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Calculating Optimum Gear Ratios of Two Step Bevel Helical Reducer

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Abstract

The article focuses on determining optimum gear ratios of a two step bevel helical reducer to obtain the minimum reducer length. An investigation was carried out into the effects of the input factors consisting of the total reducer ratio, the face width coefficient of the bevel and the helical gear sets, the allowable contact stress and the output torque. Furthermore, a simulation experiment was established with the aim of weighing their impacts on the optimum gear ratios. The obtained results contained the information on the influences of the input parameter and proposed equations which proved to be a helpful way of determining the optimum gear ratios.

1. INTRODUCTION

In designing optimum reducers, calculating optimum gear ratios plays a significant role. This is because the dimension, the mass, and therefore the cost of a reducer is dependent on the gear ratios. This can be clarified by Figure 1 which represents the relation between the gear ratio of the second step and the gear mass of a two step helical reducer. From the figure, with optimum gear ratio $u_2=2$, the gear mass (about 178 kg) is much less than that when $u_2=5$ (about 242 kg).

Up to now, there have been numerous studies for determining the gear ratios of helical reducers. For this reducer type, the gear ratios have been determined for two steps [1-4], three steps [4-8], and four steps [4, 8-10]. In addition, their calculation has been carried out by using several methods including the graph method [1, 2 and 4], the practical method [2] and the model method [5-10].

For bevel helical reducer, for determining the gear ratio of the bevel gear set u_1, we can use the graph in Figure 2 [1]. Besides, in [2] a practical method to find the optimum gear ratios of a two step bevel helical reducer was proposed. Based on the data from reducer factories, the authors noted that the reducer weight was minimum if the ratio between the diameter of the bevel gear wheel and the center distance of the helical gear d_{ae2}/a_w ranged from 1.12 to 1.4 [2]. Therefore, the optimal gear ratios of the reducer were proposed in the tabulated form.

The model method is widely used to determine the optimum gear ratios. In this method, the equations to determine the optimum gears are used for different objectives. The objectives can be the minimum reducer height [11], the minimum length of the reducer [12] or the minimum reducer cross-section area [13].

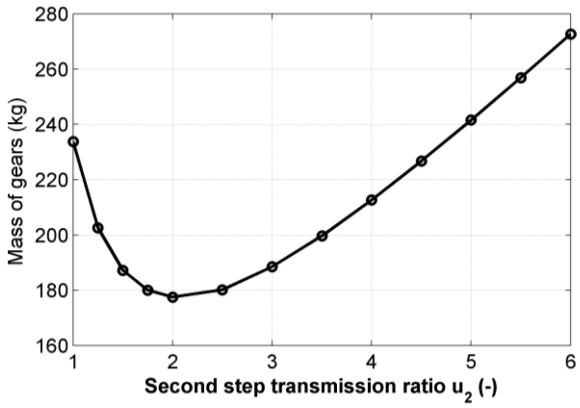

Figure 1. Partial ratios versus gear mass of a two-step helical reducer

Calculation with $u_h = 10; T_1 = 60000 Nmm; \beta = 12^0; k_H = 1.3$

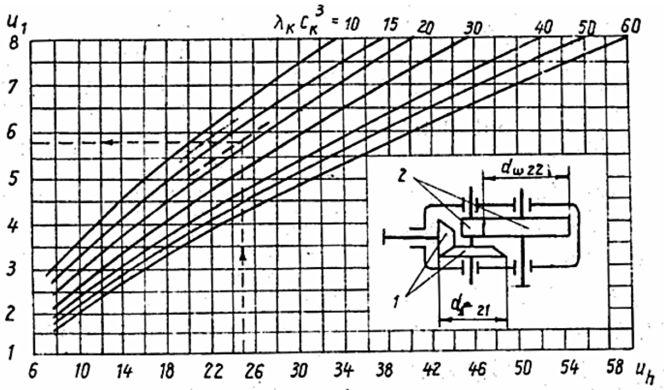

Figure 2. The gear ratio of the bevel gear set versus the total reducer ratio [1]

This paper is concerned with the determination of the optimum gear ratios of a two step bevel helical reducer to

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attain the minimum reducer length. Besides, the influence of the input parameters on the optimum gear ratios was inspected.

2. OPTIMIZATION PROBLEM

The reducer length is gained by (see Figure 3):

$$L = d_{e21}/2 + a_{w2} + d_{w22}/2 \tag{1}$$

In which, d_{e21} is the outer pitch diameter of the wheel of bevel gear set; a_{w2} and d_{w22} are the center distance and the pitch diameters of the helical gear set.

Therefore, the optimization problem can be defined as:

$$minimize\ L \tag{2}$$

With the following constraints:

$$\begin{gathered} 1 \le u_1 \le 6 \\ 1 \le u_2 \le 9 \end{gathered} \tag{3}$$

Where, u_1 is the gear ratio of the bevel gear set; u_2 is the gear ratio of the helical gear set;

From the above equations, it can be drawn that to solve the optimization problem (2), it is essential to determine d_{e21}, a_{w2} and d_{w22}.

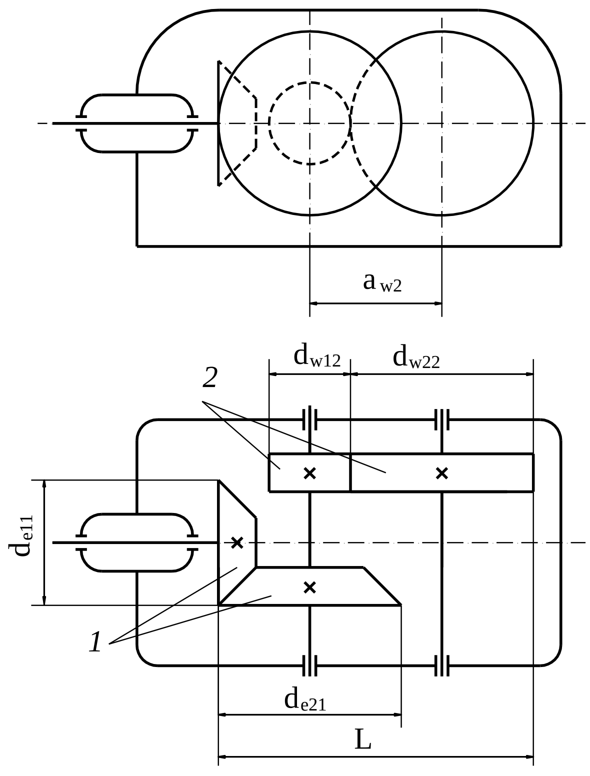

Figure 3. Calculation schema

2.1. *Determining the outer pitch diameter of the wheel*

For a straight bevel gear set, the outer pitch diameter of the wheel can be determined by:

$$d_{e11} = 2 \cdot R_e \cdot \left(1 + u_1^2\right)^{-1/2} \tag{4}$$

Wherein, R_e is the external cone distance of the bevel gear set; R_e is calculated by [14]:

$$R_e = k_R \cdot \left(u_1^2 + 1\right)^{1/2} \cdot \left(T_{11} \cdot k_{H\beta 1} / \left[\left(1 - k_{be}\right) \cdot k_{be} \cdot u_1 \cdot \left[\sigma_H\right]^2\right]\right)^{1/3} \tag{5}$$

In which, k_R is the coefficient contingent upon the gear type and gear material; $k_R = 50$ (MPa$^{1/3}$) [14]; $k_{be} = 0.25\ldots0.3$ is the face width coefficient; $\left[\sigma_{H1}\right]$ is the allowable contact stress (MPa); $K_{H\beta 1}$ is the pitting resistance contact load ratio; From the tabulated data in [14], $K_{H\beta 1}$ can be determined by the following regression equations (with the coefficient of determination $R^2 = 1$):

$$K_{H\beta 1} = 0.25 \cdot k^2 + 0.2 \cdot k + 1.02 \tag{6}$$

Where, $k = k_{be} \cdot u_1 / \left(2 - k_{be}\right)$.

$$T_{11} = T_{out} / \left(u_g \cdot \eta_{bg} \cdot \eta_{hg} \cdot \eta_b^3\right) \tag{7}$$

In which, u_g is the reducer total ratio; T_{out} is the output torque (Nmm); η_{bg} is the transmission efficiency of the bevel gear set ($\eta_{hg} = 0.95\ldots0.97$ [14]); η_{hg} is the transmission efficiency of the helical gear set ($\eta_{hg} = 0.96\ldots0.98$ [14]); η_b is the transmission efficiency of a pair of rolling bearing ($\eta_b = 0.99\ldots0.995$ [14]). Choosing $\eta_{bg} = 0.96$, $\eta_{hg} = 0.97$, $\eta_b = 0.992$ and substituting them into (7) gives:

$$T_{11} = 1.101 \cdot T_{out} / u_g \tag{8}$$

Substituting $k_R = 50$ and (8) into (5) gets:

$$R_e = 51.6296 \cdot \left(u_1^2 + 1\right)^{1/2} \cdot \left(T_{out} \cdot k_{H\beta 1} / \left[\left(1 - k_{be}\right) \cdot k_{be} \cdot u_1 \cdot u_g \cdot \left[\sigma_H\right]^2\right]\right)^{1/3} \tag{9}$$

2.2. *Determining the center distance of the second step*

For the helical gear set, the center distance a_{w2} is calculated by [14]:

$$a_{w2} = k_m \cdot \left(u_2 + 1\right) \cdot \left(T_{12} \cdot k_{H\beta} / \left(\left[\sigma_H\right]^2 \cdot u_2 \cdot \psi_{ba2}\right)\right)^{1/3} \tag{10}$$

Where, k_m is the material coefficient; k_m =43 because the gear material is steel [14]; $K_{H\beta}$ is the contact load ratio for pitting resistance; we can choose $k_{H\beta} = 1.1$ because $k_{H\beta} = 1.02 \div 1.28$ [14]; $[\sigma_H]$ is the allowable contact stress (MPa); In practice, $[\sigma_H] = 350 \ldots 420$ (MPa); ψ_{ba2} is the wheel face width coefficient of the helical gear set; $\psi_{ba2} = 0.35 \ldots 0.4$ [14];

In addition, for this reducer we have:

$$T_{out} = T_{12} \cdot \eta_{hg} \cdot \eta_b^2 \cdot u_2 \tag{11}$$

In which, T_{12} is the torque on the pinion shaft (Nmm).

Choosing $\eta_{hg} = 0.97$ and $\eta_b = 0.992$ as in Section 2.1 and substituting them into (11) gives:

$$T_{12} = 1.0476 \cdot T_{out} / u_2 \tag{12}$$

Substituting (12) and $k_{H\beta} = 1.1$ into (10) we have:

$$a_{w2} = 45.0814 \cdot (u_2 + 1) \cdot \left(T_{out} / \left([\sigma_H]^2 \cdot u_2^2 \cdot \psi_{ba2} \right) \right)^{1/3} \tag{13}$$

The pitch diameter of the helical gear set then is identified by [14]:

$$d_{w22} = 2 \cdot a_{w2} \cdot u_2 / (u_2 + 1) \tag{14}$$

2.3. *Experimental work*

A simulation experiment with 2-level full factorial experimental design was performed with the assistance of a computer program to explore the influence of the input factors on the optimum gear ratios. Additionally, the computer program was designed based on equations (2) and (3). It is also reported that 5 factors were taken into the exploration (see Table 1). Consequently, 32 numbers of runs were carried out in the test. The experimental results are illustrated in Table 2.

Table 1. Input parameters.

Factor	Code	Unit	Low	High
Total reducer ratio	u_g	-	5	30
Coefficient of the face width of bevel gear set	Kbe		0.25	0.3
Coefficient of wheel face width of helical gear set	x_{ba2}	-	0.35	0.4
Allowable contact stress	AS	MPa	350	420
Output torque	T_{out}	Nmm	10^5	10^7

Table 2. Experimental plans and output response.

StdOrder	RunOrder	CenterPt	Blocks	u_g	K_{be}	Xba2	AS (MPa)	Tout (Nm)	u_1
21	1	1	1	5	0.25	0.4	350	10000	1.75
2	2	1	1	30	0.25	0.35	350	100	4.68
30	3	1	1	30	0.25	0.4	420	10000	4.54
14	4	1	1	30	0.25	0.4	420	100	4.54
19	5	1	1	5	0.3	0.35	350	10000	1.83
17	6	1	1	5	0.25	0.35	350	10000	1.80
7	7	1	1	5	0.3	0.4	350	100	1.77
18	31	1	1	30	0.25	0.35	350	10000	4.68
25	32	1	1	5	0.25	0.35	420	10000	1.80

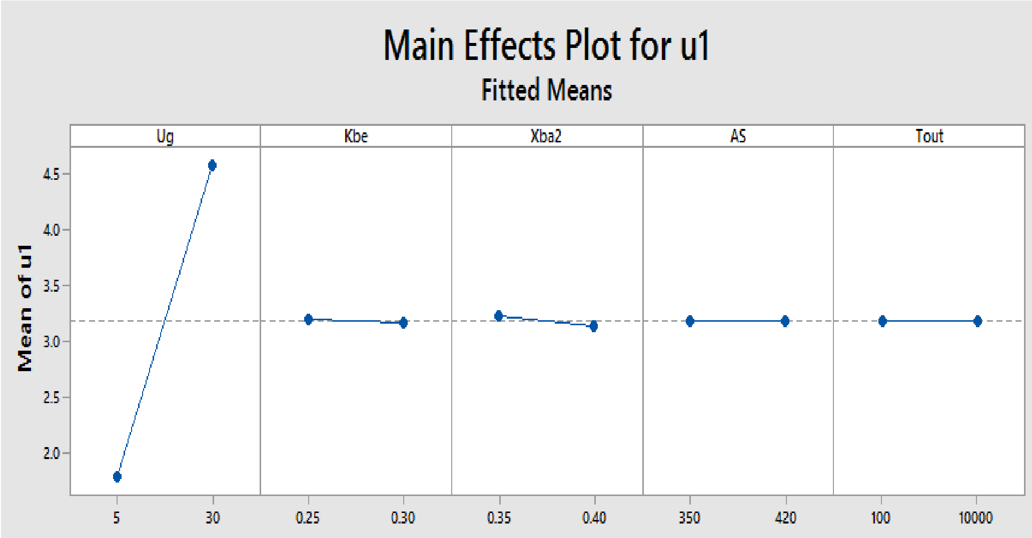

Figure 4. Main effects plot for u_1

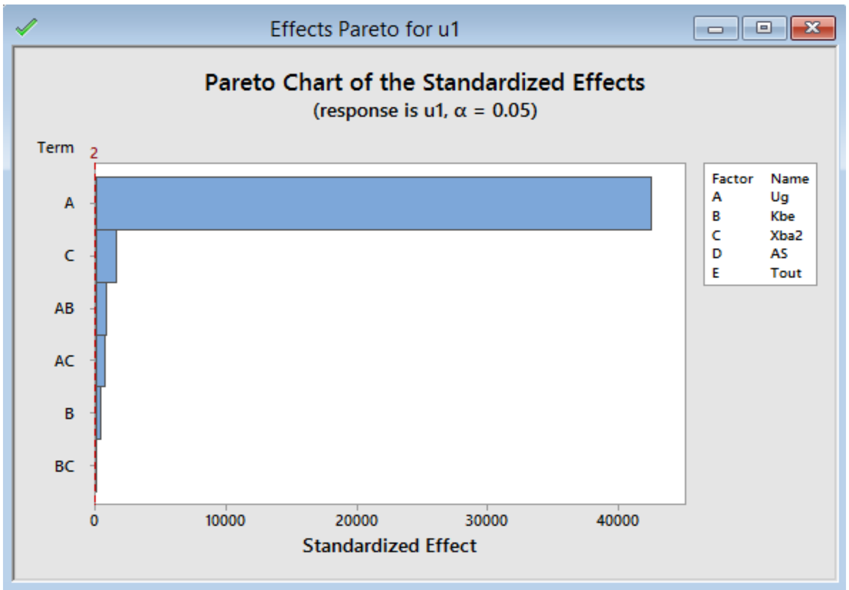

Figure 5. Pareto Chart of the Standardized Effects for u_1.

The main effects plot for u_1 is exhibited in Figure 4. It is noticeable that the optimum gear ratio is largest influenced by the total reducer ratio U_g, whereas it is almost not dependent on the allowable contact stress AS and the output torque T_{out}. However, the wheel face width coefficients of bevel gear set and helical gear set K_{be} and X_{ba2} do affect the optimum gear ratio to a low extent.

The Pareto chart of the standardized effects is demonstrated in Fgure 5. It is interpreted that those factors consisting of total reducer ratio, the wheel face width coefficients of the bevel gear set and the helical gear set are statistically significant at the 0.05 level with the response model. The indicator's signal is on the bars denoting the factors passing the reference line.

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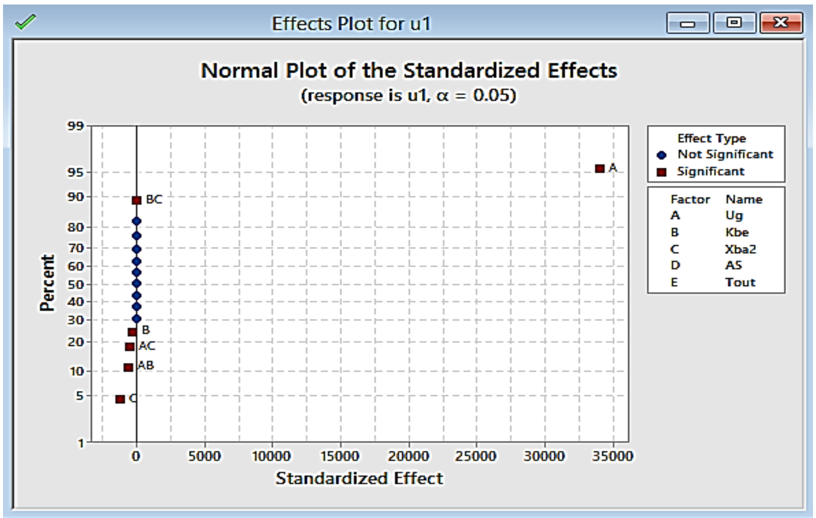

Figure 6. Normal Plot for u_1

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		3.18095	0.00003	97128.21	0.000	
Ug	2.78710	1.39355	0.00003	42551.18	0.000	1.00
Kbe	-0.022367	-0.011184	0.000033	-341.49	0.000	1.00
Xba2	-0.098243	-0.049121	0.000033	-1499.89	0.000	1.00
Ug*Kbe	-0.047402	-0.023701	0.000033	-723.70	0.000	1.00
Ug*Xba2	-0.041238	-0.020619	0.000033	-629.58	0.000	1.00
Kbe*Xba2	0.001802	0.000901	0.000033	27.52	0.000	1.00

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0001853	100.00%	100.00%	100.00%

Figure 7. Estimated Effects and Coefficients for u_1

The Normal Plot of the standardized effects on u_1 is presented in Figure 6. It can be seen from the figure that the total reducer ratio u_g (factor A) is the most significant parameter for u_1. In addition, it has a positive standardized effect on u_1. Besides, k_{be} (factor B) and ψ_{ba2} (factor C) have a negative standardized effect on u_1.

Figure 7 illustrates the estimated effects and coefficients for u_1. It is elucidated from the figure that some factors such as the total reducer ratio, the wheel face width coefficients of the bevel gear and the helical gear sets and their interactionshave P-values lower than 0.05, which signifies that they influence the response significantly. Subsequently, u_1 can be reached by:

$$u_1 = 1.44053 + 0.157084 \cdot u_g + 0.3392 \cdot k_{be} - 1.2068 \cdot \psi_{ba2} - \\ -0.075844 \cdot u_g \cdot k_{be} - 0.06598 \cdot u_g \cdot \psi_{ba2} + 1.442 \cdot k_{be} \cdot \psi_{ba2} \quad (15)$$

Figure 7 displays the adj-R^2 and pred-R^2 with the high values. Therefore, the equation (15) appropriately fits the data. This equation is used for determination of the optimum gear ratio of the bevel gear set u_1. After that, the optimum gear ratio of the helical gear set is calculated by $u_2 = u_g / u_1$.

3. CONCLUSIONS

The determination of the optimum gear ratios of a two step bevel helical reducer for the minimum reducer length was performed. Several input parameters such as the total ratio of the reducer, the wheel face width coefficients of the bevel gear and helical gear sets, the allowable contact stress and the output torque, and their impacts on the optimum gear ratios were taken into considereation. More importantly, to achieve the minimum reducer length, some equations for calculating the optimum gear ratios were recommended.

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