International Journal of Applied Engineering Research, ISSN 0973-4562 Volume 13, Number 8 (2018) pp. 6463-6471

Centralized Sensing and Scheduling in Multi-User Cognitive Radio Networks

V. Hamsadhwani, Ph.D. Research Scholar and Dr. K. Lakshmi
Periyar Maniammai Institute of Science and Technology, Thanjavur, India.

Dr. Arun Pachai Kannu
Indian Institute of Technology Madras, India.

Abstract: We study the behavior of Queues in multi-user cognitive radios for the case when the centralized coordinator is controlling the channel sensing and scheduling process. We consider three different scheduling policies: maximum-rate scheduling, maximum-weight scheduling and proportionally fair scheduling. For each policy, we present the queue stability analysis and characterize the corresponding stability regions. We also study the cases where users have identical SNR statistics and unequal SNR statistics. For the case of equal SNR among users, we obtain the optimal centralized sensing order and study the stability performance of queues. For the case of unequal SNR among users, we consider intuitive sensing order and study the queue behavior. Our results show insights on the performance of the joint channel sensing and scheduling policies.

Keywords: cognitive radios, queues, average delay, channel sensing order, stability.

INTRODUCTION

With the increase in wireless data, usable RF spectrum is becoming heavily congested and the cognitive radios provide a means for enhancing the efficiency of spectrum usage. A cognitive radio is a smart radio that can intelligently detect available wireless channels in its vicinity and dynamically configure its transmission or reception parameters so as to make the best use of these spectrum holes. Cognitive radios are capable of communicating on licensed spectrum without causing interference to the primary/licensed users of the bands, and therefore hold great potential for improving the efficiency of the usage of licensed spectrum that is otherwise poorly utilized due to static frequency allocations.

However, the spectrum that a cognitive radio would be allowed to operate on can be expected to be scattered and heterogeneous in general. In other words, a cognitive radio would need to search over multiple portions of the licensed spectrum, possibly having different bandwidths and primary user characteristics, in order to select the best free channel for its use. Also, these cognitive radios are usually small devices with hardware limitations, they can not simultaneously sense more than one portion of the spectrum quickly, efficiently and reliably. Hence, each Cognitive User (CU) in a cognitive radio network (CRN) would need to have a sensing-order i.e., an order in which it will sequentially sense the different channels until it finds a suitable channel for its communication.

In general, each user's data is buffered in the corresponding Queue before suitable channel is selected for transmission. In this work, we study the Queue behavior of multi-user cognitive radio networks with several channel sensing procedures and scheduling mechanisms.

NETWORK MODEL

Centralized Sensing

We consider a CRN with M CUs, N licensed channels and a coordinator. The frame structure is divided into slots of duration T [1], [2], [3]. In any given slot, channel i is free from primary users with probability θ_i and it is used by primary users with probability $(1-\theta_i)$. We refer θ_i as the *primary-free probability* of channel i. We label the channels $1, 2, \ldots, N$ in descending order of their primary-free probabilities such that $\theta_1 > \theta_2 > \cdots > \theta_N$.

We assume that the signal-to-noise ratio (SNR) represented as γ, seen by a CU to its receiver on a given channel remains constant over a time slot and changes independently at the beginning of the next time slot. We also assume that the SNR of m^{th} cognitive user on any channel is independent and identically distributed across time slots and independent of other users with the probability density function denoted by $f_{\Gamma^m}(\gamma)$.

The coordinator specifies a sensing-order, also referred to as *central sensing-order*, by $\boldsymbol{s} \triangleq (s_1, s_2, \ldots, s_N)$, which is a permutation of $(1, 2, \ldots, N)$ and all the CUs sense the channels in that order. Note that s_i denotes the i^{th} channel in the sensing-order $\boldsymbol{s}$ of the coordinator. At the beginning of each time slot, the coordinator requests the CUs to sense the channels as per its sensing-order and collects the local sensing results from the waiting CUs, i.e., CUs who have not yet been allotted a channel for transmission. Using data fusion techniques, coordinator makes a decision on whether the sensed channel is free or not. This mechanism of cooperative centralized sensing is illustrated in Fig. 1.

Spectrum sensing is performed locally by the individual cognitive users. The coordinator fuses the local sensing reports to make a centralized decision on whether the channel being sensed is free to be used by the cognitive users [4]. When the sensed channel is free, the coordinator allots to one of the waiting CUs, depending on a suitable criterion, which will be described subsequently.

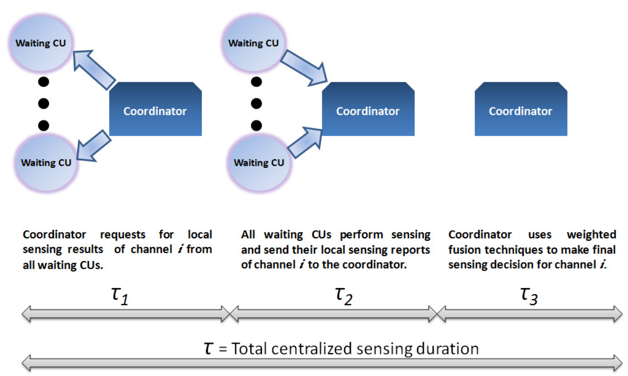

FIGURE 1. Model for Centralized Sensing

Rate Adaptation

The CUs use opportunistic transmissions, i.e. they adapt their transmission rates according to the channel quality using rate-adaptation techniques. In this paper, we denote the transmission rate used by m^{th} CU on channel i by $r(\gamma_i^m)$, where γ_i^m is the instantaneous SNR seen on channel i by m^{th} CU to its receiver and $r(.)$ is a non-decreasing function of γ_i^m. When we say that the CUs do not use rate-adaptation, we mean that they use a fixed rate R (i.e., $r(\gamma_i^m) = R\, \forall i, m$) whenever they transmit on a channel, irrespective of the channel quality.

Let τ denote the total sensing duration, i.e. the time required by the coordinator to sense any channel with the desired accuracy. If a CU is granted access to a channel after k sensings of the coordinator, the total sensing duration is $k\tau$ and hence, the time available for data transmission is $T - k\tau$. The *effectiveness* of transmission in a slot, as defined in [1] and [2], is the ratio of the data transmission duration to the total slot duration. If a CU starts transmitting data after k sensings of the coordinator, the effectiveness c_k is defined as

$$c_k \triangleq \frac{T - k\tau}{T} = 1 - k\frac{\tau}{T} \tag{1}$$

A summary of the slot structure is illustrated in Fig. 2.

Queuing model

Let Q_m denote the (buffer) Queue containing the data corresponding to CU m. Let the data arrive at Q_m with rate λ_m and we do not assume any particular probability distribution for data arrival. For convenience, we collect the arrival rates of all users in a vector as $\boldsymbol{\lambda} = [\lambda_1, \ldots, \lambda_M]$. Let $q_m(t)$ denote the number of packets in Q_m at the beginning of t^{th} time slot. If $v_m(t)$ and $b_m(t)$ denote the data arrived and

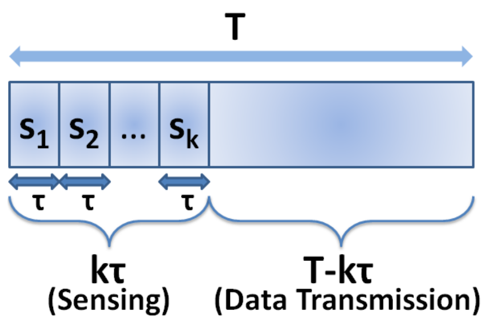

FIGURE 2. Slot Structure

transmitted (successfully) *during* the t^{th} slot, we have

$$q_m(t+1) = q_m(t) + v_m(t) - b_m(t) \tag{2}$$

We are interested in studying the behavior of queues $q_j(t)$ as a function of time t. Specifically, we are interested in the asymptotic queue behavior as $t \to \infty$. Before we proceed for that, we discuss briefly about the optimal channel sensing order chosen by the central coordinator.

OPTIMAL SENSING ORDER

Sensing order used by the coordinator can be chosen in many ways. One particular option is to maximize the total throughput of the cognitive users, and we refer to the corresponding sensing order as the optimal sensing order. In this section, we briefly discuss about finding the optimal sensing order [4].

Reward Function

First, we present a general procedure for computing the total cognitive throughput as a function of the sensing-order $\boldsymbol{s} = (s_1, s_2, \ldots, s_N)$ of the coordinator. Let v_i^j denote the sum-throughput at channel s_i given that the channel is sensed to be primary-free while there are j *waiting-CUs* i.e. CUs who are yet to be allotted a channel by the coordinator for transmission. Let V_{i+1}^j denote the expected sum-throughput if the coordinator proceeds from channel s_i to sense channel s_{i+1} with j waiting-CUs. We can write V_{i+1}^j as follows.

$$V_{i+1}^j = \begin{cases} 0, & j = 0 \\ \theta_{s_{i+1}} \operatorname{E}[v_{i+1}^j], & j \neq 0, i = N-1 \\ \theta_{s_{i+1}} \operatorname{E}[v_{i+1}^j] + (1-\theta_{s_{i+1}}) V_{i+2}^j, & \text{otherwise} \end{cases} \tag{3}$$

In order to write an expression for v_i^j, we make use of the following observations. If $j = 0$, then $v_i^j = 0$. Otherwise, in the N^{th} (final) round of sensing with j (> 0) waiting-CUs, when the coordinator senses channel s_N to be primary-free, in order to achieve the maximum possible reward, it should let the waiting-CU seeing the best channel quality on channel s_N, access the channel. If $w_1, w_2, \ldots, w_j$ denotes the j waiting-CUs, and δ_i^j denotes the maximum SNR seen by any of the j waiting-CUs on channel s_i, i.e.

$$\delta_i^j = \max(\gamma_{s_i}^{w_1}, \gamma_{s_i}^{w_2}, \ldots, \gamma_{s_i}^{w_j}), \tag{4}$$

then the coordinator should let the waiting-CU seeing SNR δ_N^j access channel s_N. The corresponding reward would be $c_i r(\delta_N^j)$.

In any other i^{th} ($1 \le i < N$) round of sensing with j (> 0) waiting-CUs, when the coordinator senses channel s_i to be primary-free, it can choose, based on the choice that results in a higher reward, to either

(i) let the waiting-CU seeing the maximum SNR *access* this channel, and earn the corresponding reward $c_i r(\delta_i^j) + V_{i+1}^{j-1}$, or
(ii) *skip* the channel, i.e. not let any waiting-CU access this channel and proceed to sense the next channel in its sensing-order, and earn the corresponding reward V_{i+1}^j.

Thus, we can write $v_i^j (j > 0)$ as

$$v_i^j = \begin{cases} c_i r(\delta_i^j), & \text{if } i = N \\ & \text{(coordinator grants access to } s_N) \\ c_i r(\delta_i^j) + V_{i+1}^{j-1}, & \text{if } i \neq N \;\&\; c_i r(\delta_i^j) + V_{i+1}^{j-1} > V_{i+1}^j \\ & \text{(coordinator grants access to } s_i) \\ V_{i+1}^j, & \text{otherwise (coordinator skips } s_i) \end{cases} \tag{5}$$

Here, $c_i r(\delta_i^j) + V_{i+1}^{j-1} > V_{i+1}^j$ is the *access rule* for the coordinator, analogous to the traditional stopping rule in the single-user case [1], and can be similarly shown to be optimal as in [2].

Using equations (3) and (5), we can recursively compute the matrix $\{V_i^j\}_{N\times M}$ ($1 \le i \le N, 1 \le j \le M$).We start with V_N^1, then compute $\{V_N^j\}_{j=2}^M$ and then repeat the procedure for $i = N-1, N-2, \ldots, 1$ to eventually compute V_1^M. When the coordinator uses a given sensing-order $\boldsymbol{s}$, it starts off by proceeding to sense channel s_1 with M waiting-CUs, and the reward associated with that step is, by definition, V_1^M. Therefore, V_1^M is the reward function that we had set out to formulate, and can be derived using the above procedure. Using an exhaustive search procedure, we can determine the sensing-order $\boldsymbol{s}$ that maximizes V_1^M.

Computing Optimal Sensing Order

We discuss the optimal sensing order under some special cases below. When the CUs in the M-user CRN do not use rate-adaptation and transmit at a fixed rate R whenever they are granted access to a channel by the coordinator, i.e. $r(\delta_i^m) = R \ \forall i, m$, the optimal sensing order is the descending order of channels based on their primary-free probabilities, $\boldsymbol{s}_{\text{opt}} = [1, 2, \cdots, N]$ if $\theta_1 > \theta_2 > \cdots > \theta_N$. For subsequent use, we call this sensing order as the *intuitive* sensing order $\boldsymbol{s}_{\text{int}} = [1, 2, \ldots, N]$. In general, when the CUs use rate adaptation, intuitive sensing order need not be optimal.

Let us consider Rayleigh-fading channels where Γ^m denotes the mean SNR seen on each channel by the CU m. Hence, the instantaneous SNR γ_i^m seen on any channel i by any CU m follows the same exponential distribution with the pdf given by $f_{\Gamma^m}(\gamma) = \frac{1}{\Gamma^m} e^{-\frac{\gamma}{\Gamma^m}}, \gamma \geq 0$. Let the transmission rate used on channel i by CU m be $r(\gamma_i^m) = \ln(1 + \gamma_i^m)$.

For the special case of a 2-user CRN ($M = 2$), with 3 channels ($N = 3$), we give a closed-form expression [4] for the reward V_1^2 as a function of the sensing-order $\boldsymbol{s} = (s_1, s_2, \ldots, s_N)$ of the coordinator, under the assumption that all the users have the same fading statistics $f_{\Gamma^m}(\gamma) = f_\Gamma(\gamma), \forall\ m$. Reward functions are written in terms of the function $\psi(\,\cdot\,)$ which is defined as $\psi(x) = \int_x^\infty \frac{e^{-t}}{t} dt$.

$$
\begin{aligned}
V_3^1 &= \theta_{s_3} c_3 e^{1/\Gamma} \psi\left(\frac{1}{\Gamma}\right) \\
V_2^1 &= V_3^1 + \theta_{s_2} c_2 e^{1/\Gamma} \psi\left(\frac{e^{V_3^1/c_2}}{\Gamma}\right) \\
V_1^1 &= V_2^1 + \theta_{s_1} c_1 e^{1/\Gamma} \psi\left(\frac{e^{V_2^1/c_1}}{\Gamma}\right) \\
V_3^2 &= 2\theta_{s_3} c_3 e^{1/\Gamma} \psi\left(\frac{1}{\Gamma}\right) - \theta_{s_3} c_3 e^{2/\Gamma} \psi\left(\frac{2}{\Gamma}\right) \\
V_2^2 &= V_3^2 + 2\theta_{s_2} c_2 e^{\frac{1}{\Gamma}} \psi\left(\frac{e^{(V_3^2 - V_3^1)/c_2}}{\Gamma}\right) \\
&\quad - \theta_{s_2} c_2 e^{\frac{2}{\Gamma}} \psi\left(\frac{2e^{(V_3^2 - V_3^1)/c_2}}{\Gamma}\right) \\
V_1^2 &= V_2^2 + 2\theta_{s_1} c_1 e^{\frac{1}{\Gamma}} \psi\left(\frac{e^{(V_2^2 - V_2^1)/c_1}}{\Gamma}\right) \\
&\quad - \theta_{s_1} c_1 e^{\frac{2}{\Gamma}} \psi\left(\frac{2e^{(V_2^2 - V_2^1)/c_1}}{\Gamma}\right)
\end{aligned}
$$

The sensing-order $\boldsymbol{s}$ that maximizes V_1^2 is the optimal sensing-order and the corresponding V_1^2 is the optimal cognitive throughput.

However, when the channel fading statistics are not identical across the users, finding the optimal sensing order is still an open problem.

SCHEDULING ALGORITHMS

In this section, we give details of the scheduling algorithms considered in our work. Multiuser scheduling has been extensively studied in both academia and industry. The scheduling schemes have been motivated to tackle the unique features in wireless networks such as scarcity of resources, mobility of users, interference and time-varying channel conditions due to fading. Hence, good scheduling schemes in wireless networks should opportunistically seek to exploit channel conditions to achieve higher network performance.We consider three different scheduling algorithms, maximum-rate scheduling, maximum-weight scheduling and proportionally fair scheduling.

In order to describe these algorithms, we need to introduce notations regarding the data arrival, departure and queuing state. Also, we first describe the schedulers for the case when there is only a single channel available. The general case of multi-user cognitive radio networks is discussed in the next section. As already mentioned, each user m has a buffer Q_m which has $q_m(t)$ amount of data at a given time slot t. We denote the amount of data $b_m(t)$ successfully transmitted by the CU m at the time slot t. Hence the cumulative amount of data sent by CU m upto time slot t is given by

$$D_m(t) = \sum_{n=1}^{t} d_m(n) \tag{6}$$

where $d_m(n)$ denotes the amount of data.

Also, the empirical throughput of user m upto time slot t is given by

$$T_m(t) = \frac{D_m(t)}{t} \tag{7}$$

The set of users who have non-empty queues at time t is denoted as $\mathcal{A}(t) = \{m : q_m(t) > 0\}$. Also, based on the channel conditions, let $r_m(t)$ denote the rate at which user m can transmit data reliably at the given slot t. Given various information such as $\{q_m(t), r_m(t), T_m(t)\}$, different scheduling algorithms chooses an user $j \in \{1, \cdots, M\}$ who gets to access the channel at time slot t, based on their own scheduling criterion.

Maximum-Rate Scheduling

Maximum-rate scheduler picks the user j with non-empty queue to be allotted the channel resource in the t^{th} slot where

$$j = \arg \max_{m \in \mathcal{A}(t)} r_m(t) \tag{8}$$

This scheduler picks the user who has the highest data rate in that given slot. Hence, this is an opportunistic scheduling scheme. Intuitively, this scheduler tries to maximize the overall throughput of the system. If the channel quality statistics are same across all the users, then, every user would get the same transmission time. Therefore the overall data rate of the system will be the theoretical maximum data rate.

However, this scheduler does not ensure fairness. For instance, an user with poor channel statistics may never get channel assigned in his favor. Also, the quality of service (QoS) for any user might be poor. For instance, an user, though may have a large number of data waiting in his queue, may not get assigned the channel due to his channel condition not being the best. Hence, the overall queuing delay of the packets can be high. Though this scheduler is simple to implement and tries to maximize the utility of the channel, it has drawbacks on the overall QoS performance.

Maximum-Weight Scheduling

Maximum-weight scheduler picks the user j with non-empty queue to be allotted the channel resource in the t^{th} slot where

$$j = \arg \max_{m \in \mathcal{A}(t)} q_m(t) r_m(t) \tag{9}$$

This scheduler picks the user who has the highest product of data rate and queue length, in that given slot. This is also an opportunistic scheduling scheme. As opposed to maximum-rate scheduler, maximum-weight scheduler uses the amount of data available at present in the queue. This scheduler tries to strike a balance between users with high data rates and users with long queues. This scheduler has been shown to have some optimality properties [5] regarding the stochastic stability of queues. For instance, as long as the data arrival rates of the users are within the overall capacity region of the system, maximum-weight scheduler ensures that all the users' queues remain stable, that is, they do not grow unbounded.

Proportionally Fair Scheduling

The proportionally fair scheduler exploits the random variations in channel quality, but at the same time, tries to ensure fairness among users. Its metric for allocation is not based on the actual data rate but its ratio to the average data rate. The proportionally fair scheduler chooses user j to access channel at time slot t where

$$j = \arg \max_{m \in \mathcal{A}(t)} \frac{r_m(t)}{T_m(t-1)} \tag{10}$$

Here, the ratio of the data rate at the present slot with respect to the empirical throughput of the user upto the previous slot is used as the metric to select the user for the present slot. The scheduler has to calculate the ratio for each user and then compare the ratios to determine the user with the highest ratio to schedule him for transmission in that slot. The overall throughput of the proportionally fair scheduler is usually higher than that of round-robin method, since the users are not chosen in a apriori fixed order. Instead, the user whose data rate is farthest above its average rate will be selected. However, the throughput will be lower than the total throughput of the maximum-rate scheduler. Another issue is the computation of the ratio $\frac{r_m(t)}{T_m(t-1)}$ and the empirical throughput needs to be continually updated. Nevertheless, the proportionally-fair scheduler avoids the situation of an user with poor channel statistics never getting a channel assigned in his favor.

JOINT SENSING AND SCHEDULING

In this section, we describe the details of the centralized sensing and scheduling procedure for our multi-user cognitive radio networks. The scheduling algorithms need to take into the fact that there are multiple channels and each channel has to be sensed for availability before allocation to users.

Procedure Description

We describe the centralized sensing and scheduling procedure at the t^{th} time slot. Towards that the queue information of the cognitive users $q_m(t), m \in \{1, \ldots, M\}$ and their empirical throughput until the previous slot $T_m(t-1), m \in \{1, \ldots, M\}$ are made available to the coordinator. Also, the set of users with non-empty queue is obtained as

$\mathcal{A}(t) = \{m : q_m(t) > 0\}$. Let $\boldsymbol{s}$ be an $N \times 1$ vector denoting the sensing order used by the coordinator. Here $\boldsymbol{s}$ is a permutation of $[1, \ldots, N]$. Note that, $\boldsymbol{s}$ may be the optimal sensing order, or the intuitive sensing order or any arbitrary order. We introduce the set $\mathcal{B}(t, k)$, which denotes the set of CUs for which channels have been already assigned after doing k sensings in the t^{th} time slot.The step by step procedure is as follows

1. Initialize the variables as $k = 1$ and $\mathcal{B}(t, 0) = \emptyset$.
2. Get the set of waiting CUs who have non-empty queues and who are not yet assigned channel after $k-1$ sensings as $\mathcal{W}(t, k-1) = \mathcal{A}(t) \setminus \mathcal{B}(t, k-1)$. Check if $\mathcal{W}(t, k-1) == \emptyset$. If *true*, then go to the last step. If *not true*, continue with the following step.
3. Pick the k^{th} channel to be sensed from the sensing order $\boldsymbol{s}$. Denote the channel as $[\boldsymbol{s}]_k$.
4. Ask the waiting CUs in $\mathcal{W}(t, k-1)$ to perform spectrum sensing on the channel $[\boldsymbol{s}]_k$ and obtain the sensing report from those CUs.
5. If $[\boldsymbol{s}]_k$ is *not* primary-free, then set $\mathcal{B}(t, k) = \mathcal{B}(t, k-1)$ and go to step 2. On the other hand, if $[\boldsymbol{s}]_k$ is primary-free, proceed to the following step.
6. From the sensing report, obtain the instantaneous SNR in the channel $[\boldsymbol{s}]_k$ for the waiting CUs and denote them as $\gamma_m(t, k)$ for $m \in \mathcal{W}(t, k-1)$.
7. Obtain the instantaneous data rates of waiting CUs using the effectiveness of the time slot as $r_m(t, k) = c_k \log(1 + \gamma_m(t, k))$
8. Based on the instantaneous data rates, we allot the channel $[\boldsymbol{s}]_k$ to an user $j \in \mathcal{W}(t, k-1)$, according to the scheduling mechanism. For each scheduler, the user selection is done as follows:
   - Maximum-Rate Scheduler
   $$j = \arg \max_{m \in \mathcal{W}(t,k-1)} r_m(t, k) \tag{11}$$
   - Maximum-Weight Scheduler
   $$j = \arg \max_{m \in \mathcal{W}(t,k-1)} q_m(t) r_m(t, k) \tag{12}$$
   - Proportionally fair Scheduler
   $$j = \arg \max_{m \in \mathcal{W}(t,k-1)} \frac{r_m(t, k)}{T_m(t-1)} \tag{13}$$
9. When user j gets assigned to the channel $[\boldsymbol{s}]_k$, the sets and Queue buffer gets updated as follows. We have
   $$\mathcal{B}(t, k) = \mathcal{B}(t, k-1) \cup j \tag{14}$$
   Now, the amount of data transmitted by the user j in the k^{th} time slot is obtained as
   $$b_j(t) = \min\left(q_j(t), r_j(t, k)\right) \tag{15}$$
   Now the queue length of user j gets updated as
   $$q_j(t+1) = q_j(t) - b_j(t) + v_j(t) \tag{16}$$
   where $v_j(t)$ denote the amount of data arrived in the t^{th} time slot for user j.
10. Increment $k = k + 1$.
11. If $k \leq N$ then go to step 2. Else, go to the following step.
12. Perform Queue updates for users who were not assigned any channel in the t^{th} time slot
   $$q_j(t+1) = q_j(t) + v_j(t), \forall j \in \mathcal{A}(t) \setminus \mathcal{B}(t, N). \tag{17}$$
13. Go to the next time slot $t + 1$.

Stability Analysis

We introduce the notion of stability of queues and related performance metrics. We say that the queue is stable if the average number of packets/amount of data present in the queue remains bounded. For instance, user m queue is stable if

$$\limsup_{t \to \infty} \mathrm{E}\left(q_m(t)\right) < \infty \tag{18}$$

where $\mathrm{E}(\cdot)$ denotes expectation. An arrival rate vector $\boldsymbol{\lambda} = [\lambda_1, \ldots, \lambda_M]$ is called a stable arrival vector, if all the users' $m \in \{1, \ldots, M\}$ queues are stable for that arrival vector.

Now, the stability region $\mathcal{R}$ of the queuing system is the collection of all the stable rate vectors, such that

$$\mathcal{R} = \{\boldsymbol{\lambda} : \boldsymbol{\lambda} \text{ is a stable arrival vector}\} \tag{19}$$

Note that, the stability region depends on the sensing order and the scheduler used. We perform the stability analysis for various sensing orders and the scheduling mechanisms. We also introduce the following parameters related to the stability region. Typically, when $\boldsymbol{\lambda}$ is in the stability region, then $\boldsymbol{\lambda} - \boldsymbol{\epsilon}$ is also in the stability region where $\epsilon = [\epsilon_1, \ldots, \epsilon_M]$ with $0 \leq \epsilon_i \leq \lambda_i$. Now, the volume of the largest cuboid within the stability region is obtained as

$$A^{\max} = \max_{\boldsymbol{\lambda} \in \mathcal{R}} \Pi_{m=1}^{M} \lambda_m \tag{20}$$

International Journal of Applied Engineering Research, ISSN 0973-4562 Volume 13, Number 8 (2018) pp. 1–9

The maximum total throughput within the stability region is defined as

$$T^{\max} = \max_{\lambda \in \mathcal{R}} \sum_{m=1}^{M} \lambda_m \tag{21}$$

Now, the maximum stable arrival rates for each user is obtained as

$$\lambda_m^{\max} = \max_{\lambda \in \mathcal{R}} \lambda_m \tag{22}$$

The behavior of $A^{\max}$, $T^{\max}$ and $\lambda_m^{\max}$ are studied for various sensing orders and scheduling mechanisms in the following section.

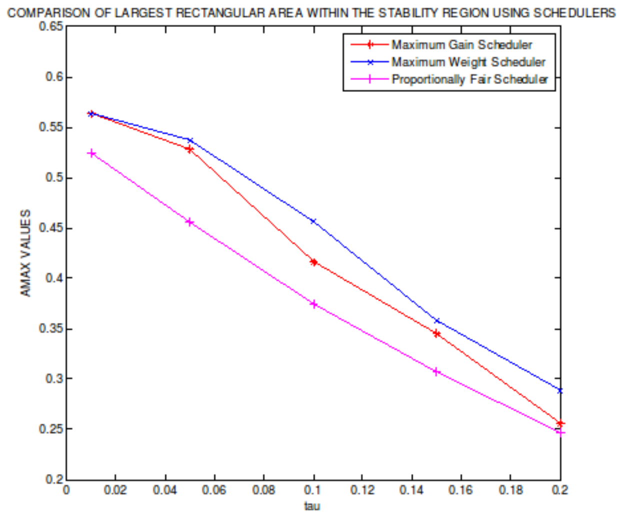

FIGURE 3. $A^{\max}$ versus τ

Simulation Results

We consider the case of 2 users $M = 2$ and 4 channels $N = 4$. In the first study, we consider unequal SNR statistics between the users. Specifically, the SNR of both users are exponentially distributed with means $\Gamma_1 = 1.5$ and $\Gamma_2 = 1$. In such case, finding the optimal sensing order is an open problem. Hence, we perform the stability analysis with intuitive sensing order for various scheduling algorithms.

In Fig. 3, we show the $A^{\max}$ versus τ for the schedulers. We note that the maximum-weight scheduler has the best performance in terms of the largest rectangle inside the stability region. This can be explained from the fact that the maximum-weight scheduler has optimal properties regarding the stochastic stability of queues [5]. The plot clearly shows:

1. The performance of the maximum-rate scheduler is poorer than the maximum-weight but better than the proportionally fair algorithm.
2. Proportionally fair algorithm tries to ensure fairness among users having different SNR statistics. Hence, its overall stability region is typically smaller than that of the maximum-weight scheduling scheme.

In Fig. 4, we show the $T^{\max}$ versus τ for the schedulers. Again, we note that the maximum-weight scheduler has the maximum total throughput within its stability region. This can be explained from the fact that the maximum-weight scheduler typically has larger stability region (infer from Fig. 3) than the other two schedulers. The plot clearly shows:

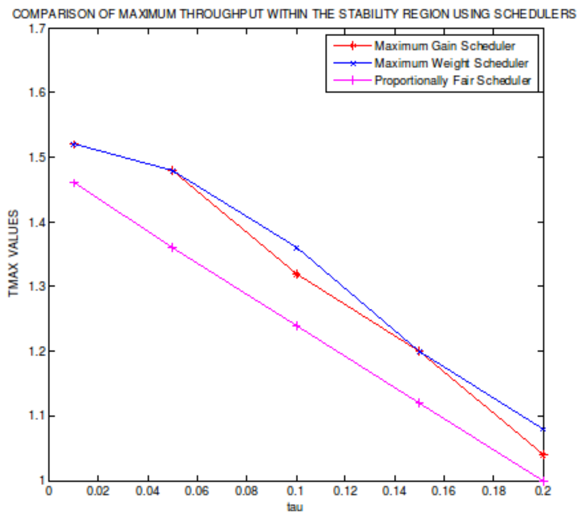

FIGURE 4. $T^{\max}$ versus τ

1. The performance of the maximum-rate scheduler is quite close to that of the maximum-weight algorithm since it tries to maximize the overall throughput.
2. Again, proportionally fair algorithm performs poorer than the other two schedulers.

In Figures 5 and 6, we plot the maximum data rate for the first and second user respectively, within the stability region. We note that:

1. The first user who has the higher mean SNR has higher data rates within the stability region when compared with the second user.

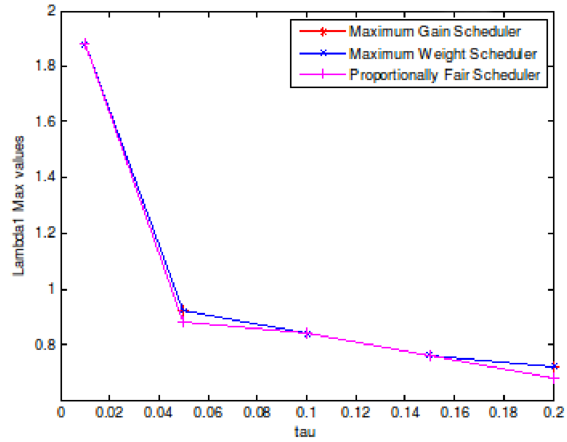

FIGURE 5. λ_1^{max} versus τ

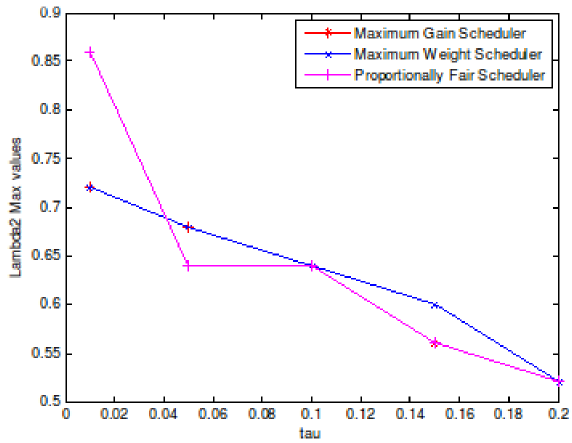

FIGURE 6. λ_2^{max} versus τ

2. Also, all the three algorithms have nearly the same maximum data rate for the first user.
3. On the other hand, the second user who has lesser mean SNR has poorer rates compared to the first user.
4. Interestingly, the proportionally fair scheduler has higher rate for the second user for $\tau = 0.01$ when compared with the other schedulers. This clearly indicates the proportionally fair algorithm tries to ensure fairness by allocating channel resources to the second user more often than the first two schedulers.
5. However, since its overall stability region is small, proportionally fair algorithm may have smaller rates for the second user as well compared to the other algorithms for some values of τ.

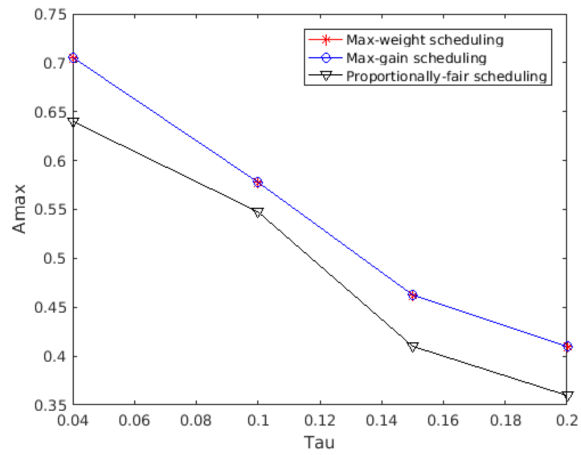

FIGURE 7. A^{max} versus τ

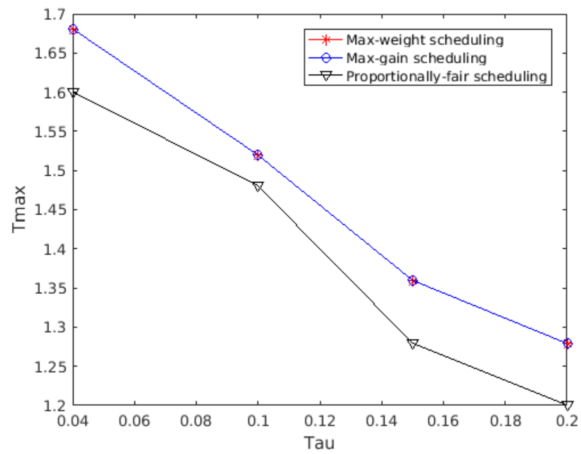

FIGURE 8. T^{max} versus τ

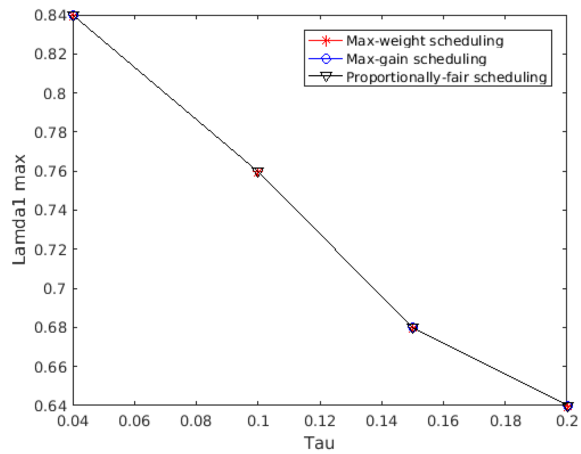

FIGURE 9. λ_1^{max} versus τ

Next, we consider the case where the two users have the same SNR ($\Gamma = 1$). For this case, we find the optimal sensing order, as explained previously. The stability parameters

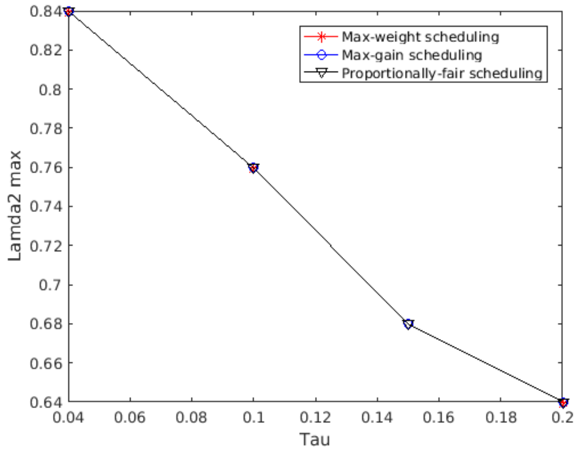

FIGURE 10. $\lambda_2^{\max}$ versus τ

for various schedulers are obtained and shown in the following plots. In Fig. 7, we show the $A^{\max}$ versus τ for the schedulers with optimal sensing order. We note that:

1. The maximum-weight scheduler and maximum-rate scheduler have nearly the same performance. Since the users have identical SNR statistics, both these schedulers (on the average) end up giving equal channel allocation between the users and have similar performance.
2. Proportionally fair algorithm tries to ensure fairness among users by comparing instantaneous SNR statistics and pays the penalty in the overall stability region area compared to the other two schedulers.

In Fig. 8, we show the $T^{\max}$ versus τ for the schedulers. Again, we note that the performance between the schedulers is very similar to the $A^{\max}$ performance.

In Figures 9 and 10, we plot the maximum data rate for the first and second user respectively, within the stability region. Since both users have identical SNR, the behavior of $\lambda_1^{\max}$ and $\lambda_2^{\max}$ are similar. In addition, all the three schedulers have same behavior for the maximum rates within the stability region.

CONCLUSIONS

We studied the behavior of Queues in multi-user cognitive radios for the case when the centralized coordinator is controlling the channel sensing and scheduling process. We considered three different scheduling policies: maximum-rate scheduling, maximum-weight scheduling and proportionally fair scheduling. Our results show that the maximum-weight scheduler has the largest stability region. Performance of maximum-rate scheduler is typically in between the maximum-weight and proportionally fair schedulers. Proportionally fair algorithm has the smallest stability region. However, it ensures that users who have poor SNR conditions enjoy better data rates than the other two schedulers.

REFERENCES

[1] H. Jiang, L. Lai, R. Fan, and H. V. Poor, "Optimal selection of channel sensing order in cognitive radio," *IEEE Transactions on Wireless Communications*, vol. 8, no. 1, pp. 297–307, 2009.

[2] R. Fan and H. Jiang, "Channel sensing-order setting in cognitive radio networks: A two-user case," *IEEE Transactions on Vehicular Technology*, vol. 58, no. 9, pp. 4997–5008, 2009.

[3] H. T. Cheng and W. Zhuang, "Simple channel sensing order in cognitive radio networks," *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 4, pp. 1–13, 2011.

[4] R. Misra and A. P. Kannu, "Optimal sensing-order in cognitive radio networks with cooperative centralized sensing," in *IEEE International Conference on Communications (ICC)*, pp. 1566–1570, June 2012.

[5] X. Lin, N. B. Shroff, and R. Srikant, "A tutorial on cross-layer optimization in wireless networks," *IEEE Journal on Selected Areas in Communications*, vol. 24, pp. 1452–1463, Aug 2006.