

Negative Sequence Impedance of a Synchronous Machine under Complicated Fault Conditions

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Abstract

The paper presents a method of obtaining a formula for the calculation of negative-sequence impedance of synchronous machine when complicated faults occurred in external network.

The mathematical manipulations in this paper is based upon the mathematical model of synchronous machines under complicated fault conditions.

Numerical values are obtained for representative machines by applying this formula and by using the traditional method of calculating negative-sequence reactance.

Keywords: negative-sequence impedance; negative-sequence reactance; complicated faults.

Introduction

When the method of symmetrical components is used to calculate the short circuit current, the synchronous machine is represented in negative- sequence network by its negative – sequence impedance, which is the ratio of the negative-sequence fundamental frequency voltage to the currents [1], and is equal to:

$$Z_2 = r_2 + jX_2$$

Where:

r_2 indicates the power associated with the negative sequence current.

X_2 is the negative sequence reactance, and if we ignore the third harmonic [2], it may be calculated from sub-transient reactances as follows:

$$X_2 = \frac{X_d'' + X_q''}{2} \quad (1)$$

Or

$$X_2 = 1.22X_d'' \quad (1-1)$$

The model of a synchronous machine presented in [3] will be used to obtain an equation for the calculation of negative sequence impedance of the machine.

Let us obtain equations of rotor and amortissur windings when negative sequence voltage V_2 is applied to stator circuit.

The differential equation of amortteuser winding in q-axis is:

$$\frac{d\psi_{rq}}{dt} = \omega_s \cdot \rho_{rq} \cdot E_{rd} \quad (2)$$

This equation may be expressed in the following

$$\text{form: } \frac{d(Re \sum_{k=0}^{k=\infty} \psi_{rqk} \cdot e^{jk\gamma})}{dt} = Re \sum_{k=0}^{k=\infty} e^{jk\gamma} \cdot \frac{d\psi_{rqk}}{dt} + Re \sum_{k=0}^{k=\infty} jk\omega_s \cdot \psi_{rqk} \cdot e^{jk\gamma} = Re. [\omega_s \cdot \rho_{rq} \sum_{k=0}^{k=\infty} \dot{E}_{rdk} \cdot e^{jk\gamma}] \quad (3)$$

Now, assume $k=2$ and equate members with the same exponent $e^{j2\gamma}$ in the two sides of above equation, we obtain:

$$\frac{d\psi_{rq2}}{dt} = \omega_s \rho_{rq} E_{rd2} - 2j\omega_s \cdot \psi_{rq2} \quad (4)$$

Neglect emf transformation $\frac{d\psi}{dt}$, yields:

$$\dot{\psi}_{rq2} = j \frac{r_{rq}}{2X_{rq}} \cdot E_{rd2} \quad (5)$$

Substitute Eq. (5) into Eq. (19) Part I and convert to qd coordinate system, yields:

$$-j \frac{r_{rq}}{2X_{rq}} \cdot \dot{E}_{rd} = \frac{X_{aq}^2}{X_{rq}} \cdot \dot{I}_2 - \dot{E}_{rd2}$$

Finally:

$$-j \dot{E}_{rd2} = \frac{X_{aq}^2}{(\frac{r_{rq}}{2} + jX_{rq})} \cdot \dot{I}_2 \quad (6)$$

Similarly, for d axis, neglecting high order harmonics, and convert to qd coordinate system, we obtain the following two equations:

$$\dot{\psi}_{r2} = j \frac{X_{ad}^2}{X_r} \cdot \dot{I}_2 + \dot{E}_{q2} + \frac{X_{ad}}{X_r} \cdot \dot{E}_{rq2} \quad (7)$$

$$\dot{\psi}_{rd2} = j \frac{X_{ad}^2}{X_{rd}} \cdot \dot{I}_2 + \frac{X_{ad}}{X_{rd}} \cdot \dot{E}_{q2} + \dot{E}_{rd2} \quad (8)$$

Now, differential equations in d-axis, considering that negative sequence emf e_{r2} is equal to zero, will be:

$$\frac{d\psi_{r2}}{dt} = -\omega_s \cdot \rho_r \cdot E_{q2} \quad (9)$$

$$\frac{d\psi_{rd2}}{dt} = -\omega_s \cdot \rho_{rd} \cdot E_{rq2} \quad (10)$$

Expressing differentiation in seires according to Eq. (20) Part I, and equate members with the same exponent $j2\gamma$ in both sides of equation, and considering Eqs. 9 and 10, we obtain:

$$\frac{d\dot{\psi}_{r2}}{dt} = -\omega_s \cdot \rho_r \cdot E_{q2} - j2\omega_s \cdot \dot{\psi}_{r2}$$

$$\frac{d\dot{\psi}_{rd2}}{dt} = -\omega_s \cdot \rho_{rd} \cdot E_{rq2} - j2\omega_s \dot{\psi}_{rd2}$$

Neglecting transformation emf, finally obtain:

$$j2\omega_s \cdot \dot{\psi}_{r2} = -\omega_s \cdot \rho_r \cdot \dot{E}_{q2}$$

$$j2\omega_s \cdot \dot{\psi}_{rd2} = -\omega_s \cdot \rho_{rd} \cdot \dot{E}_{rq2}$$

Substitute Eqs. 7 and 8 in the abovementioned equations yields:

$$j2\omega_s \left(j \frac{X_{ad}^2}{X_r} \cdot \dot{I}_2 + \dot{E}_{q2} + \frac{X_{ad}}{X_r} \cdot \dot{E}_{rq2} \right) = -\omega_s \cdot \frac{r_r}{X_r} \cdot \dot{E}_{q2}$$

$$j2\omega_s \left(j \frac{X_{ad}^2}{X_{rd}} \cdot \dot{I}_2 + \frac{X_{ad}}{X_{rd}} \dot{E}_{q2} + \dot{E}_{rq2} \right) = -\omega_s \cdot \frac{r_{rd}}{X_{rd}} \cdot \dot{E}_{rq2}$$

After some manipulations, we obtain:

$$\dot{E}_{q2} \left(\frac{r_r}{2} + jX_r \right) + jX_{ad} \cdot \dot{E}_{rq2} = X_{ad}^2 \cdot \dot{I}_2 \quad (11)$$

$$jX_{ad} \cdot \dot{E}_{q2} + \left(\frac{r_{rd}}{2} + jX_{rd} \right) \cdot \frac{r_r}{2} + jX_r = X_{ad}^2 \cdot \dot{I}_2 \quad (12)$$

From Eq. (11) we find;

$$\dot{E}_{q2} = \frac{X_{ad}^2}{\frac{r_r}{2} + jX_r} \cdot \dot{I}_2 - \frac{jX_{ad}}{\frac{r_r}{2} + jX_r} \cdot \dot{E}_{rq2}$$

Substitute \dot{E}_{q2} in Eq. (12), yields:

$$\frac{jX_{ad}^3}{\frac{r_r}{2} + jX_r} \cdot \dot{I}_2 + \frac{X_{ad}^2}{\frac{r_r}{2} + jX_r} \cdot \dot{E}_{rq2} + \left(\frac{r_{rd}}{2} + jX_{rd} \right) \cdot \dot{E}_{rq2} = X_{ad}^2 \cdot \dot{I}_2$$

Therefore, we can obtain the value of:

$$\dot{E}_{rq2} = \frac{X_{ad}^2 \left(\frac{r_r}{2} + jX_r \right) - jX_{ad}^3}{\left(\frac{r_{rd}}{2} + jX_{rd} \right) \cdot \left(\frac{r_r}{2} + jX_r \right) + X_{ad}^2} \cdot \dot{I}_2 \quad (13)$$

From Eq. 912) we find:

$$\dot{E}_{rq2} = \frac{X_{ad}^2}{\frac{r_r}{2} + jX_r} \cdot \dot{I}_2 - \frac{jX_{ad}}{\frac{r_{rd}}{2} + jX_{rd}} \cdot \dot{E}_{q2}$$

Substitute the value of \dot{E}_{rq2} from the last equation into Eq. (11), we obtain:

$$\dot{E}_{q2} = \frac{X_{ad}^2 \cdot \left(\frac{r_{rd}}{2} + jX_{rd} \right) - jX_{ad}^3}{\left(\frac{r_r}{2} + jX_r \right) \cdot \left(\frac{r_{rd}}{2} + jX_{rd} \right) + X_{ad}^2} \cdot \dot{I}_2 \quad (14)$$

Now substitute Eqs. (6), (130, and ((14) into eq. (30), we obtain;

$$X \dot{I}_2 - j \frac{X_{ad} \left(\frac{r_{rd}}{2} + jX_{rd} \right) + X_{ad} \cdot \left(\frac{r_r}{2} + jX_r \right) - 2jX_{ad}^3}{2 \left[\left(\frac{r_r}{2} + jX_r \right) \cdot \left(\frac{r_{rd}}{2} + jX_{rd} \right) + X_{ad}^2 \right]} \cdot \dot{I}_2$$

$$- j \frac{X_{ad}^2}{\left(\frac{r_{rd}}{2} + jX_{rd} \right)} \cdot \dot{I}_2 = j(V_2 + r_a \cdot \dot{I}_2)$$

Taking into consideration the following relationships:

$$X_{rd} = X_{srd} + X_{ad}$$

$$X_r = X_{sr} + x_{ad}$$

And after some manipulations we can obtain an equation for the calculation of negative sequence complex impedance of a synchronous machine with amortisseur windings:

$$Z_2 = r_a + jX_s + \frac{jX_{ad} \cdot \left(\frac{r_r}{2} + jX_{sr} \right) \cdot \left(\frac{r_{rd}}{2} + jX_{srd} \right)}{2 \left[\left(\frac{r_r}{2} + jX_{sr} \right) \cdot \left(\frac{r_{rd}}{2} + jX_{srd} \right) + jX_{ad} \cdot \left[\left(\frac{r_r}{2} + jX_{sr} \right) + \left(\frac{r_{rd}}{2} + jX_{srd} \right) \right] \right]} + \frac{jX_{aq} \cdot \left(\frac{r_{rq}}{2} + jX_{srq} \right)}{2 \left(\frac{r_{rq}}{2} + jX_{srq} \right)} \quad (15)$$

Fig. 1 shows equivalent diagram of a synchronous machine with amortisseur windings in qd coordinate system.

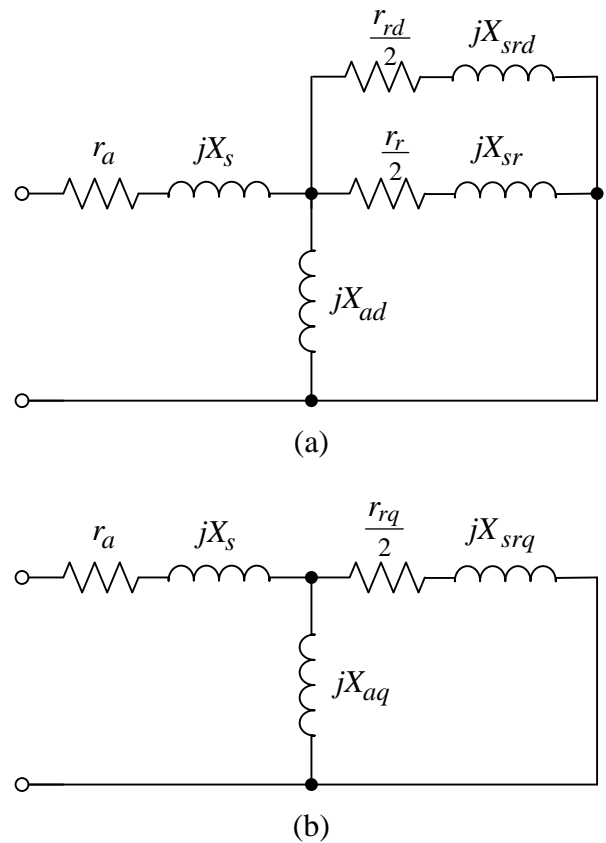


Figure 1. Equivalent circuits of a synchronous machine with amortisseur winding in dq coordinate system to determine negative sequence-impedance.

In the abovementioned formula we ignore the skin effect in the conductors of rotor circuits. This effect will change the values of resistances due to the flowing of negative sequence current.

Table 1 shows results of negative sequence impedance \dot{Z}_2 calculated by formula (15) and negative sequence reactance X_2 for some types of machines.

Table 1. Negative sequence complex impedance

Machine type	$\dot{Z}_2(\text{p.u})$		$X_2 = \frac{x_d''+x_q''}{2}$ (p.u)	$X_2 = 1.22X_d''$ (p.u)
	r_2	X_2		
Turbogenerator 200 MW	0.260	0.212	0.191	0.232
Turbogenerator 300 MW	0.252	0.164	0.176	0.211
Turbogenerator 600 MW	0.282	0.258	0.248	0.272
Hydrogenerator 64 MW	0.0074	0.276	0.331	-

CONCLUSION

The paper demonstrates a new approach for obtaining a formula to calculate the negative sequence impedance of synchronous machine under complicated fault conditions. The results are more accurate than the previous values used till now in the calculations.

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