International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 6 (2018) pp. 3670-3672

On Some Properties of β-open sets

Belal Nairat

Applied Sciences Private University, Amman-Jordan.

Abstract

In this paper we introduce and study the concepts of β-open set, β-continuous functions, then we also study the concepts of β -compact subsets and study some new characterizations of β -separation axioms such as β -T_2 . Then we discuss the relations between the β -continuous functions and these concepts .

Keywords: β-open set, β-compact, β-open cover, β-closed sets, β-continuous

INTRODUCTION

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Levine [7] introduced the notion of semi-open sets and semi-continuity in topological spaces. Andrijevic [2] introduced a class of generalized open sets in topological spaces. Mashhour [9] introduced pre open sets in topological spaces. Monsef et al. [1] initiated the study of β-open sets and β-continuity in a topological spaceThe class of β-open sets is contained in the class of semi-open and pre-open sets.In this paper we discuss the covering properties of β-sets and β-continuous functions. All through this paper(X ,τ) and (Y,σ)stand for topological spaces with no separation assumed, unless otherwise stated. the closure of A and the interior of A will be denoted by Cl(A) and Int(A) , respectively.

PRELIMINARIES

Definition 3.1 A subset A of a space X is said to be [2],[10]:

1. Semi-open if A ⊆ Cl(Int(A))

2. Pre open if A ⊆ Int(Cl(A))

3. α -open if A ⊆ Int(Cl(Int(A)))

4. b-open if A ⊆ Cl(Int(A)) ∪ Int(Cl(A))

5. β-open if A ⊆ Cl(Int(Cl(A)))

Definition 3.2. A function f : X → Y is called . [1] , [9]:

1. semi continuous if f^{-1}(V) is semi open in X for each open set V of Y .

2. pre continuous iff^{-1}(V) is pre open in X for each open set V of Y .

3. α -continuous if f^{-1}(V) isα - open in X for each open set V of Y .

4. b-continuous if f^{-1}(V) is b-open in X for each open set V of Y .

5. β-continuous if f^{-1}(V) is β -open in X for each open set V of Y .

Definition 3.3 [10] A space X is a β-T_2 space iff for each x, y ∈ X such that x ≠ y there are

β-open sets U, V ⊂ X so that x ∈ U, y ∈ V and U ∩ V = ∅ **.**

COVERING PROPERTIES

Definition 4.1

Let $\{G_\alpha : \alpha \in \Delta\}$ be a family of β-open sets of the space X . the family $\{G_\alpha : \alpha \in \Delta\}$ covers X if $X \subseteq \bigcup_{\alpha\in\Delta} G_\alpha$.

Definition 4.2

A space X is called a β-compact space if each β-open cover of X has a finite subcover for X.

Theorem 4.3

Let A be a b-compact subset of the β-T_2 space X and ∉ A . then there exist two disjoint

β-open sets U and V containing x and A , respectively.

Proof :

Let $y \in A$, since X is β -T_2 space there exist two β -open sets $U_x, V_y \in X$ such that

$x \in U_x, y \in V_y, U_x \bigcap V_y = \phi$, the family $\bigcup \{A \bigcap V_y : y \in A\}$ is β -open cover of A

has a finite β-subcover $\{A\bigcap V_{y_1}, A\bigcap V_{y_2}, \ldots, A\bigcap V_{y_n}\}$, thus $U = U_{y_1} \bigcup U_{y_2} \bigcup \ldots \bigcup U_{y_n}$.

Theorem 4.4

If X is β-T_2 space and A is a β -open subset , if A is b-compact then A is a β-closed.

Proof:

Let $x \in X - A$,by the theorem 4.3 there exist two β -open sets U and V such that $x \in U, A \subseteq V, U \bigcap V = \phi$, thus $x \in U \subseteq X - V \subseteq X - A$,which implies $X - A$ is β-open so that A is β-closed .

International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 6 (2018) pp. 3670-3672
© Research India Publications. http://www.ripublication.com

Theorem 4.5

Let A and B be a two β - compact subsets of the β -T$_2$ space X , then there exist disjoint

β -open sets U and V containing A and , receptively .

Proof:

Let b ∈ B , since A is a β -compact subset and β -open in X , there exist two β -open sets

U$_b$, V$_b$ such that $U_b \cap V_b = \phi ; b \in V_b , A \subseteq U_b$, **so** $\beta = \{B \cap V_b ; b \in B\}$ is a β -open cover of

B , since B is β -compact subset there exist finite subcover $\{B \cap V_{in} ; 1 \le i \le n\}$ from β .

Let $U = \bigcap_{i=1}^{n} U_{b_i} , V = \bigcup_{i}^{n} V_{b_i}$, thus $A \subseteq U, B \subseteq V, U \cap V = \phi$.

Theorem 4.5

Let $f : (X,\tau) \to (Y,\rho)$ be a continuous surjection open function , if X is a β -compact then Y is a b-compact .

Proof:

Let $\beta = \{V_\alpha : \alpha \in \Delta\}$ be a β -open cover of Y , then $L = \{f^{-1}(V_\alpha) : \alpha \in \Delta\}$ is a β -open cover of X .since X is a β -compact space , there exist a finite β- subcover from L to the space X . such that

$X \subseteq \bigcup_{i=1}^{n} f^{-1}(V_{\alpha i})$, thus

$$Y = f(X) \subseteq f\left(\bigcup_{i=1}^{n} f^{-1}(V_{\alpha i})\right) = f\left(f^{-1}\left(\bigcup_{i=1}^{n} (V_{\alpha i})\right)\right) = \bigcup_{i=1}^{n} (V_{\alpha i})$$

Hence $Y \subseteq \bigcup_{i=1}^{n} (V_{\alpha i})$, this shows Y is a β-compact .

Corollary 4.6

β -compactness is a topological property

Proof:

The proof from theorem Theorem 4.5 .

Definition 4.7:

A family of sets $\mathcal{J}$ has "finite intersection property" if every finite subfamily of $\mathcal{J}$ has a nonempty intersection.

Theorem 4.5

A topological space is β- compact if and only if any collection of its β- closed sets having the finite intersection property has non-empty intersection.

Proof:

Suppose X is β -compact, i.e., any collection of β -open subsets that cover X has a finite collection that also cover X. Further, suppose $\{G_\alpha : \alpha \in \Delta\}$ is an arbitrary collection of β -closed subsets with the finite intersection property. We claim that $\bigcap_{\alpha \in \Delta} G_\alpha \neq \phi$ is non-empty. Suppose otherwise, i.e., suppose $\bigcap_{\alpha \in \Delta} G_\alpha = \phi$.

Then $\bigcup_{\alpha \in \Delta} (X - G_\alpha) = X - \left(\bigcap_{\alpha \in \Delta} G_\alpha\right) = X - \phi = X$. Since each G$_\alpha$ is β -closed, the collection $\{X - G_\alpha : \alpha \in \Delta\}$ is an β -open cover for X. By β- compactness, there is a finite β- subcover L such that

$X = \bigcup_{i=1}^{n} (X - G_{\alpha i})$. But then

$$\bigcap_{i=1}^{n} G_{\alpha_i} = \bigcap_{i=1}^{n} (X - (X - G_{\alpha_i})) = X - \left(\bigcup_{i=1}^{n} (X - G_{\alpha_i})\right) = X - X = \phi,$$

which contradicts the finite intersection property of $\{G_\alpha : \alpha \in \Delta\}$.

Conversely , take the hypothesis that every family of a β -closed sets in X having the finite intersection property has a nonempty intersection .we are to show X is β- compact. let $\{G_\alpha : \alpha \in \Delta\}$ be any β-open cover of X. then $\{X - G_\alpha : \alpha \in \Delta\}$ is a family of β -closed sets such that

$$\cap X - G_\alpha = X - \left(\bigcup_{\alpha \in \Delta} G_\alpha\right) = X - X = \phi .$$

Consequently , our hypothesis implies the family $\{X - G_\alpha : \alpha \in \Delta\}$ does not have the finite intersection property . Therefore , there is some finite β-subcollection $\{X - G_{\alpha_i} : i = 1,2,3,\dots,n\}$ such that $\bigcap_{i=1}^{n} X - G_{\alpha_i} = \phi$ and hence

$$X = \bigcup_{i=1}^{n} G_{\alpha_i} = \bigcup_{i=1}^{n} (X - (X - G_{\alpha_i})) = X - \left(\bigcap_{i=1}^{n} (X - G_{\alpha_i})\right) = X - \phi = X$$

Thus $X = \bigcup_{i=1}^{n} G_{\alpha_i}$, implying X is β -compact .

ACKNOWLEDGEMENTS

The author acknowledges Applied Science Private University, Amman, Jordan, for the fully financial support granted of this research article.

REFERENCES

[1] M. E. Abd El-Monsef, S. N. El-Deeb, R. A. Mahmoud, *β-open sets and β-continuous mappings*, Bull. Fac. Sci. Assint Univ. **12**(1983), 77–90

[2] Andrijevic, D., On b-Open Sets, Mat Vesn., Vol.48, 1996, PP.59-64.

[3] Calads, Georgiou D.N and Jafari S., Characterizarions of Low Separation Axioms Via α -open Sets and α -Closure Operation, Bol. Soc. Paran. Mat., Vol.21, 2003, PP.1-14.

[4] Crossley, S. G., and Hildebrand S. K., Semi – Closure, Texas J.Sci., Vol.22, 1971, PP.99-112

[5] Dugundji, J., Topology. Boston, Masachusetts, Allyn and Bacon, Inc., 1966.

[6] Jafari, S., On a Weak Separation Axioms, Far East J. Math. Sci., to appear.

[7] 7.Levine, N., Semi-Open Sets and Semi-Continuity in Topological Spaces, Amer. Math Monthly, Vol.70, 1963, PP.36-41.

[8] Maheshwari, S. N., and Prasad, R., Some New Separation Axioms, Ann. Soc. Sci Bruxelles, Vol.89, 1975, PP.395-402.

[9] Mashhuor, A. S., and Abd El-Monsef M.F., and El-Deeb, S. N., On Precontinuous and Weak Precontinuous Mapping. Proc. Math. Phys. Soc. Egypt, Vol.53, 1982, PP.47-53.

[10] Mustafa, J.M, Some Separation Axioms by b-open Sets, Mutah Lil-Buhuth Wad-Dirasat, Vol.20, No.3, 2005, PP.57-64.

[11] Njastad, O., On Some Classes of Nearly Open sets, pacific J. Math., Vol.15, 1965, PP.961-970.

[12] Noiri, T., El-Deeb, S. N., Hasanein, I. A., and Mashhour A. S., On p-regular Spaces, Bukk. Math. Soc. Math. R. S. Roumantie, Vol.27, No.25, 1983, PP. 311-315.

[13] Willard, S., General Topology, Addison-Wesley, 1970.