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Analysis of a Power Supported System with Two Types of Repair Pattern and Forced Reduction in its Operational Time

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Abstract

In this paper, the comparative study of the two repair models of a system comprising a main unit and two supportive units, viz. electricity and uninterrupted power supply (UPS), has been conducted taking into consideration the propounded aspect of forced reduction in operational time. Model 1 considers the situation wherein repair of supportive units is done on first come first served (FCFS) pattern, whereas Model 2 considers the situation wherein priority for repair is given to the electricity unit over the UPS. In order to analyze the system, various measures of system effectiveness like mean time to system failure (MTSF), availability, expected number of occurrences of forced reduction in operational time, expected number of visits of repairman and expected busy period of the repairman etc. have been obtained by using the regenerative point technique and semi-Markov processes. Comparative analysis of the two models with reference to performability measures and profitability of system has been done by considering a particular case.

Keywords: main unit, supportive unit, forced reduction in operational time, measures of system effectiveness, profit, regenerative point technique.

1. Introduction

With the ever-increasing importance of reliable systems in the field of industry, the reliability models have always caught the attention of many researchers. Considering further the role of standby redundancy in increasing the reliability of the system, standby systems have been studied by the various authors [1-5] under different set of assumptions and situations including random interchange of operative and standby units; switchover time; varying demand; arbitrary distributions for life, repair and waiting time; degradation etc. Further, models have also been developed taking into consideration the various aspects regarding operational time. Kumar and Kapoor [6] evaluated a base transceiver system considering various operational modes and catastrophic failures. Gupta and Bhardwaj [7] analyzed a two-unit standby system with two operating modes. Kadyan [8] dealt with reliability and profit analysis of a single unit system with preventive maintenance subject to maximum operation time. But, in the field of reliability, none of the authors has yet considered the aspect of forced reduction in operational time which may be observed in practical situations.

In this paper, a system comprising a main unit, with electricity and UPS as supportive units, has been considered. Main unit may be a computer or any similar machine/device which works on electricity with UPS as standby for electricity unit. The reliability and cost- benefit analysis of the system has been carried out in the light of the posed aspect of forced reduction in operational time. The situation of forced reduction in operational time may be faced when there is failure of the electricity unit and consequently the main unit operates on UPS only. At this stage, the main unit, after saving the data/files, has to be shut down (i.e., operational time is reduced) before the completion of the backup time of UPS. Such shut down is nothing but the forced reduction in operational time and is done to avoid any loss of data /system files.

There is provision of single repair facility for the system which becomes available as and when any of the considered units fails. Two different situations regarding the repair facility have been analyzed under the two models (Model 1 and Model 2). In Model 1, the repair discipline of supportive units is First come First Served i.e. no priority for repair is given to any of the supportive units over the other. In Model 2, the priority for repair is given to the electricity unit over the UPS, i.e., if UPS is under repair and during that time there is failure of electricity unit, the repair of the latter is undertaken keeping that of former in abeyance, subject to the condition that this is done not more than once during the whole repair of a fault in the former.

2. Materials and Methods

In the present study, performability and profit analysis of the system has been carried out by obtaining the characteristics such as MTSF, availability, expected busy period of repairman, expected downtime of the system, expected number of occurrences of forced reduction in operational time etc., using the regenerative point technique, semi-Markov processes, Laplace transform and Laplace - Stieltjes Transform. Further, software like Mathematica and MATLAB have been used for the purpose of calculations and comparative analysis.

3. System Description and Assumptions

- There is provision of single repair facility which becomes available, as and when required, to repair the main unit as well as the supportive units, one at a time.

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- Failures of main unit and electricity unit are self-announcing while that of UPS is not self-announcing and is revealed on the failure of electricity unit.
- Failure times of the main unit, electricity unit and UPS follow exponential distribution with different parameters.
- All the repair times follow general distribution. Also, the distribution of time, for which main unit can be made operative on UPS only (before occurrence of forced reduction in its operational time), is assumed to be general.
- All the faults/failures of the main unit as well as supportive units are repairable. The defective/damaged parts or components, if any, are replaced during the repair and the system works as good as new after each repair.
- There is provision of repairing the UPS during the electricity failure.
- Repair of UPS is considered to be complete only after ensuring its functionality using electricity. In case there is electricity failure before the completion of repair of UPS and the remaining repair work is done during the electricity failure, then after restoring the electricity, testing of UPS is conducted to check its functionality.
- Main unit does not fail during the time when either of the supportive units is under repair or UPS is under testing.
- There does not occur any electricity failure during the repair of main unit.
- Main unit, when operative on UPS only, is safely put to down mode before the completion of backup time of UPS and UPS does not fail during this time.
- Main unit is made operative on electricity, only on the completion of repair/testing of UPS.
- Electricity supply is continuous unless there occurs some fault in it.

4. Nomenclature

λ / λ_1 / λ_2	constant failure rate of the main unit / electricity unit / UPS
g(t) / G(t)	p.d.f. (probability density function) / c.d.f. (cumulative distribution function) of repair time of main unit
$g_1(t)$ / $G_1(t)$	p.d.f. / c.d.f. of repair time of electricity unit
$g_{21}(t)$ / $G_{21}(t)$	p.d.f. / c.d.f. of repair time of UPS
$g_{22}(t)$ / $G_{22}(t)$	p.d.f. / c.d.f. of the time to repair UPS after the resumption of repair
f(t) / F(t)	p.d.f./ c.d.f. of time to test the functionality of UPS
p / q	probability that, after testing, UPS is found / not found to be operable
h(t) / H(t)	p.d.f. /c.d.f. of time for which the main unit can safely operate on UPS only, before the occurrence of forced reduction in its operational time
$q_{ij}(t)$ / $Q_{ij}(t)$	p.d.f. / c.d.f. of first passage time of the system from a regenerative state S_i to a regenerative or failed state S_j, without visiting any other regenerative state in the time interval (0, t]
$q_{ij}^{(k)}(t)$ / $Q_{ij}^{(k)}(t)$	p.d.f. / c.d.f. of first passage time of the system from regenerative state S_i to a regenerative or failed state S_j, visiting state S_k once in the time interval (0, t]
$\phi_i(t)$	c.d.f. of the first passage time of the system from regenerative state S_i to a failed state
$A_i(t)$ / $AE_i(t)$	probability that the system / electricity unit is in operative state at instant t, given that the system entered the regenerative state S_i at t = 0
$D_i(t)$	probability that the system is in down state at instant t, given that the system entered regenerative state S_i at t = 0
$BM_i(t)$ / $BE_i(t)$	probability that repairman is busy in repairing the main unit / electricity unit at instant t, given that the system entered regenerative state S_i at t = 0
$BU_i(t)$ / $BT_i(t)$	probability that repairman is busy in repairing / testing UPS at instant t, given that the system entered regenerative state S_i at t = 0
$FR_i(t)$	the expected number of occurrences of forced reduction in operational time in the time interval (0, t], given the system entered the regenerative state S_i at t = 0
$V_i(t)$	the expected number of visits of repairman in the time interval (0, t], given the system entered the regenerative state S_i at t = 0
$M_i(t)$ / $Y_i(t)$	probability that system / electricity unit initially up in regenerative state S_i, is up at time t, given that the system neither enters any other regenerative state nor returns to S_i through one or more non-regenerative states in the time interval (0, t]
$Z_i(t)$	probability that system initially down in regenerative state S_i is down at time t, given that the system neither enters any other regenerative state nor returns to S_i through one or more non-regenerative states in the time interval (0, t]
$W_i(t)$	probability that the repairman initially busy in regenerative state S_i, is busy at time t, given that the system neither enters any other regenerative state nor returns to S_i through one or more non-regenerative states in the time interval (0, t]
**/*	symbol for Laplace - Stieltjes transform / Laplace transform
® / ©	symbol for Laplace - Stieltjes convolution / Laplace convolution

5. Symbols for the States of the System

M_o / E_o	main unit / electricity unit is in operative state
U_o	UPS is in operative state with the main unit operative on it only
M_d / U_d	main unit / UPS is in down state

M_{fr} / E_{fr} / U_{fr}	main unit / electricity unit / UPS is under repair after being failed
E_{FR} / U_{FR}	repair of electricity unit / UPS continues from previous state
E_{fw} / U_{fw}	Electricity unit / UPS is waiting for repair after being failed
U_s	UPS is in state of standby of electricity unit
U_t	UPS is under testing to check its functionality
U_{ra}	repair of UPS is put in abeyance
U_{rr}	repair of UPS resumes after abeyance /suspension
U_{RR}	repair of UPS continues after being resumed in previous stage

6. Description of Model 1

In light of the symbols and assumptions mentioned in Sections **3** and **5**, possible states of the system for Model 1 are:

$\mathbf{S_0}: (\mathbf{M_o}; \mathbf{E_o}, \mathbf{U_s})$, $\mathbf{S_1}: (\mathbf{M_0}; \mathbf{E_{fr}}, \mathbf{U_o})$, $\mathbf{S_2}: (\mathbf{M_d}; \mathbf{E_{fr}}, \mathbf{U_{fw}})$,
$\mathbf{S_3}: (\mathbf{M_{fr}}; \mathbf{E_o}, \mathbf{U_s})$, $\mathbf{S_4}: (\mathbf{M_d}; \mathbf{E_{FR}}, \mathbf{U_d})$, $\mathbf{S_5}: (\mathbf{M_d}; \mathbf{E_o}, \mathbf{U_{fr}})$,
$\mathbf{S_6}: (\mathbf{M_d}; \mathbf{E_{fw}}, \mathbf{U_{FR}})$, $\mathbf{S_7}: (\mathbf{M_d}; \mathbf{E_{fr}}, \mathbf{U_d})$, $\mathbf{S_8}: (\mathbf{M_d}; \mathbf{E_o}, \mathbf{U_t})$.

The epochs of entry into the states S_0, S_1, S_2, S_3, S_5, S_7, S_8 are regenerative points and thus these are regenerative states. States S_0 and S_1 are operative states of the system. States S_2, S_4, S_5, S_6, S_7 and S_8 are down states of the system and state S_3 represent the system in failed state. Table 1, given below, shows all the possible state transitions for Model 1.

7. Analysis of Model 1

7.1. Transition Times and Probabilities

By the simple arguments of the probability theory, the p.d.f. of transition times i.e. $q_{ij}(t)$ are given by:

$$
\begin{aligned}
q_{01}(t) &= \lambda_1 e^{-(\lambda+\lambda_1+\lambda_2)t},\\
q_{02}(t) &= \lambda_1 e^{-(\lambda+\lambda_1)t}(1-e^{-\lambda_2 t}),\\
q_{03}(t) &= \lambda e^{-(\lambda+\lambda_1)t},\\
q_{10}(t) &= g_1(t)\overline{H(t)},\\
q_{10}^{(4)}(t) &= g_1(t)\ H(t),\\
q_{25}(t) &= g_1(t),\\
q_{30}(t) &= g(t),\\
q_{50}(t) &= e^{-\lambda_1 t} g_{21}(t),\\
q_{57}^{(6)}(t) &= (1-e^{-\lambda_1 t})g_{21}(t)\\
q_{78}(t) &= g_1(t),\\
q_{80}(t) &= p\, e^{-\lambda_1 t} f(t),\\
q_{85}(t) &= q\, e^{-\lambda_1 t} f(t),\\
q_{87}(t) &= \lambda_1 e^{-\lambda_1 t}\ \overline{F(t)}.
\end{aligned}
$$

(1) - (13)

Using the relation $p_{ij} = q_{ij}^*(0) = \int_0^\infty q_{ij}(t)\,dt$, the following steady state transition probabilities, p_{ij}'s, have been obtained from the equations (1) - (13):

$$
\begin{aligned}
p_{01} &= \lambda_1/(\lambda+\lambda_1+\lambda_2),\\
p_{02} &= \lambda_1\lambda_2/(\lambda+\lambda_1)(\lambda+\lambda_1+\lambda_2),\\
p_{03} &= \lambda/(\lambda+\lambda_1),\\
p_{10} &= \int_0^\infty g_1(t)\overline{H(t)}\,dt,\\
p_{10}^{(4)} &= \int_0^\infty g_1(t)\ H(t)\,dt,\\
p_{25} &= p_{30} = 1,\\
p_{50} &= g_{21}^*(\lambda_1),\\
p_{57}^{(6)} &= 1-g_{21}^*(\lambda_1),\\
p_{78} &= 1,\\
p_{80} &= p\, f^*(\lambda_1),\\
p_{85} &= q\, f^*(\lambda_1),\\
p_{87} &= 1-\ f^*(\lambda_1).
\end{aligned}
$$

(14) - (25)

From the equations (14) – (25), we obtain:

$$p_{01}+p_{02}+p_{03} = p_{10}+p_{10}^{(4)} = p_{50}+p_{57}^{(6)} = p_{80}+p_{85}+p_{87} = 1$$

(26)

7.2. Sojourn Times

If T_i is the random variable representing the time of stay in regenerative state S_i, then the mean sojourn time in state S_i, denoted by μ_i, is given by

$$\mu_i = \int_0^\infty P(T_i > t)\,dt$$

(27)

Therefore, we get

$$
\begin{aligned}
\mu_0 &= 1/(\lambda+\lambda_1),\\
\mu_1 &= \int_0^\infty \overline{G_1(t)}\ \overline{H(t)}\,dt,\\
\mu_2 &= \mu_7 = -g_1^{*\prime}(0),\\
\mu_3 &= -g^{*\prime}(0),\\
\mu_5 &= \{1-g_{21}^*(\lambda_1)\}/\lambda_1,\\
\mu_8 &= \{1-\ f^*(\lambda_1)\}/\lambda_1
\end{aligned}
$$

(28) – (33)

The unconditional mean time taken by the system to transit to regenerative state S_j, if the time is counted from the epoch of entry in to the state S_i, is given by

$$m_{ij} = \int_0^\infty t\, q_{ij}(t)\,dt = -q_{ij}^{*\prime}(0)$$

(34)

Thus, we have

$$
\begin{aligned}
m_{01} &= \lambda_1/(\lambda+\lambda_1+\lambda_2)^2,\\
m_{02} &= \lambda_1[\{1/(\lambda+\lambda_1)^2\}-\{1/(\lambda+\lambda_1+\lambda_2)^2\}],\\
m_{03} &= \lambda/(\lambda+\lambda_1)^2,\\
m_{10} &= \int_0^\infty t\, g_1(t)\overline{H(t)}\,dt,\\
m_{10}^{(4)} &= \int_0^\infty t\, g_1(t)\ H(t)\,dt,\\
m_{25} &= m_{78} = -\ g_1^{*\prime}(0),\\
m_{30} &= -g^{*\prime}(0),\\
m_{50} &= -\ g_{21}^{*\prime}(\lambda_1),\\
m_{57}^{(6)} &= g_{21}^{*\prime}(\lambda_1)-g_{21}^{*\prime}(0),\\
m_{80} &= -p\, f^{*\prime}(\lambda_1),\\
m_{85} &= -q\, f^{*\prime}(\lambda_1),\\
m_{87} &= f^{*\prime}(\lambda_1)+\{1-\ f^*(\lambda_1)\}/\lambda_1.
\end{aligned}
$$

(35) – (46)

From the equations (35) – (46), we get the following relations:

$$
\begin{aligned}
m_{01}+m_{02}+m_{03} &= \mu_0,\\
m_{10}+m_{10}^{(4)} &= m_{25} = \mu_2,\\
m_{30} &= \mu_3,\\
m_{50}+m_{57}^{(6)} &= -g_{21}^{*\prime}(0) = K_5(\text{say}),\\
m_{78} &= \mu_7,\\
m_{80}+m_{85}+m_{87} &= \mu_8.
\end{aligned}
$$

(47) – (52)

Table 1: Possible State Transitions (Model 1)

From (Regenerative State)	**S_0**			**S_1**		**S_2**	**S_3**	**S_5**		**S_7**	**S_8**		
To (Regenerative State)	S_1	S_2	S_3	S_0	S_0	S_5	S_0	S_0	S_7	S_8	S_0	S_5	S_7
Via (Non - regenerative State)	--	--	--	--	S_4	--	--	--	S_6	--	--	--	--

7.3. Measures of System Effectiveness (Model 1)

7.3.1. Reliability Function and Mean Time to System Failure

Considering the failed state as absorbing state and using the Laplace - Stieltjes convolution, we have the following recursive relations for $\phi_i(t)$:

$$\phi_0(t) = Q_{01}(t) ® \phi_1(t) + Q_{02}(t) ® \phi_2(t) + Q_{03}(t)$$
$$\phi_1(t) = Q_{10}(t) ® \phi_0(t) + Q_{10}^{(4)}(t) ® \phi_0(t)$$
$$\phi_2(t) = Q_{25}(t) ® \phi_5(t)$$
$$\phi_5(t) = Q_{50}(t) ® \phi_0(t) + Q_{57}^{(6)}(t) ® \phi_7(t)$$
$$\phi_7(t) = Q_{78}(t) ® \phi_8(t)$$
$$\phi_8(t) = Q_{80}(t) ® \phi_0(t) + Q_{85}(t) ® \phi_5(t) + Q_{87}(t) ® \phi_7(t)$$

(53) – (58)

Now, taking Laplace -Stieltjes transform of the above equations and solving them for $\phi_0^{**}(s)$ by determinant method, we have

$$\phi_0^{**}(s) = N_1(s)/D_1(s)$$

(59)

Where,

$$N_1(s) = q_{03}^*(s)\{ 1 - q_{78}^*(s)\ q_{87}^*(s) - q_{57}^{(6)^*}(s)\, q_{78}^*(s)\ q_{85}^*(s)\}$$
$$D_1(s) = \{1 - q_{78}^*(s)\, q_{87}^*(s) - q_{57}^{(6)^*}(s)\, q_{78}^*(s)\, q_{85}^*(s)\}$$
$$\{1 - q_{01}^*(s)\, q_{10}^{(4)^*}(s) - q_{01}^*(s)\, q_{10}^*(s)\} - q_{02}^*(s)\, q_{25}^*(s)$$
$$\{ q_{50}^*(s) - q_{50}^*(s)\, q_{78}^*(s)\, q_{87}^*(s) + q_{57}^{(6)^*}(s)\, q_{78}^*(s)\ q_{80}^*(s)\}$$

(60) – (61)

If $R_i(t)$ denote the reliability function, i.e. $R_i(t)$ is the probability that the system is operative during the time interval (0, t], given that it starts from state S_i at t = 0, then we have

$$R_0^*(s) = \{1 - \phi_0^{**}(s)\}/s$$

(62)

Taking the inverse Laplace transform of the equation (62), we may get the reliability function $R_0(t)$. The mean time to system failure, when the system starts from the state S_0, is given by

$$MTSF^{(1)} = \lim_{s\to 0} [\{1 - \phi_0^{**}(s)\}/s] = N_1^{(1)}/D_1^{(1)}$$

(63)

Where, $N_1^{(1)} = (1-p_{87} - p_{57}^{(6)}\ p_{85})\{\mu_0 + (p_{01} + p_{02})\mu_2\}$
$+ p_{02}\{(1 - p_{87})\ K_5 + p_{57}^{(6)}\ (\mu_7 + \mu_8)\}$

$D_1^{(1)} = p_{03}(1-p_{87} - p_{57}^{(6)}\ p_{85})$

(64) – (65)

7.3.2. Availability Analysis

Using the Laplace convolution and simple arguments of probability theory, the following recursive relations for A_i (t) have been obtained:

$$A_0(t) = M_0(t) + q_{01}(t)©A_0(t) + q_{02}(t)©A_2(t) + q_{03}(t)©A_3(t)$$
$$A_1(t) = M_1(t) + q_{10}(t) © A_0(t) + q_{10}^{(4)}(t) © A_0(t)$$
$$A_2(t) = q_{25}(t) © A_5(t)$$
$$A_3(t) = q_{30}(t) © A_0(t)$$
$$A_5(t) = q_{50}(t) © A_0(t) + q_{57}^{(6)}(t) © A_7(t)$$
$$A_7(t) = q_{78}(t) © A_8(t)$$
$$A_8(t) = q_{80}(t) © A_0(t) + q_{85}(t) © A_5(t) + q_{87}(t) © A_7(t)$$

(66) – (72)

Where,

$M_0(t) = e^{-(\lambda+\lambda_1)t}$, $M_1(t) = \overline{G_1(t)}\ \overline{H(t)}$

(73) – (74)

Taking Laplace transform of the equations (66) – (72) and solving them for $A_0^*(s)$ by determinant method, we get

$$A_0^*(s) = N_2(s)/D_2(s)$$

(75)

Where,

$$N_2(s) = \{1 - q_{78}^*(s)\, q_{87}^*(s) - q_{57}^{(6)^*}(s)\, q_{78}^*(s)\, q_{85}^*(s)\}\{M_0^*(s) + q_{01}^*(s)\, M_1^*(s)\}$$
$$D_2(s) = \{1 - q_{78}^*(s)\, q_{87}^*(s) - q_{57}^{(6)^*}(s)\, q_{78}^*(s)\, q_{85}^*(s)\}$$
$$\{1 - q_{01}^*(s)\, q_{10}^{(4)^*}(s) - q_{01}^*(s)\, q_{10}^*(s) - q_{03}^*(s)q_{30}^*(s)\}$$
$$- q_{02}^*(s)\, q_{25}^*(s)\, \{ q_{50}^*(s) - q_{50}^*(s)\, q_{78}^*(s)\, q_{87}^*(s)$$
$$+ q_{57}^{(6)^*}(s)\, q_{78}^*(s)\ q_{80}^*(s)\}$$

(76) – (77)

In steady state, the fraction of time for which the system is available, is given by

$$A_0^{(1)} = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} s\, A_0^*(s) = N_2^{(1)}/D_2^{(1)}$$

(78)

Where, $N_2^{(1)} = (1-p_{87} - p_{57}^{(6)}\ p_{85})(\mu_0 + p_{01}\ \mu_1)$

$D_2^{(1)} = (1-p_{87} - p_{57}^{(6)}\ p_{85})\{\mu_0 + (p_{01} + p_{02})\mu_2 + p_{03}\ \mu_3\}$
$+ p_{02}\{(1 - p_{87})K_5 + p_{57}^{(6)}(\mu_7 + \mu_8)\}$

(79) – (80)

7.3.3. Expected Downtime of the System

Applying the arguments of probability theory, we get the recursive relations for D_i (t) as given below:

$$D_0(t) = q_{01}(t) © D_1(t) + q_{02}(t) © D_2(t) + q_{03}(t) © D_3(t)$$
$$D_1(t) = q_{10}(t) © D_0(t) + q_{10}^{(4)}(t) © D_0(t)$$
$$D_2(t) = Z_2(t) + q_{25}(t) © D_5(t)$$
$$D_3(t) = q_{30}(t) © D_0(t)$$
$$D_5(t) = Z_5(t) + q_{50}(t) © D_0(t) + q_{57}^{(6)}(t) © D_7(t)$$
$$D_7(t) = Z_7(t) + q_{78}(t) © D_8(t)$$
$$D_8(t) = Z_8(t) + q_{80}(t)©D_0(t) + q_{85}(t)©D_5(t) + q_{87}(t)©D_7(t)$$

Where,

$Z_2(t) = Z_7(t) = \overline{G_1(t)}$, $Z_5(t) = \overline{G_{21}(t)}$, $Z_8(t) = e^{-\lambda_1 t}\ \overline{F(t)}$

(81) – (90)

Taking Laplace transform of the equations (81) – (87) and solving them for $D_0^*(s)$, we get

$$D_0^*(s) = N_3(s)/D_2(s)$$

Where,

$$N_3(s) = \{1 - q_{78}^*(s)\, q_{87}^*(s) - q_{57}^{(6)^*}(s)\, q_{78}^*(s)\, q_{85}^*(s)\}$$
$$q_{02}^*(s)\, Z_2^*(s) + q_{02}^*(s)q_{25}^*(s)[\{1 - q_{78}^*(s)\, q_{87}^*(s)\}Z_5^*(s)$$
$$+ q_{57}^{(6)^*}(s)\, Z_7^*(s) + q_{57}^{(6)^*} q_{78}^*(s)\, Z_8^*(s)]$$

(91) – (92)

and $D_2(s)$ has been already specified in equation (77).

In steady state, the fraction of time for which the system is in down state, is given by

$$D_0^{(1)} = \lim_{t\to\infty} D_0(t) = \lim_{s\to 0} s\ D_0^*(s) = N_3^{(1)}/D_2^{(1)}$$

Where,

$$N_3^{(1)} = (1-p_{87} - p_{57}^{(6)}\ p_{85})\ p_{02}\ \mu_2 + p_{02}\ \{(1-p_{87})\ K_5 + p_{57}^{(6)}(\mu_7+\mu_8)\}$$

(93) – (94)

and $D_2^{(1)}$ has been already specified in equation (80).

7.3.4. Analysis of Busy Period of Repairman for Repairing the Main Unit

For analyzing the busy period of repairman for repairing the main unit, we first derive the following recursive relations for $BM_i(t)$ by using the probabilistic arguments and Laplace convolution:

$$BM_0(t) = q_{01}(t)©BM_0(t) + q_{02}(t)©BM_2(t) + q_{03}(t)©BM_3(t)$$
$$BM_1(t) = q_{10}(t) © BM_0(t) + q_{10}^{(4)}(t) © BM_0(t)$$
$$BM_2(t) = q_{25}(t) © BM_5(t)$$
$$BM_3(t) = W_3(t) + q_{30}(t) © BM_0(t)$$
$$BM_5(t) = q_{50}(t) © BM_0(t) + q_{57}^{(6)}(t) © BM_7(t)$$
$$BM_7(t) = q_{78}(t) © BM_8(t)$$
$$BM_8(t) = q_{80}(t)©BM_0(t) + q_{85}(t)©BM_5(t) + q_{87}(t)©BM_7(t)$$

Where, $W_3(t) = \overline{G(t)}$

(95) – (102)

Now, taking Laplace transform of the equations (95) – (101) and solving them for $BM_0^*(s)$, we get

$$BM_0^*(s) = N_4(s)/D_2(s)$$

Where, $N_4(s) = \{1 - q_{78}^*(s)\ q_{87}^*(s) - q_{57}^{(6)*}(s)\ q_{78}^*(s)\ q_{85}^*(s)\}\ q_{03}^*(s)\ W_3^*(s)$

(103) – (104)

In steady state, the fraction of time for which the main unit is under repair, is given by

$$BM_0^{(1)} = \lim_{t\to\infty} BM_0(t) = \lim_{s\to 0} s\ BM_0^*(s) = N_4^{(1)}/D_2^{(1)}$$

Where, $N_4^{(1)} = (1-p_{87} - p_{57}^{(6)}\ p_{85})p_{03}\ \mu_3$

(105) – (106)

7.3.5. Analysis of Busy Period of Repairman for Repairing the Electricity Unit

Proceeding as in Section **7.3.4**, the following relations have been obtained for $BE_i(t)$:

$$BE_0(t) = q_{01}(t)©BM_0(t) + q_{02}(t)©BE_2(t) + q_{03}(t)©BE_3(t)$$
$$BE_1(t) = W_1(t) + q_{10}(t) © BE_0(t) + q_{10}^{(4)}(t) © BE_0(t)$$
$$BE_2(t) = W_2(t) + q_{25}(t) © BE_5(t)$$
$$BE_3(t) = q_{30}(t) © BE_0(t)$$
$$BE_5(t) = q_{50}(t) © BE_0(t) + q_{57}^{(6)}(t) © BE_7(t)$$
$$BE_7(t) = W_7(t) + q_{78}(t) © BE_8(t)$$
$$BE_8(t) = q_{80}(t)©BE_0(t) + q_{85}(t)©BE_5(t) + q_{87}(t)©BE_7(t)$$

Where, $W_1(t) = W_2(t) = W_7(t) = \overline{G_1(t)}$

(107) – (114)

Taking Laplace transform of the equations (107) – (113) and solving them for $BE_0^*(s)$, we get

$$BE_0^*(s) = N_5(s)/D_2(s)$$

Where, $N_5(s) = \{1 - q_{78}^*(s)q_{87}^*(s) - q_{57}^{(6)*}(s)q_{78}^*(s)q_{85}^*(s)\}\ \{q_{01}^*(s)W_1^*(s) + q_{02}^*(s)W_2^*(s)\} + q_{02}^*(s)q_{25}^*(s)q_{57}^{(6)*}(s)W_7^*(s)$

(115) – (116)

In steady state, the fraction of time for which the electricity unit is under repair, is given by

$$BE_0^{(1)} = \lim_{t\to\infty} BE_0(t) = \lim_{s\to 0} s\ BE_0^*(s) = N_5^{(1)}/D_2^{(1)}$$

Where,

$$N_5^{(1)} = (1-p_{87} - p_{57}^{(6)}\ p_{85})(p_{01}+p_{02})\ \mu_2 + p_{02}\ p_{57}^{(6)}\mu_7$$

(117) – (118)

7.3.6. Analysis of Busy Period of Repairman for Repairing the UPS

Applying the arguments as in Section **7.3.4**, $BU_i(t)$ can be shown to satisfy the following recursive relations**:**

$$BU_0(t) = q_{01}(t)©BU_0(t) + q_{02}(t)©BU_2(t) + q_{03}(t)©BU_3(t)$$
$$BU_1(t) = q_{10}(t) © BU_0(t) + q_{10}^{(4)}(t) © BU_0(t)$$
$$BU_2(t) = q_{25}(t) © BU_5(t)$$
$$BU_3(t) = q_{30}(t) © BU_0(t)$$
$$BU_5(t) = W_5(t) + q_{50}(t) © BU_0(t) + q_{57}^{(6)}(t) © BU_7(t)$$
$$BU_7(t) = q_{78}(t) © BU_8(t)$$
$$BU_8(t) = q_{80}(t)©BU_0(t) + q_{85}(t)©BU_5(t) + q_{87}(t)©BU_7(t)$$

Where, $W_5(t) = \overline{G_{21}(t)}$

(119) – (126)

Taking Laplace transform of the equations (119) – (125) and solving them for $BU_0^*(s)$, we get

$$BU_0^*(s) = N_6(s)/D_2(s)$$

Where, $N_6(s) = \{1 - q_{78}^*(s)\ q_{87}^*(s)\}\ q_{02}^*(s)\ q_{25}^*(s)\ W_5^*(s)$

(127) – (128)

In steady state, the fraction of time for which UPS is under repair, is given by

$$BU_0^{(1)} = \lim_{t\to\infty} BU_0(t) = \lim_{s\to 0} s\ BU_0^*(s) = N_6^{(1)}/D_2^{(1)}$$

Where, $N_6^{(1)} = p_{02}(1-p_{87})K_5$

(129) – (130)

7.3.7. Analysis of Busy Period of Repairman for Testing of the UPS

Applying the probabilistic arguments as above, we get the following recursive relations for $BT_i(t)$:

$$BT_0(t) = q_{01}(t)©BT_0(t) + q_{02}(t)©BT_2(t) + q_{03}(t)©BT_3(t)$$
$$BT_1(t) = q_{10}(t) © BT_0(t) + q_{10}^{(4)}(t) © BT_0(t)$$
$$BT_2(t) = q_{25}(t) © BT_5(t)$$
$$BT_3(t) = q_{30}(t) © BT_0(t)$$
$$BT_5(t) = q_{50}(t) © BT_0(t) + q_{57}^{(6)}(t) © BT_7(t)$$
$$BT_7(t) = q_{78}(t) © BT_8(t)$$
$$BT_8(t) = W_8(t) + q_{80}(t) © BT_0(t) + q_{85}(t) © BT_5(t) + q_{87}(t) © BT_7(t)$$

Where, $W_8(t) = e^{-\lambda_1 t}\ \overline{F(t)}$

(131) – (138)

Taking Laplace transform of the equations (131) – (137) and solving them for $BT_0^*(s)$, we get

$$BT_0^*(s) = N_7(s)/D_2(s)$$

Where, $N_7(s) = q_{02}^*(s)\ q_{25}^*(s)q_{57}^{(6)*}(s)\ q_{78}^*(s)W_8^*(s)$

(139) – (140)

In steady state, the fraction of time for which the UPS is under testing to check its functionality, is given by

$$BT_0^{(1)} = \lim_{t\to\infty} BT_0(t) = \lim_{s\to 0} s\ BT_0^*(s) = N_7^{(1)}/D_2^{(1)}$$

Where, $N_7^{(1)} = p_{02}p_{57}^{(6)}\mu_8$

(141) – (142)

7.3.8. Expected Number of Visits by Repairman

Using Laplace - Stieltjes convolution and simple probabilistic arguments, the following recursive relations for $V_i(t)$ have been obtained:

$$V_0(t) = Q_{01}(t) ® \{1 + V_1(t)\} + Q_{02}(t) ® \{1 + V_2(t)\} + Q_{03}(t) ® \{1 + V_3(t)\}$$
$$V_1(t) = Q_{10}(t) ® V_0(t) + Q_{10}^{(4)}(t) ® V_0(t)$$
$$V_2(t) = Q_{25}(t) ® V_5(t)$$
$$V_3(t) = Q_{30}(t) ® V_0(t)$$
$$V_5(t) = Q_{50}(t) ® V_0(t) + Q_{57}^{(6)}(t) ® V_7(t)$$

$V_7(t) = Q_{78}(t) ® V_8(t)$
$V_8(t) = Q_{80}(t) ® V_0(t) + Q_{85}(t) ® V_5(t) + Q_{87}(t) ® V_7(t)$

(143) – (149)

Taking Laplace - Stieltjes transform of the above equations and solving them for $V_0^{**}(s)$, we get

$$V_0^{**}(s) = N_8(s)/D_2(s)$$

Where,

$$N_8(s) = \{ 1 - q_{78}^*(s)q_{87}^*(s) - q_{57}^{(6)^*}(s)\, q_{78}^*(s)\, q_{85}^*(s)\}\{q_{01}^*(s) + q_{02}^*(s) + q_{03}^*(s)\}$$

(150) – (151)

In steady state, the expected number of visits of repairman per unit time is, given by

$$V_0^{(1)} = \lim_{t\to\infty} [V_0(t)/t] = \lim_{s\to 0} s\, V_0^{**}(s) = N_8^{(1)}/D_2^{(1)}$$

Where, $N_8^{(1)} = 1 - p_{87} - p_{57}^{(6)}\ p_{85}$

(152) – (153)

7.3.9. Expected Number of Occurrences of Forced Reduction in Operational Time

The following relations have been obtained by proceeding in the similar fashion as above:

$FR_0(t) = Q_{01}(t)®FR_1(t) + Q_{02}(t)®FR_2(t) + Q_{03}(t)®FR_3(t)$
$FR_1(t) = Q_{10}(t) ® FR_0(t) + Q_{10}^{(4)}(t) ® \{1 + FR_0(t)\}$
$FR_2(t) = Q_{25}(t) ® FR_5(t)$
$FR_3(t) = Q_{30}(t) ® FR_0(t)$
$FR_5(t) = Q_{50}(t) ® FR_0(t) + Q_{57}^{(6)}(t) ® FR_7(t)$
$FR_7(t) = Q_{78}(t) ® FR_8(t)$
$FR_8(t) = Q_{80}(t)®FR_0(t) + Q_{85}(t)®FR_5(t) + Q_{87}(t)®FR_7(t)$

(154) – (160)

Taking Laplace - Stieltjes transform of the above equations and solving them for $FR_0^{**}(s)$ we get

$$FR_0^{**}(s) = N_9(s)/D_2(s)$$

Where, $N_9(s)$ is given by

$$N_9(s) = \{ 1 - q_{78}^*(s)q_{87}^*(s) - q_{57}^{(6)^*}(s)q_{78}^*(s)q_{85}^*(s)\}\, q_{01}^*(s)q_{10}^{(4)^*}(s)$$

(161) – (162)

In steady state, the expected number of occurrences, per unit time, of forced reduction in operational time, is given by

$$FR_0^{(1)} = \lim_{t\to\infty} [FR_0(t)/t] = \lim_{s\to 0} s\, FR_0^{**}(s) = N_9^{(1)}/D_2^{(1)}$$

Where, $N_9^{(1)} = \left(1 - p_{87} - p_{57}^{(6)}\ p_{85}\right) p_{01}\, p_{10}^{(4)}$

(163) – (164)

7.3.10. Expected Uptime of Electricity Unit

Using the arguments as in Section **7.3.2**, $AE_i(t)$ can be shown to satisfy the recursive relations given below**:**

$AE_0(t) = Y_0(t) + q_{01}(t) © AE_0(t) + q_{02}(t) © AE_2(t) + q_{03}(t) © AE_3(t)$
$AE_1(t) = q_{10}(t) © AE_0(t) + q_{10}^{(4)}(t) © AE_0(t)$
$AE_2(t) = q_{25}(t) © AE_5(t)$
$AE_3(t) = Y_3(t) + q_{30}(t) © AE_0(t)$
$AE_5(t) = Y_5(t) + q_{50}(t) © AE_0(t) + q_{57}^{(6)}(t) © AE_7(t)$
$AE_7(t) = q_{78}(t) © AE_8(t)$
$AE_8(t) = Y_8(t) + q_{80}(t) © AE_0(t) + q_{85}(t) © AE_5(t) + q_{87}(t) © AE_7(t)$

Where,

$Y_0(t) = e^{-(\lambda+\lambda_1)t}$, $Y_3(t) = \overline{G(t)}$, $Y_5(t) = e^{-\lambda_1 t}\,\overline{G_{21}(t)}$,
$Y_8(t) = e^{-\lambda_1 t}\ \overline{F(t)}$

(165) – (175)

Taking Laplace transform of the equations (165) – (171) and solving them for $AE_0^*(s)$ by determinant method, we get

$$AE_0^*(s) = N_{10}(s)/D_2(s)$$

Where,

$$N_{10}(s) = \{ 1 - q_{78}^*(s)\, q_{87}^*(s) - q_{57}^{(6)^*}(s)\, q_{78}^*(s)\ q_{85}^*(s)\}\, \{Y_0^*(s) + q_{03}^*(s)\, Y_3^*(s)\} + q_{02}^*(s)\ q_{25}^*(s)[\{1 - q_{78}^*(s)\, q_{87}^*(s)\}Y_5^* + q_{57}^{(6)^*} q_{78}^*(s)\, Y_8^*(s)]$$

(176) – (177)

In steady state, the fraction of time for which the electricity unit is in operative state, is given by

$$AE_0^{(1)} = \lim_{t\to\infty} AE_0(t) = \lim_{s\to 0} s\, AE_0^*(s) = N_{10}^{(1)}/D_2^{(1)}$$

Where, $N_{10}^{(1)} = (1 - p_{87} - p_{57}^{(6)}\ p_{85})\,\{\mu_0 + p_{03}\,\mu_3\} + p_{02}\{(1 - p_{87})\mu_5 + p_{57}^{(6)}\ \mu_8\}$

(178) – (179)

8. Description of Model 2

The possible states of the system for Model 2 are:

$\mathbf{S_0}: (\mathbf{M_o}; \mathbf{E_o}, \mathbf{U_s})$, $\mathbf{S_1}: (\mathbf{M_o}; \mathbf{E_{fr}}, \mathbf{U_o})$, $\mathbf{S_2}: (\mathbf{M_d}; \mathbf{E_{fr}}, \mathbf{U_{fw}})$,
$\mathbf{S_3}: (\mathbf{M_{fr}}; \mathbf{E_o}, \mathbf{U_s})$, $\mathbf{S_4}: (\mathbf{M_d}; \mathbf{E_{FR}}, \mathbf{U_d})$, $\mathbf{S_5}: (\mathbf{M_d}; \mathbf{E_o}, \mathbf{U_{fr}})$,
$\mathbf{S_6}: (\mathbf{M_d}; \mathbf{E_{fr}}, \mathbf{U_{ra}})$, $\mathbf{S_7}: (\mathbf{M_d}; \mathbf{E_o}, \mathbf{U_{rr}})$, $\mathbf{S_8}: (\mathbf{M_d}; \mathbf{E_{fw}}, \mathbf{U_{RR}})$,
$\mathbf{S_9}: (\mathbf{M_d}; \mathbf{E_{fr}}, \mathbf{U_d})$, $\mathbf{S_{10}}: (\mathbf{M_d}; \mathbf{E_o}, \mathbf{U_t})$.

The epochs of entry into the states S_0, S_1, S_2, S_3, S_5, S_6, S_7, S_9 and S_{10} are the regenerative points and thus these are regenerative states. States S_0 and S_1 are operative states of the system. States S_2, S_4, S_5, S_6, S_7, S_8, S_9 and S_{10} are down states of the system and state S_3 represents the system in failed state. The possible state transitions are as shown below in Table 2.

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9. Analysis of Model 2

9.1. Transition Times and Probabilities

Using the arguments as used in Model 1, the p.d.f. of transition times and transition probabilities are given by:

$q_{56}(t) = \lambda_1 e^{-\lambda_1 t}\ \overline{G_{21}(t)}$
$q_{67}(t) = g_1(t)$,
$q_{70}(t) = e^{-\lambda_1 t}\, g_{22}(t)$,
$q_{79}^{(8)}(t) = \left(1 - e^{-\lambda_1 t}\right) g_{22}(t)$,
$q_{9,10}(t) = g_1(t)$,
$q_{10,0}(t) = p\, e^{-\lambda_1 t}\, f(t)$,
$q_{10,7}(t) = q\, e^{-\lambda_1 t}\, f(t)$,
$q_{10,9}(t) = \lambda_1 e^{-\lambda_1 t}\ \overline{F(t)}$

(180) – (187)

The values of $q_{01}(t), q_{02}(t), q_{03}(t), q_{10}(t), q_{10}^{(4)}(t), q_{25}(t), q_{30}(t)$ and $q_{50}(t)$ for this model are same as obtained for Model 1 in equations (1) – (8). Further, the transition probabilities p_{ij}'s, are given by

$p_{56} = 1 - g_{21}^*(\lambda_1)$,
$p_{67} = p_{9,10} = 1$,
$p_{70} = g_{22}^*(\lambda_1)$,
$p_{79}^{(8)} = 1 - g_{22}^*(\lambda_1)$,
$p_{10,0} = p\, f^*(\lambda_1)$,
$p_{10,7} = q\, f^*(\lambda_1)$,
$p_{10,9} = 1 - f^*(\lambda_1)$.

(188) – (194)

Other probabilities are same as that for Model 1 and given by equations (14) – (20).

Table 2: Possible State Transitions (Model 2)

From (Regenerative State)	S_0			S_1		S_2	S_3	S_5		S_6	S_7		S_9	S_{10}		
To (Regenerative State)	S_1	S_2	S_3	S_0	S_0	S_5	S_0	S_0	S_6	S_7	S_0	S_9	S_{10}	S_0	S_7	S_9
Via (Non - regenerative State)	--	--	--	--	S_4	--	--	--	--	--	--	S_8	--	--	--	--

From the above equations, we have obtained the following results:

$$p_{01} + p_{02} + p_{03} = p_{10} + p_{10}^{(4)} = p_{50} + p_{56} = p_{70} + p_{79}^{(8)} = p_{10,0} + p_{10,7} + p_{10,9} = 1 \quad (195)$$

9.2. Sojourn Times

Proceeding in the similar fashion as in Model 1, we have

$$\mu_6 = \mu_9 = -g_1^{*'}(0),$$
$$\mu_7 = \{1 - g_{22}^{*}(\lambda_1)\}/\lambda_1,$$
$$\mu_{10} = \{1 - f^{*}(\lambda_1)\}/\lambda_1$$
$$m_{56} = g_{21}^{*'}(\lambda_1) + \{1 - g_{21}^{*}(\lambda_1)\}/\lambda_1,$$
$$m_{67} = m_{9,10} = -g_1^{*'}(0),$$
$$m_{70} = -g_{22}^{*'}(\lambda_1),$$
$$m_{79}^{(8)} = g_{22}^{*'}(\lambda_1) - g_{22}^{*'}(0),$$
$$m_{10,0} = -p\, f^{*'}(\lambda_1),$$
$$m_{10,7} = -q\, f^{*'}(\lambda_1),$$
$$m_{10,9} = f^{*'}(\lambda_1) + \{1 - f^{*}(\lambda_1)\}/\lambda_1. \quad (196) - (205)$$

The values of $\mu_0, \mu_1, \mu_2, \mu_3, \mu_5, m_{01}, m_{02}, m_{03}, m_{10}, m_{10}^{(4)}, m_{25}, m_{30}$, and m_{50} obtained for Model 1, given by equations (28) – (32) and (35) – (42), work for Model 2 also.

Further, in addition to equations (47) – (49), the following relations have been obtained from the above expressions:

$$m_{50} + m_{56} = \mu_5,$$
$$m_{67} = \mu_6,$$
$$m_{70} + m_{79}^{(8)} = -g_{22}^{*'}(0) = K_7(\text{say}),$$
$$m_{9,10} = \mu_9,$$
$$m_{10,0} + m_{10,7} + m_{10,9} = \mu_{10}. \quad (206) - (210)$$

9.3. Measures of System Effectiveness (Model 2)

Proceeding as in Model 1, the following measures of system effectiveness have been obtained for Model 2:

- Mean time to system failure ($\mathbf{MTSF}^{(2)}$) $= \mathbf{N_1^{(2)}}/\mathbf{D_1^{(2)}}$
- Steady-state availability ($\mathbf{A_0^{(2)}}$) $= \mathbf{N_2^{(2)}}/\mathbf{D_2^{(2)}}$
- Expected fraction of time for which the system is in down state ($\mathbf{D_0^{(2)}}$) $= \mathbf{N_3^{(2)}}/\mathbf{D_2^{(2)}}$
- Expected fraction of time for which the main unit is under repair ($\mathbf{BM_0^{(2)}}$) $= \mathbf{N_4^{(2)}}/\mathbf{D_2^{(2)}}$
- Expected fraction of time for which the electricity unit is under repair ($\mathbf{BE_0^{(2)}}$) $= \mathbf{N_5^{(2)}}/\mathbf{D_2^{(2)}}$
- Expected fraction of time for which the UPS is under repair ($\mathbf{BU_0^{(2)}}$) $= \mathbf{N_6^{(2)}}/\mathbf{D_2^{(2)}}$
- Expected fraction of time for which the UPS is under testing to check its functionality ($\mathbf{BT_0^{(2)}}$) $= \mathbf{N_7^{(2)}}/\mathbf{D_2^{(2)}}$
- Expected number of visits, per unit time, of repairman ($\mathbf{V_0^{(2)}}$) $= \mathbf{N_8^{(2)}}/\mathbf{D_2^{(2)}}$
- Expected number of occurrences, per unit time, of forced reduction in operational time ($\mathbf{FR_0^{(2)}}$) $= \mathbf{N_9^{(2)}}/\mathbf{D_2^{(2)}}$
- Expected fraction of time for which the electricity unit is in operative state ($\mathbf{AE_0^{(2)}}$) $= \mathbf{N_{10}^{(2)}}/\mathbf{D_2^{(2)}}$

(211) – (220)

Where,

$$\mathbf{N_1^{(2)}} = (1 - p_{10,9} - p_{79}^{(8)}\ p_{10,7})\{\mu_0 + (p_{01} + p_{02})\mu_2 + p_{02}(\mu_5 + p_{56}\,\mu_6)\} + p_{02}\,p_{56}\{(1 - p_{10,9})K_7 + p_{79}^{(8)}(\mu_9 + \mu_{10})\},$$
$$\mathbf{D_1^{(2)}} = p_{03}(1 - p_{10,9} - p_{79}^{(8)}\ p_{10,7}),$$
$$\mathbf{N_2^{(2)}} = (1 - p_{10,9} - p_{79}^{(8)}\ p_{10,7})(\mu_0 + p_{01}\,\mu_1),$$
$$\mathbf{D_2^{(2)}} = (1 - p_{10,9} - p_{79}^{(8)}\ p_{10,7})\{\mu_0 + (p_{01} + p_{02})\mu_2 + p_{03}\,\mu_3 + p_{02}(\mu_5 + p_{56}\,\mu_6)\} + p_{02}\,p_{56}\{(1 - p_{10,9})K_7 + p_{79}^{(8)}(\mu_9 + \mu_{10})\},$$
$$\mathbf{N_3^{(2)}} = p_{02}\,(1 - p_{10,9} - p_{79}^{(8)}\ p_{10,7})(\mu_2 + \mu_5 + p_{56}\,\mu_6) + p_{02}\,p_{56}\{(1 - p_{10,9})K_7 + p_{79}^{(8)}(\mu_9 + \mu_{10})\}$$
$$\mathbf{N_4^{(2)}} = (1 - p_{10,9} - p_{79}^{(8)}\ p_{10,7})p_{03}\mu_3,$$
$$\mathbf{N_5^{(2)}} = (1 - p_{10,9} - p_{79}^{(8)}\ p_{10,7})\{(p_{01} + p_{02})\,\mu_2 + p_{02}\,p_{56}\,\mu_6\} + p_{02}\,p_{56}\,p_{79}^{(8)}\mu_9,$$
$$\mathbf{N_6^{(2)}} = (1 - p_{10,9} - p_{79}^{(8)}\ p_{10,7})p_{02}\,\mu_5 + p_{02}\,p_{56}(1 - p_{10,9})K_7,$$
$$\mathbf{N_7^{(2)}} = p_{02}p_{56}\,p_{79}^{(8)}\mu_{10},$$
$$\mathbf{N_8^{(2)}} = 1 - p_{10,9} - p_{79}^{(8)}\ p_{10,7},$$
$$\mathbf{N_9^{(2)}} = (1 - p_{10,9} - p_{79}^{(8)}\ p_{10,7})\ p_{01}\,p_{10}^{(4)},$$
$$\mathbf{N_{10}^{(2)}} = (1 - p_{10,9} - p_{79}^{(8)}\,p_{10,7})\{\mu_0 + p_{03}\,\mu_3 + p_{02}\mu_5\} + p_{02}p_{56}\{(1 - p_{10,9})\mu_7 + p_{79}^{(8)}\mu_{10}\}. \quad (221) - (232)$$

10. Profit Equation

Profit per hour generated by the system is given by

Profit per hour
= **revenue per hour**
− **expenditure per hour on repair of main unit**
− **expenditure per hour on repair of electricity unit**
− **expenditure per hour on repair of UPS**
− **expenditure per hour on testing of UPS**
− **expenditure per hour on visiting charges of repairman**
− **expenditure per hour on electricity consumption**

Therefore, in steady state, the expected profit per hour (P_k) generated by the system for k^{th} model (k= 1, 2), is given by the following equation:

$$\mathbf{P_k = (C_0)\, A_0^{(k)} - \{(C_1)\, BM_0^{(k)} + (C_2)\, BE_0^{(k)} + (C_3)\, BU_0^{(k)} + (C_4)\, BT_0^{(k)} + (C_5)\, V_0^{(k)} + (C_6)\,(L)\, AE_0^{(k)}\}} \quad (233)$$

Where,

C_0 = revenue per hour for which the system is operative
C_1 = cost per hour of repairing the main unit
C_2 = cost per hour of repairing the electricity unit

C_3 = cost per hour of repairing UPS
C_4 = cost per hour of testing of UPS
C_5 = cost per visit of repairman
C_6 = cost per KWH of electricity consumption
L= power of the system in KWs

11. Comparative Analysis of the Two Models

In this section, we shall carry out the comparative analysis of the studied models to find out which and when one of the considered repair patterns is better than the other. For this purpose, the following particular case has been considered:

Table 3: Assumed Distributions & Numerical Values

Probability Density functions		Revenue / Costs (INR)		Rates / Parameters	
g(t)	$\alpha e^{-\alpha t}$	C_0	700	**α**	0.25
$g_1(t)$	$\beta e^{-\beta t}$	C_1	800	**β**	0.7
h(t)	$\gamma e^{-\gamma t}$	C_2	500	**γ**	3
$g_{21}(t)$	$\delta_1 e^{-\delta_1 t}$	C_3	400	δ_1	0.8
$g_{22}(t)$	$\delta_2 e^{-\delta_2 t}$	C_4	200	δ_2	4
f(t)	$\delta e^{-\delta t}$	C_5	250	δ	5
		C_6	7	**p**	0.8
				λ	.0003
				λ_1	.0002
				λ_2	.0004
				L	4

The comparative study of the two models with reference to performability measures and the profit has been done as given below:

11.1. Comparison with Reference to MTSF

Taking the numerical values as assumed in Table 3 and using the equations (63) and (211), we get the following expression for $MTSF^{(1)} - MTSF^{(2)}$ in terms of λ:

$$MTSF^{(1)} - MTSF^{(2)} = \frac{2.89941 \times 10^{-11}}{\lambda\,(0.0006 + \lambda)} \quad (234)$$

Equation (234) further implies that

$$\mathbf{MTSF^{(1)} > MTSF^{(2)}, \text{ for all } \lambda > 0} \quad (235)$$

11.2. Comparison with Reference to Availability

Using the equations (78) and (212) and taking the assumed values of the parameters and rates, the following expression of $A_0^{(2)} - A_0^{(1)}$ in terms of λ_1 has been obtained:

$$A_0^{(2)} - A_0^{(1)} = Y_1/Y_2$$

Where,

$$Y_1 = (9.94501\,x\,10^{-7} + 0.001422\,\lambda_1 + 0.002418\,\lambda_1^2 + 0.0008506\,\lambda_1^3 + 0.00008133\,\lambda_1^4)\lambda_1^2$$

$$Y_2 = 9.6251\,x\,10^{-7} + 0.002756\,\lambda_1 + 1.98089\,\lambda_1^2 + 10.4576\,\lambda_1^3 + 21.1173\,\lambda_1^4 + 19.8423\,\lambda_1^5 + 8.20432\,\lambda_1^6 + \lambda_1^7 \quad (236)-(238)$$

It follows from the above equations that

$$\mathbf{A_0^{(2)} > A_0^{(1)}, \text{ for all } \lambda_1 > 0} \quad (239)$$

11.3. Comparison with Reference to Expected Number of Occurrences of Forced Reduction in Operational Time

Taking the numerical values as per Table 3 and using the equations (163) and (219), the following numerical outcomes have been obtained:

(a) Expressing $FR_0^{(2)} - FR_0^{(1)}$ as a function of γ, we have

$$FR_0^{(2)} - FR_0^{(1)} = \frac{3.56811 \times 10^{-12}\,\gamma}{0.7 + \gamma}$$

Thus, we have

$$\mathbf{FR_0^{(2)} > FR_0^{(1)}, \text{ for all } \gamma > 0} \quad (240)-(241)$$

(b) Writing $FR_0^{(2)} - FR_0^{(1)}$ in terms of β, we get

$$FR_0^{(2)} - FR_0^{(1)} = \frac{(2.95388 \times 10^{-16} + 1.071 \times 10^{-11}\,\beta)\beta}{1.19713 \times 10^{-7} + 0.0011986\,\beta + 3.0004\,\beta^2 + \beta^3}$$

which further implies

$$\mathbf{FR_0^{(2)} > FR_0^{(1)}, \text{ for all } \beta > 0} \quad (242)-(243)$$

11.4. Comparison with Reference to Profit

Using the equation (233) and taking the assumed values of various parameters, rates and costs, we get the following results:

(a) Expressing $P_2 - P_1$ as a function of δ_2, we have

$$P_2 - P_1 = \frac{-4.7452 \times 10^{-9} - 0.0000243489\,\delta_2 + 0.0000397302\,\delta_2^2}{4.43625 \times 10^{-12} + 0.000160022\,\delta_2 + \delta_2^2} \quad (244)$$

It follows from the equation (244) that

$$P_2 \geq P_1, \text{ if } \delta_2 \geq 0.613051 \quad (245)$$

Also, it is obvious that $\delta_2 > \delta_1$ and $\delta_1 = .8$ in the case under consideration, therefore we must have, $\delta_2 > .8$
Therefore, it follows from the result (245) that

$$\mathbf{P_2 > P_1, \text{ for all possible values of } \delta_2} \quad (246)$$

(b) Writing the profit difference in terms of C_1, we get

$$P_2 - P_1 = 0.0000336722 - 3.85356 \times 10^{-11}\,C_1$$

Therefore, we have

$$\mathbf{P_2 \geq P_1, \text{ if } C_1 \leq 873794} \quad (247)-(248)$$

(c) Expressing $P_2 - P_1$ in terms of γ, we obtain

$$P_2 - P_1 = \frac{0.000023551 + 0.0000336407\,\gamma}{0.7 + \gamma}$$

Above equation further implies that

$$\mathbf{P_2 > P_1, \text{ for all } \gamma > 0} \quad (249)-(250)$$

(d) Writing the profit difference in terms of C_6, we get

$$P_2 - P_1 = 0.0000344175 - 1.10879 \times 10^{-7}\,C_6$$

Thus, we have

$$\mathbf{P_2 \geq P_1, \text{ if } C_6 \leq 310.406} \quad (251)-(252)$$

12. Discussion

From the numerical outcomes of above section, the following points have been observed:

- For all possible values of failure rate of main unit (**λ**), it takes the system longer to fail in case of FCFS repair pattern (Model 1) as compared to the case wherein the electricity unit is given preference in repair over the UPS (Model 2).
- Giving priority in repair to the electricity unit results in higher availability of the system as compared to the FCFS repair pattern, for all possible values of failure rate of electricity unit (**λ_1**).
- Number of occurrences of forced reduction in operational time is less in case of Model 1 as compared to Model 2, for all possible values of **γ** and repair rate of electricity unit **(β)**.
- The system has higher profit in case of Model 2 as compared to Model 1,
  - for all possible values of repair rate of UPS after resumption of repair **(δ_2)**
  - if cost per hour of repairing the main unit **(C_1)** is less than 873.794 x 10^3 (INR).
  - for all possible values of **γ.**
  - if cost per KWH of electricity consumption **(C_6)** is less than 310.406 (INR).

13. Conclusions

In the present study, aspect of forced reduction in operational time has been propounded and taken into account for developing and analyzing the two reliability models (corresponding to two repair patterns) of the system under consideration. In addition to the reliability and profit analysis of the individual models, their comparative study has also been accomplished by finding the conditions under which one model is better than the other. We have carried out the comparative analysis under some assumptions and by assigning some numerical values to the involved costs, rates and parameters. The operators of such systems can do so in the similar manner by taking the numerical values as per their practical conditions. This will help them choose the better repair pattern in terms of performability and profitability of the system.

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