

# The Effect of Internal Heat Generation on the Onset of Rayleigh–Bénard Electro Convection in a Micropolar Fluid

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## Abstract:

The effect of internal heat generation and electric field on the onset of Rayleigh–Bénard convection in a micropolar fluid are studied by performing a linear stability analysis. The eigenvalue of the problem are obtained for rigid-rigid, rigid-free, and free-free velocity boundary combinations with isothermal and adiabatic temperature boundaries using the Galerkin technique. The microrotation is assumed to vanish at the boundaries. The impact of various micropolar fluid parameters, electric Rayleigh number, and the internal Rayleigh number on the onset of convection is analyzed. The linear theory is based on normal mode analysis. The expression of Rayleigh number is obtained as a function of the electric Rayleigh number, internal Rayleigh number, and other micropolar fluid parameters. It is observed that the increasing internal Rayleigh number destabilizes the system upon an infinitesimal disturbance on it. The control of the onset of electroconvection is possible with the help of the internal heat generation, electric field, and micropolar fluid.

**Keywords:** Electric field, Internal heat source, Micropolar fluid, Rayleigh number.

## List of Symbols:

$l, m$  - Horizontal wave numbers

$d$  - Distance between the two plates

$\hat{k}$  - Unit vector in the vertical direction

$a$  - Wave number

$p$  - Hydrodynamic pressure

$\vec{q}$  -  $(U, V, W)$  Velocity

$t$  - Time

$T$  - Temperature

$z$  - Vertical coordinate

$x$  - Horizontal coordinate

$\vec{g}$  - Acceleration due to gravity

$\sigma$  - Growth rate of perturbation

$\rho$  - Density

$\rho_o$  - Density at temperature  $T = T_o$

$\nabla$  -  $\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k}$  Vector differential operator

$\nabla^2$  -  $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$  Three-dimensional Laplacian operator

$\nabla_1^2$  -  $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$  Two-dimensional Laplacian operator

$\mu$  - Dynamic viscosity of the fluid

$\vec{\omega}$  - Angular velocity

$\Omega$  -  $z$  - component of  $\vec{\omega}$

$\phi$  - Electric potential

$\chi$  - Thermal conductivity

$\lambda'$  and  $\eta'$  - Bulk and shear spin viscosity coefficients

$\beta$  - Micropolar heat conduction coefficient

$C_v$  - Specific heat

$\eta$  - Shear kinematic viscosity coefficient

$\zeta$  - Coupling viscosity coefficient

$I$  - Moment of inertia

$\chi_e$  - Electric susceptibility

$\alpha$  - Coefficient of thermal expansion

$\xi$  - Coupling viscosit coefficient or vortex viscosity

$E_o$  - Root mean quare value of the electric field at the lower surface

$\vec{E}$  - Electric field

$\vec{P}$  - Dielectric polarization

$\epsilon_o$  - Electric permeability of free space

$\epsilon_r$  - Dielectric constant

$N_5 = \frac{\beta}{\rho_o c_v d^2}$  - Micropolar heat conduction parameter

$N_3 = \frac{\eta'}{d^2(\eta + \xi)}$  - Couple stress parameter

$$N_2 = \frac{I}{d^2} - \text{Inertia parameter}$$

$$N_1 = \frac{\xi}{\eta + \xi} - \text{Coupling Parameter}$$

$$L = \frac{\epsilon_0 e^2 E_1^2 (\Delta T)^2 d^2}{(1 + \chi_e)(\xi + \eta)\chi} - \text{Electric Rayleigh Number}$$

$$\frac{1}{Pr} = \frac{\rho_o \chi}{\xi + \eta} - \text{Prandtl number}$$

$$R_i = \frac{Qd^2}{\chi} - \text{Internal Rayleigh number}$$

$$R = \frac{\rho_o \alpha g \Delta T d^3}{(\xi + \eta)\chi} - \text{Rayleigh number}$$

**Subscripts:**

*c* - Critical value

*b* - Basic state

**Superscripts:**

\* - Dimensionless quantity

' - Perturbed quantity

**1. INTRODUCTION**

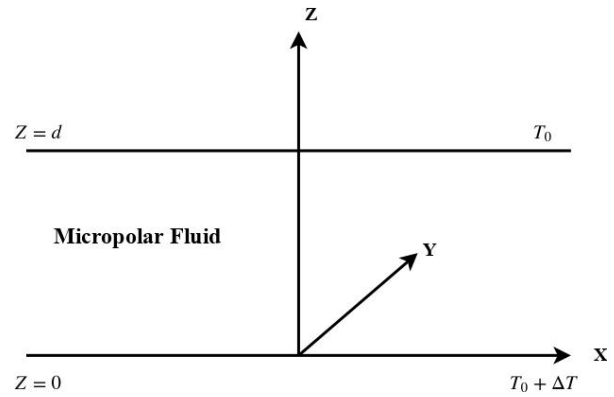
The study of interaction of the electromagnetic field within the fluid is finding its wide application in the fields like chemical engineering, nuclear fusion, noiseless printing and in medicine. To precisely define the behaviour of fluids with micro-structures have a diverse form and tend to shrink and enlarge as well as rotate on their axis freely without a direction and gyration. Micropolar fluids are the fluids having a microstructure. They are included in a class of fluids that have non-symmetric stress tensor which are widely known as the polar fluids. Micropolar fluid model can be effectively used to determine the flow taking place between parallel plates and flowing through channels. The effect of internal heat generation has a prominent role in various applications including combustion and fire studies, storage of materials that are radioactive and many others. It is found that in many situations, the material tends to offer its own source of heat, leading to an alternate manner by which a convective flow can be set up through the local heat generation inside the layer. Hence, the significance of internal heat generation turns out to be very apparent in various applications that includes storage of radioactive materials, geophysics, and reactor safety analysis. The movement of any fluid element can be distinguished by their unique feature which are translation rate of deformation and rotation taking place in the space. When we consider the rotation of the fluid with some velocity we see that the pressure rapidly increases in such a way that the

pressure is independent of the direction of the angular velocity generated by the rotating fluid. Roberts [1] studied the convection that takes place in horizontal layers and in the presence of internal heat. A theoretical study was made out of an experiment in which convective motions generated when place between the horizontal layer of water and was heated uniformly. It was noted that when such a layer is calculated, the critical Rayleigh number that is obtained turned out to be 2772 for which the patterns of the wave number is 2.63 times the reciprocal depth. Ahmadi [2] determined whether the layer having micropolar fluid is stable when it is being heated from the below. This was determined by the means of linear theory plus an energy method using which it was proven that principle of exchange of stability is valid. The critical Rayleigh number has been obtained. It was observed that when the fluid is heated from below is shown to be more stable in comparison with the viscous fluid. It is observed that when the under finite disturbances, the energy method was advantageous in studying the stability of the system. Alloui and Vasseur [3] studied the Rayleigh–Bénard MHD convection in an electrically conducting micropolar fluid layer in the presence of uniform magnetic field. A fluid layer which is bounded by the horizontal rigid boundary that is subjected to the thermal boundary conditions is considered. The critical Rayleigh number is predicted by the parallel flow approximation and is found that the onset of the convection depends on the Hartmann number and the materials parameters as well as the microrotation. The stability of the system is determined using a linear theory. Joseph et al. [4] studied the effects of an electric field as well as the effect of non-uniform temperature gradient of the Rayleigh–Bénard Marangoni Convection in a micropolar fluid. This was carried out with the help of Galerkin technique. The stability of the system is determined using the linear theory and the eigen values are obtained for different boundaries. Graphically, the influence of several parameters as well as the electric Rayleigh number on the onset of the convection were presented. Char and Chiang [5] studied the Bénard Marangoni problem that deals with the instability of a liquid layer where the upper surface is free and is heated from below in the presence of an electric field. It was determined that the various effects that include thermal conductivity and electric conductivity have an immense effect on the onset of convection in this fluid layer. The stability of the system was carried out using the linear stability analysis. It is found that with a higher electric conductivity and larger thickness tend to stabilize the system. It was also observed that there is an increase in the electric field as the critical Rayleigh number, the a.c critical Rayleigh number and the critical Marangoni number get smaller. The effect of rotation on Rayleigh Bénard convection was studied extensively by Professor Chandrasekhar [6] wherein plenty of results have been derived that has been extensively used in the study of the paper. The natural convection taking place in a rotating anisotropic porous medium in the presence of internal heat was analyzed by Bhadauria et al. [7], wherein the linear theory is performed using the normal mode analysis technique and the corresponding stability analysis derives the conditions for the stationary as well as for the oscillatory case. Joseph et al. [8], studied the linear and weakly non-linear stability of the

electrothermal convection in a micropolar fluid that is heated from below. This analyses was based the normal mode and truncated Fourier series. The impact of the parameters obtained have been studied in linear case. It is found that the system become stable when there is an increase in the concentration of particles when they are suspended and in turn there is decrease in heat transfer . Whereas the increase in the electric Rayleigh number destabilizes the system and in turn there is an increase in heat transfer. Siddheshwar and Pranesh [9] studied the Rayleigh–Bénard convection in an electrically conducting micropolar fluid wherein there is an action of uniform and vertical magnetic field considering the free-free boundaries, isothermal boundaries and spin vanishing boundaries. The impact of parameters for stationary convection has been discussed. It is found that the micropolar fluid in the existing electric field when heated from below tends to be more stable in comparison to the Newtonian fluid. In terms of its physical attributes, micropolar fluid represents the fluids that comprise rigid and random orientation of the particles which are suspended in a medium which is high in viscosity wherein the deformation of the fluid particles are ignored generally. The model for micropolar fluids was formally shown by Eringen and reviewed by Lucaszewicz [10]. It is a well-founded and a significant generalization to the classical Navier-Stokes model that cover its significance in theory as well as in applications. This model is very elegantly designed and is not complicated to mathematicians whose focus of study is its theory and on the other side the engineers and the physicists are the ones who apply the model. The main objective of the book includes the study of the theory behind the micropolar fluids. To be more specific, to study its mathematical theory. The book also includes the applications of micropolar fluid and their exact solutions and numerical methods. Takashima [11] studied the linear stability theory in order to solve the problem of the stability in a natural convection which occurs in a fluid layer in the internal heat source being uniformly distributed. The boundaries are kept constant and an equal temperatures. To obtain the eigen-values the power series method is used and then solved numerically using Muller's method. The theory of thermo-micropolar convection which is heated from below was studied by authors namely Ahmadi [2], Datta and Sastry [12], Bhattacharya and Jena [13] and Siddheshwar and Pranesh[14-18].

## 2. MATHEMATICAL FORMULATION

A Boussinesquian, micropolar fluid in infinite horizontal layer having depth  $d$  in the presence of internal heat source is considered.  $\Delta T$  is taken as the difference of temperature between the lower plate and the upper plate. Buoyancy and electric field are the body forces that are acting on the fluid. Also considering  $(x, y, z)$  as a Cartesian coordinate system where its origin is taken at the lower boundary while the  $z$ -axis is pointing vertically upwards.



**Figure 1.** Physical Configuration for the Rayleigh–Bénard situation in Micropolar fluid

The basic governing equations for the model under consideration are:

### Continuity Equation:

$$\nabla \cdot \vec{q} = 0, \quad (1)$$

### Conservation of linear Momentum:

$$\rho_o \left[ \frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} \right] = -\nabla p - \rho g \hat{k} + (2\zeta + \eta) \nabla^2 \vec{q} \left. \vphantom{\left[ \frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} \right]} \right\} + \zeta \nabla \times \vec{\omega} + (\vec{P} \cdot \nabla) \vec{E} \quad (2)$$

### Conservation of angular momentum:

$$\rho_o I \left[ \frac{\partial \vec{\omega}}{\partial t} + (\vec{q} \cdot \nabla) \vec{\omega} \right] = (\lambda' + \eta') \nabla (\nabla \cdot \vec{\omega}) + (\eta' \nabla^2 \vec{\omega}) \left. \vphantom{\left[ \frac{\partial \vec{\omega}}{\partial t} + (\vec{q} \cdot \nabla) \vec{\omega} \right]} \right\} + \zeta (\nabla \times \vec{q} - 2\vec{\omega}) \quad (3)$$

### Conservation of Energy:

$$\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T = \chi \nabla^2 T + \frac{\beta}{\rho_o c_v} (\nabla \times \vec{\omega}) \cdot \nabla T + Q(T - T_o), \quad (4)$$

### Equation of state:

$$\rho = \rho_o [1 - \alpha(T - T_o)], \quad (5)$$

### Faraday's Law:

$$\nabla \cdot \vec{E} = 0, \quad (6)$$

$$\vec{E} = -\nabla \phi, \quad (7)$$

### Equation of polarization field:

$$\nabla \cdot (\varepsilon_o \vec{E} + \vec{P}) = 0, \quad (8)$$

$$\vec{P} = \varepsilon_o (\varepsilon_r - 1) \vec{E}, \quad (9)$$

### Equation of state for dielectric constant:

$$\varepsilon_r = \varepsilon_r^o - e(T - T_o), \quad (10)$$

where  $\varepsilon_r^o = 1 + \chi_e$  and  $e > 0$ .

### 3. BASIC STATE

The fluid is at rest in this state and heat is transferred only by conduction. The fluid in this state is described by:

$$\left. \begin{aligned} p &= p_b(z), \rho = \rho_b(z), \vec{\omega}_b = 0, \vec{q}_b = 0, \\ T &= T_b(z), \vec{P} = P_b(z), \vec{E} = E_b(z). \end{aligned} \right\} \quad (11)$$

By using the condition (11) in equations (2)-(5) and equations (9)-(10), we get the following:

$$\frac{dp}{dz} = -\rho_b gk + P_b \frac{\partial \vec{E}_b}{\partial z}, \quad (12)$$

$$\chi \frac{d^2(T_b - T_o)}{dz^2} + Q(T_b - T_o) = 0, \quad (13)$$

$$\rho_b = \rho_o [1 - \alpha(T_b - T_o)], \quad (14)$$

$$E_b = \left[ \frac{(\chi_e + 1)E_o}{(\chi_e + 1) + \frac{e\Delta T}{h} z} \right], \quad (15)$$

$$P_b = \varepsilon_o E_o (\chi_e + 1) \left[ 1 - \frac{1}{(\chi_e + 1) + \frac{e\Delta T}{h} z} \right], \quad (16)$$

$$\varepsilon_r = (1 + \chi_e) - e(T_b - T_o). \quad (17)$$

Solving the differential equation (13), we get

$$T_b = T_o + \Delta T \frac{\text{Sin}(\sqrt{R_i}(1 - \frac{z}{d}))}{\text{Sin}(\sqrt{R_i})}. \quad (18)$$

### 4. LINEAR STABILITY ANALYSIS

Let the system be disturbed from the basic state. Equations in Perturbed state are given by,

$$\left. \begin{aligned} \rho &= \rho_b + \rho', p = p_b + p', \vec{\omega} = \vec{\omega}_b + \vec{\omega}', \vec{q} = \vec{q}_b + \vec{q}', \\ T &= T_b + T', \vec{P} = P_b + (P'_1, P'_3), \vec{E} = E_b + (E'_1, E'_3). \end{aligned} \right\} \quad (19)$$

Equations (8) and (9), on linearization yield,

$$P'_1 = \varepsilon_o \chi_e E'_1$$

$$P'_2 = \varepsilon_o \chi_e E'_2$$

$$P'_3 = \varepsilon_o \chi_e E'_1 - e \varepsilon_o E_o T'$$

Equation (19) is substituted into equations (1)-(10), also making use of the basic state equations (12)-(17), we get the following by neglecting the non-linear terms,

$$\nabla \cdot \vec{q}' = 0, \quad (20)$$

$$\left. \begin{aligned} \rho_o \left[ \frac{\partial \vec{q}'}{\partial t} \right] &= -\nabla p' + (2\zeta + \eta) \nabla^2 \vec{q}' + \zeta \nabla \times \vec{\omega}' \\ &+ (\vec{P}_b \cdot \nabla) \vec{E}' + (\vec{P}' \cdot \nabla) \vec{E}_b - \rho' g \hat{k}, \end{aligned} \right\} \quad (21)$$

$$\left. \begin{aligned} \rho_o I \left[ \frac{\partial \vec{\omega}'}{\partial t} \right] &= (\lambda' + \eta') \nabla (\nabla \cdot \vec{\omega}') \\ &+ (\eta' \nabla^2 \vec{\omega}') + \zeta (\nabla \times \vec{q}' - 2\vec{\omega}'), \end{aligned} \right\} \quad (22)$$

$$\frac{\partial T'}{\partial t} = \left( W' - \frac{\beta}{\rho_o c_v} (\nabla \times \vec{\omega}') \right) \frac{\Delta T}{d} f(z) + \chi \nabla^2 T', \quad (23)$$

$$\rho' = -\rho_o \alpha T', \quad (24)$$

$$(1 + \chi_e) \nabla^2 \phi' - e E'_1 \frac{\partial T'}{\partial z} = 0. \quad (25)$$

$$\text{where } f(z) = \frac{\sqrt{R_i} \text{Cos}(\sqrt{R_i}(1 - z))}{\text{Sin}(\sqrt{R_i})}.$$

Introducing the electric potential  $\phi'$ , applying curl twice on equation (21) to eliminate the pressure term, and once on equation (22) to obtain the perturbed state vorticity transport equation,

$$\left. \begin{aligned} \rho_o \frac{\partial (\nabla^2 W')}{\partial t} &= \rho_o \alpha g \nabla_1^2 T' + 2(\zeta + \eta) \nabla^4 W' + \zeta \nabla^2 (\nabla \times \vec{\omega}') \\ &+ \frac{\varepsilon_o e^2 E_o^2 \Delta T f(z) \nabla_1^2 T'}{(1 + \chi_e)} - \frac{\Delta T}{d} \varepsilon_o e E_o f(z) \nabla_1^2 D \phi', \end{aligned} \right\} \quad (26)$$

$$\left. \begin{aligned} \rho_o I \left[ \frac{\partial (\nabla \times \vec{\omega}')}{\partial t} \right] &= (\lambda' + \eta') \nabla^2 (\nabla \times \vec{\omega}') \\ &+ \eta' \nabla^2 \vec{\omega}' + \zeta (\nabla^2 W' - 2\nabla \times \vec{\omega}') \end{aligned} \right\} \quad (27)$$

We non-dimensionalize the equations (23), and (25)-(27) using the following:

$$\left. \begin{aligned} (x^*, y^*, z^*) &= \frac{1}{d} (x, y, z), W^* = \frac{W'}{\chi/d}, \omega^* = \frac{\omega'}{\chi/d^2}, t^* = \frac{t}{d^2/\chi}, \\ T^* &= \frac{T'}{\Delta T}, \phi^* = \frac{\phi'}{\left( \frac{e E_o \Delta T d}{1 + \chi_e} \right)}, \Omega^* = \frac{\nabla \times \vec{\omega}'}{\chi/d^3}. \end{aligned} \right.$$

The following is obtained after the substitutions and for simplicity, we drop the asteriks,

$$R\nabla_1^2 T + (1 + N_1)\nabla^4 W + N_1\nabla^2 \Omega + L\nabla_1^2 T f(z) - L \frac{\partial \nabla_1^2 \phi}{\partial z} f(z) = 0, \quad (28)$$

$$N_3\nabla^2 \Omega + N_1\nabla^2 W - 2N_1\Omega = 0, \quad (29)$$

$$(W - N_3\Omega)f(z) + \nabla^2 T + R_i T = 0, \quad (30)$$

$$\nabla^2 \phi - \frac{\partial T}{\partial z} = 0. \quad (31)$$

The infinitesimal perturbation  $W$ ,  $\Omega$ ,  $\phi$ , and  $T$  are assumed to be periodic waves and hence these permit a normal mode solution in the form

$$\begin{bmatrix} W \\ \Omega \\ \phi \\ T \end{bmatrix} = \begin{bmatrix} W(z) \cdot \exp(ilx + imy) \\ H(z) \cdot \exp(ilx + imy) \\ \phi(z) \cdot \exp(ilx + imy) \\ T(z) \cdot \exp(ilx + imy) \end{bmatrix}$$

Equation (32) is substituted into equations (28)-(31), we get,

$$(D^2 - a^2)^2 W(1 + N_1) - TRa^2 + (D^2 - a^2)HN_1 + Tf(z)La^2 + D\phi f(z)La^2 = 0, \quad (33)$$

$$(D^2 - a^2)HN_3 - (D^2 - a^2)WN_1 - 2HN_1 = 0, \quad (34)$$

$$(D^2 - a^2)T + (W - N_3H)f(z) + TR_i = 0, \quad (35)$$

$$(D^2 - a^2)\phi(z) - DT(z) = 0. \quad (36)$$

where,  $D = \frac{d}{dz}$ .

Galerkin technique is used to solve the differential equations (33)-(36). (33) is multiplied by  $W$ , (34) is multiplied by  $H$ , (35) by  $T$  and (36) by  $\phi$ , and integrating them from 0 to 1 with respect to  $z$  and taking  $W = AW_1$ ,  $H = BH_1$ ,  $T = CT_1$ , and  $\phi = E\phi_1$  where  $A, B, C$  and  $E$  are constants with  $W_1, H_1, T_1, \phi_1$  being the trial functions that need to satisfy the boundary conditions.

This procedure yields the expression for Rayleigh number  $R$  as follows:

$$R = \frac{F_1 F_4 F_{10}}{\langle W_1 T_1 \rangle [F_2 - F_3 + F_5] a^2} - L F_{12}. \quad (37)$$

Where,

$$F_1 = N_3 \langle H_1 (D^2 - a^2) H_1 \rangle - 2N_1 \langle H_1^2 \rangle,$$

$$F_2 = N_3 N_5 \langle H_1 (D^2 - a^2) W_1 \rangle \langle T_1 H_1 f(z) \rangle,$$

$$F_3 = N_3 \langle H_1 (D^2 - a^2) H_1 \rangle \langle T_1 W_1 f(z) \rangle,$$

$$F_4 = \langle T_1 (D^2 - a^2) T_1 \rangle + R_i \langle T_1^2 \rangle,$$

$$F_5 = N_1 \langle H_1^2 \rangle \langle T_1 W_1 f(z) \rangle,$$

$$F_6 = \langle W_1 D(\phi_1) f(z) \rangle \langle \phi_1 D(T_1) \rangle,$$

$$F_7 = \langle \phi_1 (D^2 - a^2) \phi_1 \rangle \langle T_1 (D^2 - a^2) T_1 \rangle + \langle \phi_1 (D^2 - a^2) \phi_1 \rangle R_i \langle T_1^2 \rangle,$$

$$F_8 = N_1^2 \langle W_1 (D^2 - a^2) G_1 \rangle \langle G_1 (D^2 - a^2) W_1 \rangle,$$

$$F_9 = (1 + N_1) \langle W_1 (D^2 - a^2)^2 W_1 \rangle,$$

$$F_{10} = F_9 + \frac{F_8}{F_1},$$

$$F_{11} = a^2 (F_2 - F_3 + 2F_5),$$

$$F_{12} = \frac{F_{11} [F_7 \langle W_1 T_1 f(z) \rangle - F_4 F_6]}{F_1 F_4 F_7}.$$

$\langle \dots \rangle$  in equation (37) denotes integration with respect to  $z$  from 0 to 1. The boundary combinations considered in this paper are:

Rigid-Rigid isothermal, no spin.

$$W = DW = H = D\phi = T = 0, \text{ at } z=0,1.$$

Rigid-Free isothermal, no spin.

$$W = DW = H = D\phi = T = 0, \text{ at } z=0,$$

$$W = D^2W = H = D\phi = T = 0, \text{ at } z=1.$$

Free-Free isothermal, no spin.

$$W = D^2W = H = D\phi = T = 0, \text{ at } z=0,1.$$

Rigid-Rigid adiabatic, no spin.

$$W = DW = DH = D\phi = T = 0, \text{ at } z=0,1.$$

Rigid-Free adiabatic, no spin.

$$W = DW = DH = D\phi = T = 0, \text{ at } z=0,$$

$$W = D^2W = DH = D\phi = T = 0, \text{ at } z=1.$$

Free-Free adiabatic, no spin.

$$W = D^2W = DH = D\phi = T = 0, \text{ at } z=0,1.$$

**Table 1.** The trial functions that satisfy the various boundary conditions

Trial functions	Rigid-Rigid	Rigid-Free	Free-Free
$W_1$	$z(1-2z^2+z^3)$	$z^4-\frac{5}{2}z^3+\frac{3}{2}z^2$	$z(1-2z^2+z^3)$
$H_1$	$z^2-z$	$z^2-z$	$z^2-z$
Isothermal temperature $T_1$	$z-z^2$	$z-z^2$	$z-z^2$
Adiabatic temperature $T_1$	1	1	1

### 5. RESULTS AND DISCUSSIONS

In this paper, the effect of internal heat generation on the onset of Rayleigh-Bénard electro-convection in a micropolar fluid is studied using linear stability analysis. The effects of the parameters  $N_1$ ,  $N_3$  and  $N_5$ , the internal Rayleigh number  $R_i$ , and the electric Rayleigh number  $L$  are investigated on the onset of convection. The first three parameters represents the micropolar fluid parameters,  $R_i$  comes from the internal heat generation and  $L$  comes from electric field. The thermodynamic restriction permits the values of  $N_1$ ,  $N_3$ , and  $N_5$  to lie in the following ranges:  $0 \leq N_1 \leq 1$  and  $0 \leq N_3, N_5 \leq m$  where  $m$  is finite and positive real number.

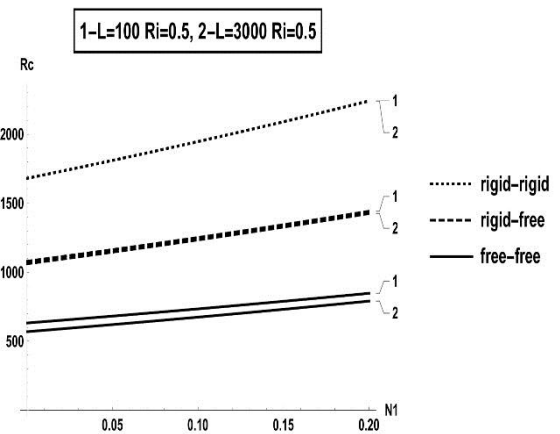
Figure 2 and 3 are the plots representing the critical Rayleigh number  $R_c$  against the parameter  $N_1$  which is the coupling parameter for various values of electric Rayleigh number  $L$  and internal Rayleigh number  $R_i$  with respect to rigid-rigid, rigid-free, free-free isothermal, no spin boundary conditions respectively. It is found that the increase in  $N_1$  results in the increase in  $R_c$ . Thus the increase in  $N_1$  results in the stabilization of the system. The reason for this is, the increase in  $N_1$  implies the increase in concentration of microelements, this result in the consumption of the energy of the system by these elements, thereby delaying the onset of the convection.

Figure 4 and 5 are the plots representing  $R_c$  against the parameter  $N_3$  which is couple stress parameter for various values of electric Rayleigh number  $L$  and internal Rayleigh number  $R_i$  with respect to rigid-rigid, rigid-free, free-free isothermal, no spin boundary conditions respectively. It is found that increasing  $N_3$  results in the decrease in  $R_c$ . Therefore, the increase in  $N_3$  leads to destabilization of the system. Increase in  $N_3$  increases the couple stresses in the fluid, hence causing a decrease in microrotation and thereby destabilizing the system.

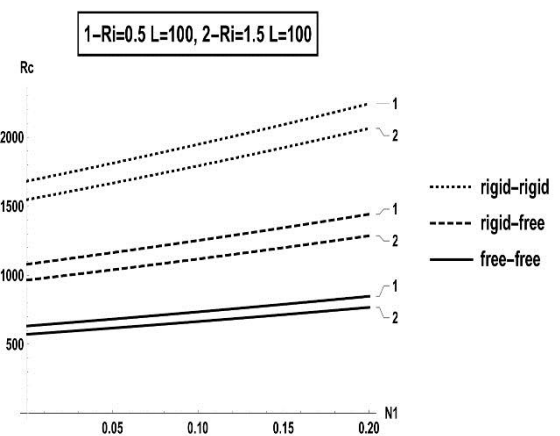
Figure 6 and 7 are the plots representing  $R_c$  against the parameter  $N_5$  which is the micropolar heat conduction parameter for various values of electric Rayleigh number  $L$  and internal Rayleigh number  $R_i$  with respect to rigid-rigid, rigid-free, free-free isothermal, no spin boundary conditions respectively. It is observed that the increase in  $N_5$  results in the increase in  $R_c$ . Thus the increase in  $N_5$  causes the system to stabilize. The increase in  $N_5$  increase the heat induced into the fluid, thereby reducing the heat transferred from the bottom to the top, which delays the onset of convection.

It is observed from these figures that increase in the electric Rayleigh number  $L$  and the internal Rayleigh number  $R_i$  decreases the  $R_c$ , thus these two parameters destabilizes the system upon increasing it. The electric Rayleigh number  $L$  is the ratio of electric force to gravitational force, thus increase in electric field makes the fluid layer more and more unstable thus it advances the onset of convection and the onset of convection is advanced by the increase in heat supply due to the increase in internal Rayleigh number  $R_i$ . Thus, the effect of electric field  $L$  and internal Rayleigh number  $R_i$  is to destabilize the system.

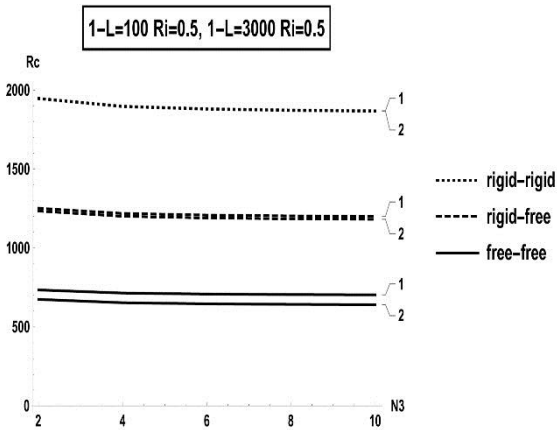
Figures 8-13 are the plots of  $R_c$  against  $N_1$ ,  $N_3$ , and  $N_5$  for different values of electric Rayleigh number  $L$  and internal Rayleigh number  $R_i$  with respect to rigid-rigid, rigid-free, and free-free, adiabatic temperature and no spin boundary conditions. Qualitatively the results of adiabatic boundaries are similar to that of isothermal boundaries. However, it is found that the  $R_c$  in case of adiabatic boundary is smaller when compared to  $R_c$  in isothermal boundaries.



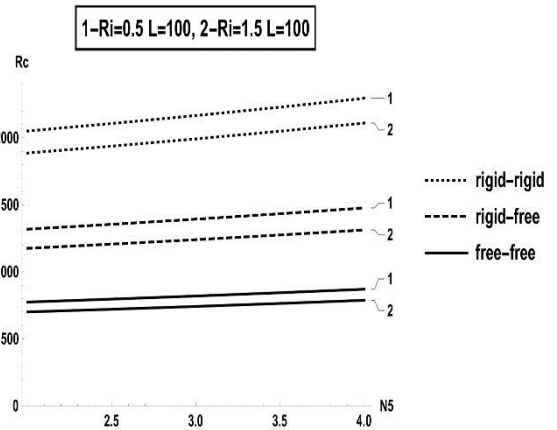
**Figure 2.**  $R_c$  against  $N_1$  for various values of  $L$  in rigid-rigid, rigid-free, free-free isothermal, no spin case for  $N_5 = 1$ ,  $N_3 = 2$ , and  $R_i = 0.5$ .



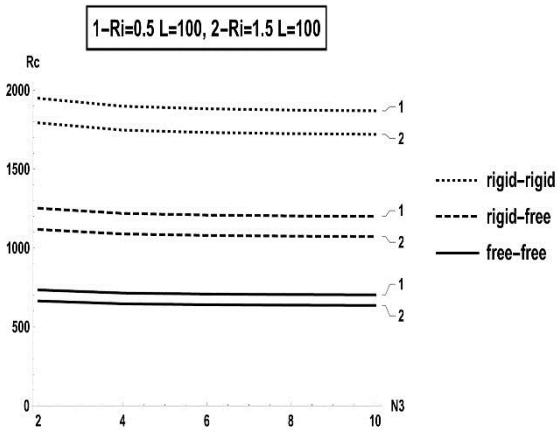
**Figure 3.**  $R_c$  against  $N_1$  for various values  $R_i$  in rigid-rigid, rigid-free, free-free isothermal, no spin case for  $N_5 = 1$ ,  $N_3 = 2$ , and  $L = 100$ .



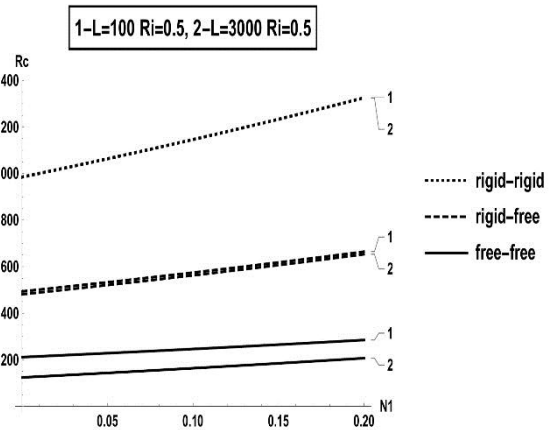
**Figure 4.**  $R_c$  against  $N_3$  for various values of  $L$  in rigid-rigid, rigi-free, free-free isothermal, no spin case for  $N_5 = 1$ ,  $N_1 = 0.1$ , and  $R_i = 0.5$ .



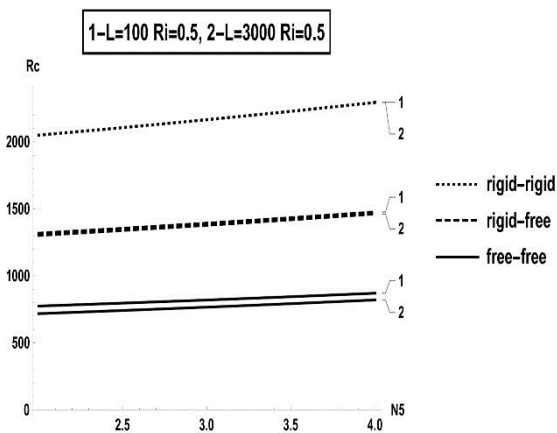
**Figure 7.**  $R_c$  against  $N_5$  for various values of  $R_i$  in rigid-rigid, rigi-free, free-free isothermal, no spin case for  $N_5 = 1$ ,  $N_3 = 2$ , and  $L = 100$ .



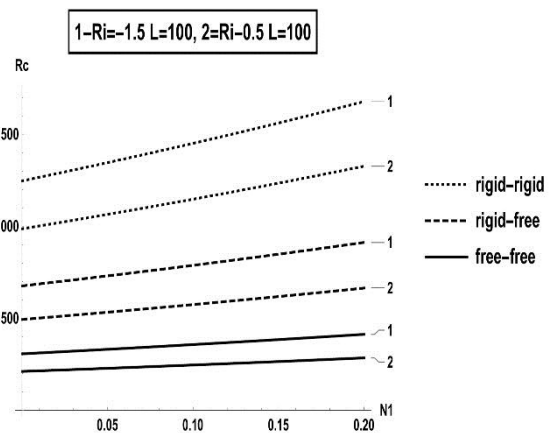
**Figure 5.**  $R_c$  against  $N_3$  for various values  $R_i$  in rigid-rigid, rigi-free, free-free isothermal, no spin case for  $N_5 = 1$ ,  $N_1 = 0.1$ , and  $L = 100$ .



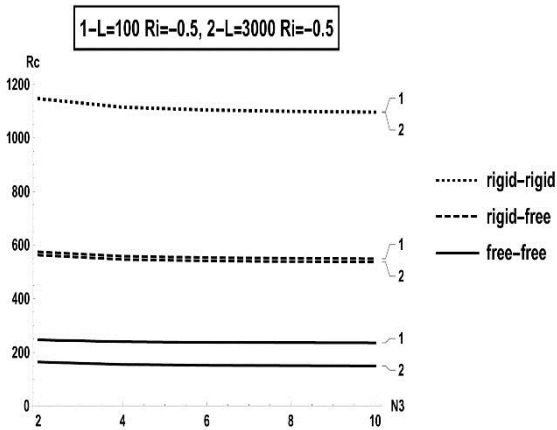
**Figure 8.**  $R_c$  against  $N_1$  for various values of  $L$  in rigid-rigid, rigi-free, free-free adiabatic, no spin case for  $N_5 = 1$ ,  $N_3 = 2$ , and  $R_i = 0.5$ .



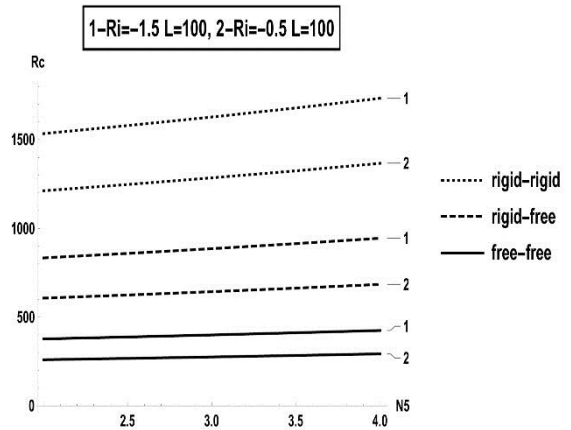
**Figure 6.**  $R_c$  against  $N_5$  for various values  $L$  in rigid-rigid, rigi-free, free-free isothermal, no spin case for  $N_3 = 2$ ,  $N_1 = 0.1$ , and  $R_i = 0.5$ .



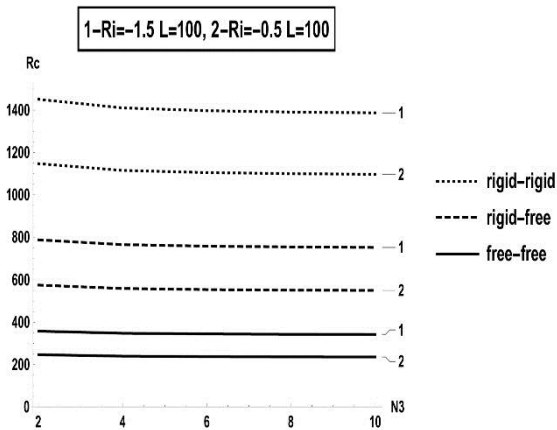
**Figure 9.**  $R_c$  against  $N_1$  for various values  $R_i$  in rigid-rigid, rigi-free, free-free adiabatic, no spin case for  $N_5 = 1$ ,  $N_3 = 2$ , and  $L = 100$ .



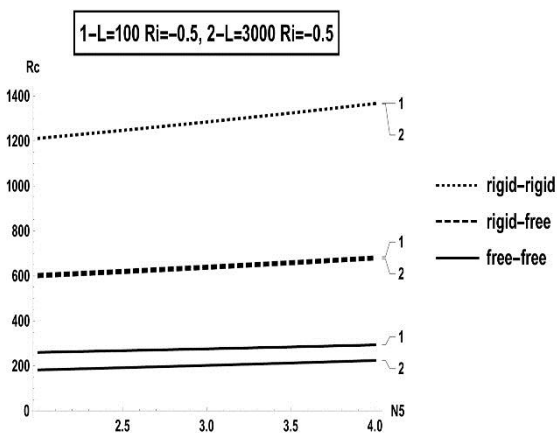
**Figure 10.**  $R_c$  against  $N_3$  for various values of  $L$  in rigid-rigid, rigi-free, free-free adiabatic, no spin case for  $N_5 = 1$ ,  $N_1 = 0.1$ , and  $R_i = 0.5$ .



**Figure 13.**  $R_c$  against  $N_5$  for various values of  $R_i$  in rigid-rigid, rigi-free, free-free adiabatic, no spin case for  $N_5=1$ ,  $N_3=2$ , and  $L=100$ .



**Figure 11.**  $R_c$  against  $N_3$  for various values  $R_i$  in rigid-rigid, rigi-free, free-free isothermal, no spin case for  $N_5 = 1$ ,  $N_1 = 0.1$ , and  $L = 100$ .



**Figure 12.**  $R_c$  against  $N_5$  for various values  $L$  in rigid-rigid, rigi-free, free-free adiabatic, no spin case for  $N_3 = 2$ ,  $N_1=0.1$ , and  $R_i = 0.5$ .

## 6. CONCLUSION

The following are the conclusions found from the study:

- (i) Both internal Rayleigh number and electric Rayleigh number destabilize the system.
- (ii) In a micropolar fluid, Stationary convection seems to be the mode of instability preferred.
- (iii) In Newtonian fluids, it is possible to delay the convection by adding micron-sized suspended particles.
- (iv)  $R_c^{RR} > R_c^{RF} > R_c^{FF}$  for both isothermal and adiabatic boundaries where subscripts represents the different velocity boundary conditions.
- (v) The control of the onset of convection is possible with the help of the internal heat generation and electric field.

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