg) |
| $Nu$ | mass flux (Kg m$^{-2}$ s$^{-1}$) |
| $n_e$ | Nusselt number |
| $p$ | electron number |
| $p_e$ | fluid pressure (N m$^{-2}$) |
| $Pr$ | electron pressure |
| $q_w$ | Prandtl number |
| $Re$ | heat flux (W m$^{-2}$) |
| $Sc$ | Reynolds number |
| $Sh$ | Schmidt number |
| $T$ | Sherwood number |
| $t_e$ | fluid temperature (K) |
| $U$ | electron collision time (s) |
| $U_1$ | composite velocity (m s$^{-1}$) |
| $U_2$ | plate velocity (m s$^{-1}$) |
| $(u, \upsilon, w)$ | free stream velocity (m s$^{-1}$) |
| | velocity components along ( x, y, z)- |
| $(x, y, z)$ | axes (m s$^{-1}$) |
| | Cartesian coordinates |

### *Symbols*

| | |
|---|---|
| $\alpha$ | Thermal diffusivity (m$^2$ s$^{-1}$) |
| $\beta$ | coefficient of thermal expansion (K$^{-1}$) |
| $\beta^*$ | coefficient of expansion with concentration (Kg $^{-1}$ m$^3$) |
| $\eta$ | similarity variable |
| $\lambda$ | wall velocity |
| $\nu$ | kinematic viscosity (m$^2$ s$^{-1}$) |
| $\xi$ | dimensionless Coriolis force |
| $\rho$ | fluid density (Kg m$^{-3}$) |
| $\sigma$ | electrical conductivity ($\Omega^{-1}$ m$^{-1}$) |
| $\psi$ | stream function (m$^2$ s$^{-1}$) |
| $\omega_e$ | electron frequency (Hz) |
| $\Omega$ | angular velocity (Rad s$^{-1}$) |

### *Subscripts*

| | |
|---|---|
| $w$ | wall condition |
| $\infty$ | free stream condition |
| 0 | constant condition |

## INTRODUCTION

Simultaneous heat and mass transfer occur in numerous engineering applications. In particular and in the field of MHD, the main applications are in the power generators, plasma studies, nuclear reactors, geothermal energy extractions. When the MHD convective flow is involved, a current of Hall can be induced in a direction normal to both the electric and the magnetic field. Hall currents are important and hence cannot be neglected for ionized gas with a low density and/or a strong magnetic field.

In the last decade, many investigations on the magnetohydrodynamic convective flow over a flat plate or from other geometries under different conditions have been published. The free convection heat and mass transfer of an electrically conducting fluid along a semi-infinite vertical flat plate in the presence of a strong magnetic field has been studied by Ghaly [1], Chamkha [2], Zueco Jordaín [3], Ibrahim et al. [4], Mohamed and Abo-Dahab [5], Sharma et al. [6], Das [7], Ziaul Haque et al. [8]. It is noted that the Hall effect is considered in a few of them.

However, this effect on the heat transfer along a vertical plate has been discussed more extensively in [9-14]. Specific works, in the same area [15- 17] take into account the Couette model flow.

The rotating system with Hall effect in the presence of an external uniform magnetic field and heat transfer has been investigated by various workers [18-23]. This is encountered in the above cited applications and also in medicine and biology. Takhar et al. [24] examined in a particular paper the combined effects of the magnetic field, Hall currents and free stream velocity on the non-similar flow over a moving horizontal surface. A rotating fluid is admitted. Hayat et al. [25] studied the effects of Hall current and heat transfer on the rotating flow of a second grade fluid past a porous plate with variable suction.

In literature, the attention has been devoted to the case of coupled heat and mass transfer during the last years. Kinyanjui et al. [26] presented their work in MHD free convection heat and mass transfer of a heat generating fluid past an impulsively started infinite vertical porous plate with Hall current and radiation absorption. Kwanza et al. [27] studied an unsteady MHD free convection flow past semi-infinite vertical porous plate subjected to constant heat flux with viscous dissipation, radiation absorption, Hall and Ion-slip currents. Aboeldahab and Elbarbary [28] investigated the heat and mass transfer past a semi-infinite vertical plate under the combined buoyancy force effects of thermal and species diffusion with Hall effect in the presence of a strong uniform magnetic field. Also, Megahed et al. [29] formed similarity equations in magnetohydrodynamics and Hall current effects and investigated a steady free convection flow and mass transfer past a semi-infinite vertical flat plate under the combined buoyancy force effects of thermal and species diffusion under a strong non-uniform magnetic field. Hassanien et al. [30] extended the last work to consider the presence of variable wall temperature and variable wall concentration. More recently, Narayana et al. [31] analyzed the effects of Hall current and radiation absorption on MHD free convection mass transfer flow of a micropolar fluid in a rotating frame of reference. Similarly, the mass diffusion of chemical species with first and higher order reactions has been discussed by Salem and Abd El-Aziz [32] from the problem of steady hydromagnetic convective heat and mass transfer adjacent to a continuous vertical plate in the presence of heat generation/absorption and Hall currents. In the same context, Elgazery [33] presented numerical study for the effects of chemical reaction, heat and mass transfer, Hall, ion-slip currents, variable viscosity and variable thermal diffusivity on magneto-micropolar fluid flow along a horizontal plate.

No attempt has been made so far to analyze the rotational effects on the MHD convective heat and mass transfer along a semi-infinite vertical and moving flat plate. So, the present study is concerned with the Coriolis force that has a significant influence on the fluid dynamics of these systems. Hence, the main objective of the present investigation is to examine the effects of the wall velocity and the Coriolis force generated on the steady magnetohydrodynamic convective flow past a moving semi-infinite vertical flat plate in a rotating fluid resulting from buoyancy forces which arise from a coupled phenomenon of temperature and species concentration effects. The fluid is viscous incompressible and electrically conducting. The uniform and strong magnetic field is applied normal to the flat plate. The coupled non-linear partial differential equations governing the flow with the boundary conditions are transformed to a system of non-linear ordinary differential equations with the appropriate boundary conditions. Searching a similarity system, a lot of new variables are used, which will be approved but not shown in this article. Furthermore, the retained equations are solved numerically by using finite difference scheme. In the following sections, the problem is formulated, analyzed and results are highlighted to the rotational effects and the wall velocity which are not represented sufficiently in the literature. It is very attractive to observe that all incorporate effects in this study are inter-linked and can produce significant deviations to classical results.

## MATHEMATICAL ANALYSIS

As mentioned above, we consider a steady coupled heat and mass transfer by hydromagnetic flow past a heated semi-infinite vertical flat plate. The coordinate system is indicated by $x$, $y$ and $z$, this let coordinate be coincident with the leading edge and with the origin at its end, Fig. 1. This plate moves with a constant velocity $U_1$ in the $z$ direction in a viscous, incompressible, electrically conducting fluid which is rotating with a constant angular velocity $\Omega$ about the $y$-axis. Also, a uniform free stream velocity $U_2$ is parallel to the $z$-axis. A strong uniform magnetic field of strength $B_0$ is imposed normally to the plate. In this case, it cannot neglect the effect of Hall currents. One reasonable assumption is that the induced magnetic field is neglected because the magnetic Reynolds number is very small, i.e $\mu_0 \bar{V} \bar{L} << 1$ where $\mu_0$ is the magnetic permeability, and $\bar{V}$ and $\bar{L}$ are characteristic velocity and length [24].

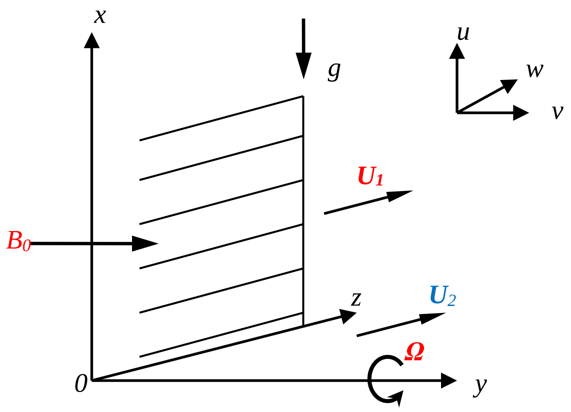


**Figure 1:** Sketch of the physical model and the coordinate system.

Mathematically, the problem can be formulated with the help of the coupled equations written in vectorial form as follows:

$$\nabla.\mathrm{V}=0 \quad (1)$$

$$(\mathrm{V}.\nabla)\mathrm{V}+2\Omega\times\mathrm{V}=-\frac{1}{\rho}\nabla p+\nu\nabla^2\mathrm{V}+\frac{1}{\rho}(\mathrm{J}\times\mathrm{B})+\vec{\mathrm{g}}[\beta(\mathrm{T}-\mathrm{T}_\infty)+\beta^*(\mathrm{C}-\mathrm{C}_\infty)] \quad (2)$$

$$(\mathrm{V}.\nabla)\mathrm{T}=\alpha\,\nabla^2\mathrm{T} \quad (3)$$

$$(\mathrm{V}.\nabla)\mathrm{C}=\mathrm{D}\,\nabla^2\mathrm{C} \quad (4)$$

$$\mathrm{J}=\sigma(E+\mathrm{V}\times\mathrm{B})-\frac{\sigma}{en_e}(\mathrm{J}\times\mathrm{B}-\nabla p_e) \quad (5a)$$

$$\nabla\times\mathrm{H}=\mathrm{J};\nabla\times\mathrm{E}=0;\nabla.\mathrm{B}=0 \quad (6)$$

These equations are continuity, momentum, energy, and mass equations, generalized Ohm's law and Maxwell's equations. V (u, v, w) is the velocity vector of the fluid, B(0, $B_0$, 0) is the applied magnetic field, J is the electric current density vector, E is the electric field vector, and H is the magnetic field strength vector. All others symbols are included in the nomenclature.

Another comprehensive assumption is that the external electric field is zero because any voltage is considered. Equation (5a) is consequently simplified by the presence additionally of a strong magnetic field imposed [29, 31] and becomes

$$\mathrm{J}=\sigma(\mathrm{V}\times\mathrm{B})-\frac{\omega_e t_e}{B_0}(\mathrm{J}\times\mathrm{B}) \quad (5b)$$

The temperature and the species concentration at the plate are $T_w$ ($>T_\infty$) and $C_w$ ($>C_\infty$), where $T_\infty$ and $C_\infty$ are the temperature and the species concentration of the free stream.

The thermoelectric pressure and ion slip are considered negligible for weakly ionized gases as fluid used here. In this paper, and without loss of generality of the formulation, it also assumed that viscous and electrical dissipation are negligible. The effects of the Coriolis force and (or) of Hall current induces a cross flow in the *z*-direction. To simplify slightly the problem, it is also assumed that there is no variation of flow, heat and mass transfer quantities in the *z*-direction, assumed valid for the infinite plate.

Explicitly, the components of the electric current density, eq. (5b) are

$$\mathrm{J}_x=\frac{\sigma B_0}{1+m^2}(mu-w)\ ;\ \mathrm{J}_y=0\ \text{ and }\ \mathrm{J}_z=\frac{\sigma B_0}{1+m^2}(u+mw)$$

In which the second equation states that the plate is electrically non-conducting.

$m$ $(=\omega_e t_e)$ is identified as the Hall parameter, the electron frequency is defined as $\omega_e$ $(=eB_0/m_e)$ while the electrical conductivity is $\sigma$ $(\sigma=e^2 n_e t_e/m_e)$.

As it can be seen, the Hall effect appeared as an additional current density linked to the corresponding and appropriate velocities.

The Boussinesq's approximation is used from the classical assumption that the fluid density is temperature-dependent.

All the considered and explained assumptions are employed and the above equations that describe the problem are converted to the scalar form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (7)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+2\Omega w=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)+g\beta^*(C-C_\infty)-\left(\frac{\sigma B_0^2}{\rho(1+m^2)}\right)(u+mw) \quad (8)$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}-2\Omega u=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\frac{\partial^2 w}{\partial y^2}-\left(\frac{\sigma B_0^2}{\rho(1+m^2)}\right)(w-mu) \quad (9)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \quad (10)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \quad (11)$$

Further, it can be performed a balance between the Lorentz and Coriolis forces far away from the surface and the pressure gradients in the *x* and *y* directions, $-\frac{1}{\rho}\frac{\partial p}{\partial x}$ and $-\frac{1}{\rho}\frac{\partial p}{\partial z}$. The corresponding equations are later included [24]:

$$-\frac{1}{\rho}\frac{\partial p}{\partial x}=2\Omega U_2+\left(\frac{\sigma B_0^2}{\rho(1+m^2)}\right)mU_2 \quad (12a)$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial z}=\left(\frac{\sigma B_0^2}{\rho(1+m^2)}\right)U_2 \quad (12b)$$

For Eqs. (7-11) with (12a-12b), the appropriate boundary conditions are expressed as

$$u=0,\quad v=0,\quad w=U_1,\quad T=T_w,\quad C=C_w \text{ at } \ y=0$$
$$u=0,\quad w=U_2,\quad T=T_\infty,\quad C=C_\infty \quad \text{as} \quad y\to\infty \quad (13)$$

Now, let us considering the following dimensionless variables which are found to give similarity equations

$$\eta=\sqrt{\frac{\Omega}{\nu}}y,\quad \psi=\sqrt{\Omega\nu}xf(\eta),\quad w=\Omega xh(\eta),$$
$$\theta(\eta)=(T-T_\infty)/(T_w-T_\infty),$$
$$\phi(\eta)=(C-C_\infty)/(C_w-C_\infty),\quad \xi=\Omega x/U,$$
$$U=U_1+U_2,\quad \lambda=U_1/U \quad (14)$$

Where $\psi$ is the stream function satisfying the continuity equation with the u-velocity and v-velocity are expressed

$$u=\frac{\partial\psi}{\partial y},v=-\frac{\partial\psi}{\partial x} \quad (15)$$

Using equations (15) and (14), we may show that:

$$u = \Omega x f'(\eta)$$
$$v = -\sqrt{\Omega \nu} f(\eta). \quad (16)$$

The results obtained in terms of the local similarity equations are:

$$f''' - f'^2 + ff'' - 2h + Gr\,\theta + Gc\,\phi - \frac{M}{(1+m^2)}\left[f' + m\left(h + \frac{(\lambda-1)}{\xi}\right)\right] + \frac{2(1-\lambda)}{\xi} = 0 \quad (17)$$

$$h'' + fh' - f'h + 2f' - \frac{M}{(1+m^2)}\left[h - mf' + \frac{(\lambda-1)}{\xi}\right] = 0 \quad (18)$$

$$\theta'' + Pr\, f\theta' = 0 \quad (19)$$

$$\phi'' + Scf\phi' = 0 \quad (20)$$

The primes indicate differentiation with respect to $\eta$, $M = \frac{\sigma B_0^2}{\rho\Omega}$ is the magnetic parameter, $Gr = \frac{g\beta(T_w - T_\infty)}{\Omega^2 x}$ is the Grashof number due to temperature differences, $Gc = \frac{g\beta^*(C_w - C_\infty)}{\Omega^2 x}$ is the Grashof number due to concentration differences, $Pr = \frac{\rho\nu c_p}{k}$ is the Prandtl number, $Sc = \frac{\nu}{D}$ is the Schmidt number, $U$ is the composite velocity, $\lambda$ is the wall velocity to the composite velocity and labeled for brevity by wall velocity and $\xi$ represents the Coriolis force.

The transformed boundary conditions are given by

$$f'(0) = 0, \quad f(0) = 0, \quad h(0) = \frac{\lambda}{\xi}, \quad \theta(0) = 1, \quad \phi(0) = 1,$$
$$f'(\infty) = 0, \quad h(\infty) = \frac{1-\lambda}{\xi}, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0. \quad (21)$$

Of special interest for this problem are the local Nusselt number and the local Sherwood number.

These physical quantities are represented by the heat flux and is given by

$$q_w = -k\left.\frac{\partial T}{\partial y}\right|_{y=0} = -k(T_w - T_\infty)(\Omega/\nu)^{1/2}\theta'(0) \quad (22)$$

And the heat transfer coefficient may be written as follows:

$$h_w = \frac{q_w}{(T_w - T_\infty)} = -k(\Omega/\nu)^{1/2}\theta'(0) \quad (23)$$

This coefficient is linked to the desired number as follows

$$Nu_x = \frac{xh_w}{k} = -Re_x^{1/2}\theta'(0)\text{ , or}$$
$$Nu_x Re_x^{-1/2} = -\theta'(0) \quad (24)$$

where $Re_x = \frac{x^2\Omega}{\nu}$ is the local Reynolds number.

Similarly, the mass flux is given by

$$m_w = -D\left.\frac{\partial C}{\partial y}\right|_{y=0} = -D(C_w - C_\infty)(\Omega/\nu)^{1/2}\phi'(0) \quad (25)$$

This corresponds to the mass transfer coefficient:

$$h_m = \frac{m_w}{(C_w - C_\infty)} = -D(\Omega/\nu)^{1/2}\phi'(0) \quad (26)$$

The local Sherwood number $Sh_x$ is given by

$$Sh_x = \frac{xh_m}{k} = -Re_x^{1/2}\phi'(0), \quad \text{or}$$
$$Sh_x Re_x^{-1/2} = -\phi'(0) \quad (27)$$

## NUMERICAL METHOD AND ACCURACY

The set of the coupled ordinary differential Eqs (17-20) is highly nonlinear and similar system are reduced to a sequence of ordinary differential in view to apply a perturbation analytical method [31]. The calculations involved with this approach are too lengthy and the accuracy remains to prove. Closed-form solutions are possible to find if many assumptions exposed here are simplified. Together with the boundary conditions (21), equations (17-20) form a two point boundary value problem which can be solved by numerical method with desired precision. The finite difference method that implements the 3-stage Lobatto collocation formula and the collocation polynomial provides a continuous solution that is fourth-order accurate uniformly in the interval of integration is used. Mesh selection and error control are based on the residual of the continuous solution. The collocation technique uses a mesh of points to divide the interval of integration into subintervals.

In order to verify the accuracy of the numerical results, the validity of the numerical code has been checked for a limiting case. A comparison is made with the similar studies, but only a part of the results of Takhar and al. [24] can be used. A lot of attention should be devoted regarding the different coordinate system. As it is seen from Fig. 2, our values are in agreement with that of the cited authors.

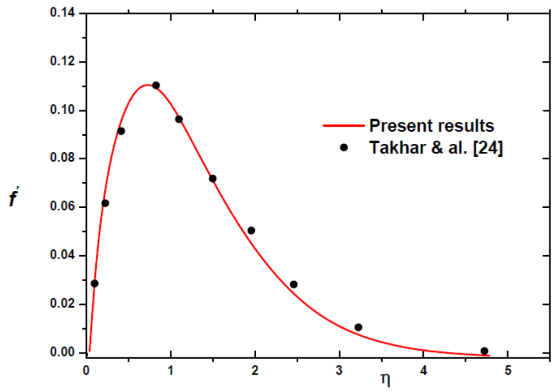


**Figure 2:** Graphical comparison with of the x-velocity $f'$ with that corresponding velocity from Takhar and al. [24] for Gr=Gc=0, Pr=Sc=0, M=1.0, $m$=1, $\lambda$ =0.25

## RESULTS

Figs. 3-6 depict the effect of the ratio of the wall velocity $\lambda$ on the behavior of the velocities of the fluid, temperature and concentration profiles $f'$, $h$, $\theta$ and $\phi$ for $M$= $m$= $\xi$ = 1. Fig. 3 shows that the x-velocity $f'$ decreases as $\lambda$ increases. If both *Gr* and *Gc* become less important, it is clear from this figure that a large value of the wall velocity leads to a reverse of the transverse x-velocity. This corresponds to a competitive role between the mechanisms involved. It is evident that a strong Buoyancy force accelerates the velocity in the x-direction. This trend is the same except that the weakly of *Gc* or *Gr* lead to a negative value of $f'$ in conjunction of strong $\lambda$, *M*, *Gr*, *Gc* and *m*. Further, in Fig. 4, it is remarked that as $\lambda$ increases the axial w-velocity $h$ increases in the region $0 \le \eta \le 0.5$ and decreases as $\eta \ge 0.5$ for *Gc* = *Gr* = 0.5, while the separate limit is around 0.6 for *Gc* = *Gr* = 2. This reverse behavior can be attributed to the presence of different forces acting in the opposite sense. It is evident that more the velocity of the plate is great more the axial velocity of the fluid near it is important. This effect becomes inverted for the far region of the plate. It can understand that a large value of $\lambda$ leads to a negative value of $h$, as explained for $f'$. Moreover, in this figure all the profiles are embedded in the limits of the boundary layer $h(0) = \lambda$ and $h(\infty) = 1 - \lambda$, ($\xi$ = 1) imposed as boundaries conditions.

On the other hand, Figs. 5 and 6 reveal that increasing in the values of $\lambda$ produce increases in the temperature distributions $\theta$ as well as in the concentration distributions $\phi$ of the fluid. The heat and mass transfer are more activated as the fluid or the plate move rapidly and when the flow and the plate are in the opposite directions. As indicated earlier, the Grashof numbers play an important role for modifying the thermal and concentration profiles.

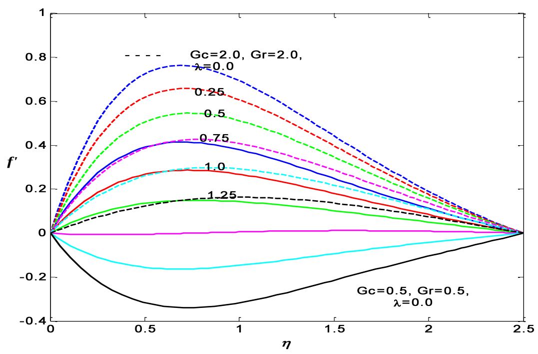


**Figure 3:** Effect of the wall velocity λ on the x- velocity profile for Pr=0.71, Sc=0.22, M=1.0, *m*=1., ξ=1.

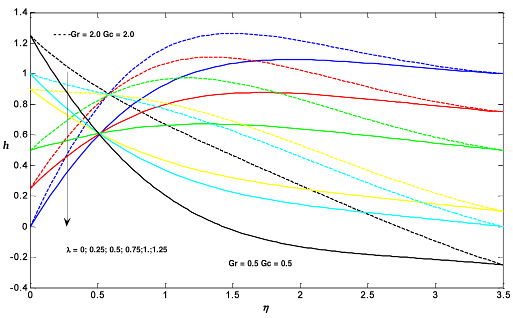


**Figure 4:** Effect of the wall velocity λ on the w- velocity profile for Pr=0.71, Sc=0.22, M=1.0, *m*=1., ξ=1.

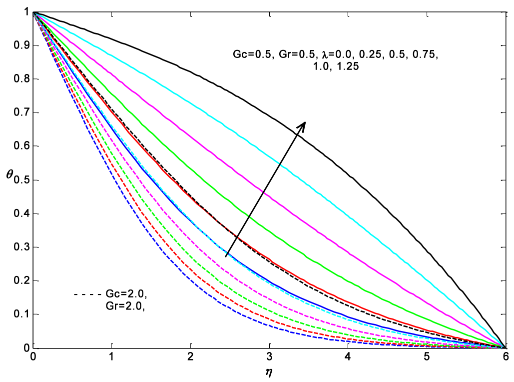


**Figure 5:** Effect of the wall velocity λ on the temperature profile for Pr=0.71, Sc=0.22, M=1.0, *m*=1, ξ=1.

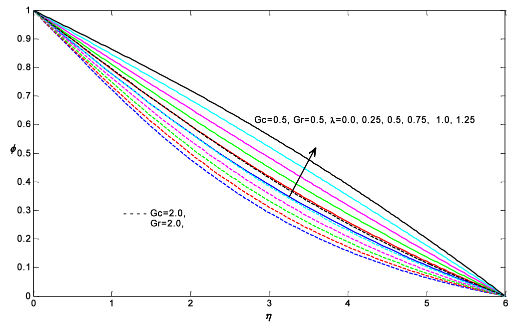


**Figure 6:** Effect of the wall velocity λ on the concentration profile for Pr=0.71, Sc=0.22, M=1.0, *m*=1., ξ=1.

The following Figs. 7-10 describe the behavior of $f'$, $h$, $\theta$ and $\phi$ with changes in the values of the Coriolis force parameter $\xi$ for fixed values $M$=1, $m$=1 and $\lambda$ =0.25. It is clear from Figs. 7 and 8 that increasing in the values of $\xi$ produces decreases in the transverse x-velocity $f'$ as well as in the axial w-velocity $h$. As physically comprehensive, the increase of the Coriolis force tends to decelerate the velocities, see Fig.1. For a fixed value of $\xi$, the variations of $h$ are precisely strong near the plate, but more is $\xi$ less is this effect for the axial velocity

International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 6 (2018) pp. 3361-3368


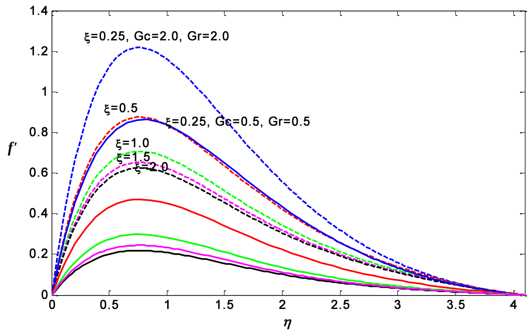


**Figure 7:** Effect of the Coriolis force ξ on the x- velocity profile for Pr=0.71, Sc=0.22, M=1.0, *m*=1., λ =0.25

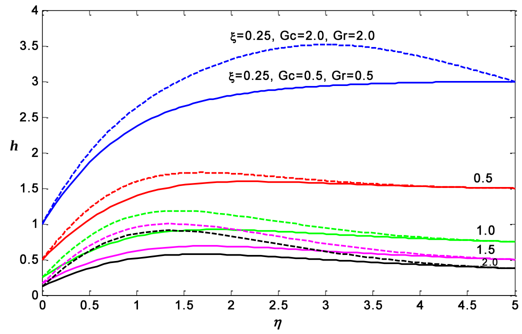


**Figure 8:** Effect of the Coriolis force ξ on the w- velocity profile for Pr=0.71, Sc=0.22, M=1.0, *m*=1, λ =0.25

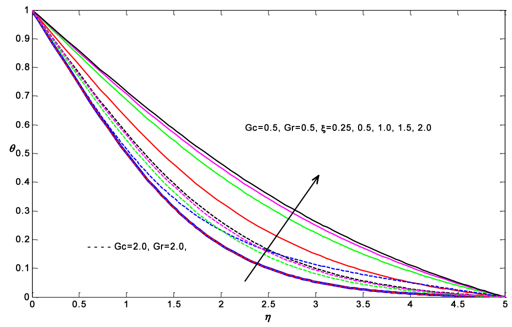


**Figure 9:** Effect of the Coriolis force ξ on the temperature profile for Pr=0.71, Sc=0.22, M=1.0, *m*=1, λ =0.25

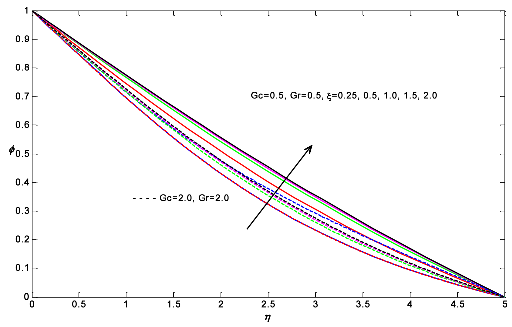


**Figure 10:** Effect of the Coriolis force ξ on the concentration profile for Pr=0.71, Sc=0.22, M=1.0, *m*=1, λ =0.25

Figs. 9 and 10 exhibit the fact that the effect of increasing the Coriolis force parameter $\xi$ is to increase $\theta$ and $\phi$. As reported in the previous analysis, the great moving of the fluid acts to exchange more heat and mass transfer between it and the plate. Consequently, the rotating fluid is viewing as an agent to enhance heat and mass transfer exchange.

Some results of the values of $\theta(0)$ and $\phi(0)$ for various values of $M$, $m$, $\lambda$ and $\xi$ are given in Table 1.

They are typical and partially representative values of the Nusselt and Sherwood numbers respectively. As reported in this table, the results showed that $\theta$ *(0)* and $\phi$ *(0)* reduce with increasing the parameter $\lambda$ while the opposite effect is found with $\xi$. It is interesting to note that an enhancement of the exchanges rates of heat and mass transfer can be obtained by a great rotary of the fluid and a slow velocity of the plate simultaneously.

**Table 1** Numerical values of *Nu* and *Sh* for various values of $\lambda$ and $\xi$ with $M=1$, $m=1$, $Gr$=0.5, $Gc$ = 0.5, $Pr$=0.71 and $Sc$=0.22.

| $\lambda$ | $\xi$ | *Nu* | *Sh* |
|---|---|---|---|
| 0 | 1 | 0.3902 | 0.2929 |
| 0.25 | 1 | 0.3485 | 0.2801 |
| 0.5 | 1 | 0.3040 | 0.2666 |
| 0.75 | 1 | 0.2573 | 0.2523 |
| 1 | 1 | 0.2096 | 0.2372 |
| 1.25 | 1 | 0.1628 | 0.2212 |
| 0.25 | 0.25 | 0.2387 | 0.2470 |
| 0.25 | 0.5 | 0.3090 | 0.2678 |
| 0.25 | 1 | 0.3485 | 0.2801 |
| 0.25 | 1.5 | 0.3622 | 0.2845 |
| 0.25 | 2 | 0.3691 | 0.2868 |

## CONCLUSIONS

The governing equations for steady MHD rotating flow, convective heat and mass transfer past a semi infinite vertical moving plate was formulated without neglecting the Hall effects. The plate velocity was maintained at a constant value and the moving-rotating flow was subjected to a uniform magnetic field. A similarity transformation was employed to change the governing partial differential equations into ordinary ones which are solved numerically. Numerical results were presented graphically to illustrate the details of the flow and heat and mass transfer characteristics and their dependence on some of the physical parameters. The following are deduced

- The increasing of the wall velocity caused reductions in the velocities, leading to the increase of the temperature and concentration profiles.
- When the Coriolis force parameter increases, the velocities, the temperature and the concentration are affected substantially in the same above trend.

- Interesting effects are observed by coupling the studied specific parameters on the heat and mass transfer coefficients which are highlighted here.

## ACKNOWLEDGMENTS

This work was supported in entire part by the Biomaterials and transport phenomena laboratory agreement N° 303 03-12-2003, at university of Medea. Z. Benharkat and M.N. Bouaziz acknowledge the financial support provided by DG-RSDT of Algeria.

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