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Reliability of Time – Dependent Stress- Strength System for Finite Mixture of Distribution

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Abstract:

In this paper the reliability is obtained for time- dependent stress – strength system for finite mixture of distribution. The general expression is obtained for reliability of stress- strength system for finite mixture of distributions for random-independent – stress and random- independent strength. In this two cases are taken i.e. stress follows mixture of lindley distribution and strength follow exponential distribution and vice-versa.

INTRODUCTION:

Reliability of a system is the probability that a system will adequately perform its intended purpose for a given period of time under stated environmental conditions [1]. In some cases system failures occur due to certain type of stresses acting on them. Thus system composed of random strengths will have its strength as random variable and the stress applied on it will also be a random variable. A system fails whenever an applied stress exceeds strength of the system. In a finite mixture model, the distribution of random quantity of interest is modelled as a mixture of a finite number of component distributions in varying proportions [2]. The flexibility and high degree of accuracy of finite mixture models have been the main reason for their successful applications in a wide range of fields in the biological physical and social sciences. The estimation of reliability based on finite mixture of pareto and beta distributions was studied by Maya, T. Nair (2007)[3]. In reliability theory, the mixture distributions are used for the analysis of the failure times of a sample of items of coherent laser used in telecommunication network.

In reliability theory, there are lots of real life situations where the concept of mixture distributions can be applied. For example, in life testing experiments, the systems will be failed due to different causes and the times to failure due to different reasons are likely to follow different distributions. Knowledge of these distributions is essential to eliminate cause of failures and thereby to improve the reliability. The problem of increasing reliability of any system become more significant in many fields of industry, transport, communications technology, etc, with the complex mechanization and automation of industrial processes. Underestimation and overestimation of factors associated with reliability may engender great losses.

Statistical model:

If X denotes the stress of the component and Y is the strength imposed on it, then the reliability of the component is

$$R = P(X < Y)$$
$$= \int_0^\infty f(x)\left(\int_x^\infty g(y)dy\right)dx$$

A finite mixture of exponential distribution with k-components can be represented in the form

$$f(x) = p_1 f_1(x) + p_2 f_2(x) + \; \ldots . + p_k f_k(x),$$
$$where \;\; p_i > 0, \qquad i = 1, 2 \ldots . k, \sum_{i=1}^{k} p_i = 1$$

In the case of Random- independent Stress and random – independent Strength, the reliability of system is

$$R_n = \prod_{i=1}^{n}\left[\int_0^\infty f_i(x)\left(\int_x^\infty g_i\,(y)dy\right)dx\right]$$

The probability density function of Lindley distribution is

$$f(x) = \frac{\lambda^2(1+x)e^{-\lambda x}}{(1+\lambda)},\ \ \lambda > 0, x > 0$$

Case i): I: Stress follows mixture of one component lindley distribution and strength follows exponential distribution

$$f_i(x) = \frac{p_1{\lambda_{i1}}^2(1+x)e^{-\lambda_{i1}x}}{(1+\lambda_{i1})}\ , \lambda > 0, x > 0$$

$$g_i(y) = \mu_i e^{-\mu_i y}, \mu > 0, y > 0$$

$$R_n = \prod_{i=1}^{n}\left[\int_0^\infty \frac{p_1{\lambda_{i1}}^2(1+x)e^{-\lambda_{i1}x}}{(1+\lambda_{i1})}\left(\int_x^\infty \mu_i e^{-\mu_i y}\,dy\right)dx\right]$$

$$R_n = \prod_{i=1}^{n}\left[\int_0^\infty \frac{p_1{\lambda_{i1}}^2(1+x)e^{-\lambda_{i1}x}}{(1+\lambda_{i1})}(e^{-\mu_i x})dx\right]$$

$$R_n = \prod_{i=1}^{n}\left[\frac{p_1{\lambda_{i1}}^2}{(1+\lambda_{i1})}\left(\frac{\lambda_{i1}+\mu_{i1}+1}{(\lambda_{i1}+\mu_i)^2}\right)\right]$$

II: Stress follows exponential distribution and strength follows mixture of one component lindley distribution

$$f_i(x) = \lambda_i e^{-\lambda_i x}\ , \lambda > 0, x > 0$$

$$g_i(y) = \frac{p_1{\mu_{i1}}^2(1+y)e^{-\mu_{1i}y}}{(1+\mu_{i1})}, \mu > 0, y > 0$$

$$R_n = \prod_{i=1}^{n}\left[\int_0^\infty \lambda_i e^{-\lambda_i x}\left(\int_x^\infty \frac{p_1{\mu_{i1}}^2(1+y)e^{-\mu_{i1}y}}{(1+\mu_{i1})}dy\right)dx\right]$$

$$R_n = \prod_{i=1}^{n}\left[\int_0^\infty \lambda_i e^{-\lambda_i x}\left(\frac{\big((1+x)\mu_{i1}+1\big)e^{-\mu_{i1}x}}{(1+\mu_{i1})}\right)dx\right]$$

$$R_n = \prod_{i=1}^{n}\left[\frac{p_1\lambda_i}{\mu_{i1}(1+\mu_{i1})}\left(\frac{{\mu_{i1}}^2+2\mu_{i1}+\lambda_i\mu_{i1}+\lambda_i}{(\lambda_i+\mu_{i1})^2}\right)\right]$$

Case ii): I: Stress follows mixture of two component lindley distribution and strength follows exponential distribution

$$f_i(x) = \frac{p_1{\lambda_{i1}}^2(1+x)e^{-\lambda_{i1}x}}{(1+\lambda_{i1})} + \frac{p_2{\lambda_{i2}}^2(1+x)e^{-\lambda_{i2}x}}{(1+\lambda_{i2})}, \lambda > 0, x > 0$$

$$g_i(y) = \mu_i e^{-\mu_i y}, \mu > 0, y > 0$$

$$R_n = \prod_{i=1}^{n}\left[\int_0^\infty \left(\frac{p_1{\lambda_{i1}}^2(1+x)e^{-\lambda_{i1}x}}{(1+\lambda_{i1})} + \frac{p_2{\lambda_{i2}}^2(1+x)e^{-\lambda_{i2}x}}{(1+\lambda_{i2})}\right)\left(\int_x^\infty \mu_i e^{-\mu_i y}\,dy\right)dx\right]$$

$$R_n = \prod_{i=1}^{n}\left[\int_0^\infty \left(\frac{p_1{\lambda_{i1}}^2(1+x)e^{-\lambda_{i1}x}}{(1+\lambda_{i1})} + \frac{p_2{\lambda_{i2}}^2(1+x)e^{-\lambda_{i2}x}}{(1+\lambda_{i2})}\right)(e^{-\mu_i x})\,dx\right]$$

$$R_n = \prod_{i=1}^{n}\left[\left[\frac{p_1{\lambda_{i1}}^2}{(1+\lambda_{i1})}\left(\frac{\lambda_{i1}+\mu_{i1}+1}{(\lambda_{i1}+\mu_i)^2}\right)\right] + \left[\frac{p_2{\lambda_{i2}}^2}{(1+\lambda_{i2})}\left(\frac{\lambda_{i2}+\mu_i+1}{(\lambda_{i2}+\mu_i)^2}\right)\right]\right]$$

II: Stress follows exponential distribution and strength follows mixture of two component lindley distribution

$$f_i(x) = \lambda_i e^{-\lambda_i x}, \lambda > 0, x > 0$$

$$g_i(y) = \frac{p_1 {\mu_{i1}}^2 (1+y) e^{-\mu_{i1} y}}{(1+\mu_{i1})} + \frac{p_2 {\mu_{i2}}^2 (1+y) e^{-\mu_{i2} y}}{(1+\mu_{i2})}, \mu > 0, y > 0$$

$$R_n = \prod_{i=1}^{n} \left[\int_0^\infty \lambda_i e^{-\lambda_i x} \left(\int_x^\infty \left(\frac{p_1 {\mu_{i1}}^2 (1+y) e^{-\mu_{i1} y}}{(1+\mu_{i1})} + \frac{p_2 {\mu_{i2}}^2 (1+y) e^{-\mu_{i2} y}}{(1+\mu_{i2})} \right) dy \right) dx \right]$$

$$R_n = \prod_{i=1}^{n} \left[\int_0^\infty \lambda_i e^{-\lambda_i x} \left[\frac{p_1 {\mu_{i1}}^2}{(1+\mu_{i1})} \left(\frac{(1+x) e^{-\mu_{i1} x}}{\mu_{i1}} + \frac{e^{-\mu_{i1} x}}{{\mu_{i1}}^2} \right) + \frac{p_1 {\mu_{i2}}^2}{(1+\mu_{i2})} \left(\frac{(1+x) e^{-\mu_{i2} x}}{\mu_{i2}} + \frac{e^{-\mu_{i2} x}}{{\mu_{i2}}^2} \right) \right] \right]$$

$$R_n = \prod_{i=1}^{n} \left[\frac{p_1 \lambda_i}{(1+\mu_{i1})} \left(\frac{{\mu_{i1}}^2 + 2\mu_{i1} + \lambda_i \mu_{i1} + \lambda_i}{(\lambda_i + \mu_{i1})^2} \right) + \frac{p_2 \lambda_i}{(1+\mu_{i2})} \left(\frac{{\mu_{i2}}^2 + 2\mu_{i2} + \lambda_i \mu_{i2} + \lambda_i}{(\lambda_i + \mu_{i2})^2} \right) \right]$$

Case iii): I: Stress follows mixture of three component lindley distribution and strength follows exponential distribution

$$f_i(x) = \frac{p_1 {\lambda_{i1}}^2 (1+x) e^{-\lambda_{i1} x}}{(1+\lambda_{i1})} + \frac{p_2 {\lambda_{i2}}^2 (1+x) e^{-\lambda_{i2} x}}{(1+\lambda_{i2})} + \frac{p_3 {\lambda_{i3}}^2 (1+x) e^{-\lambda_{i3} x}}{(1+\lambda_{i3})}$$

$$g_i(y) = \mu_i e^{-\mu_i y}$$

$$R_n = \prod_{i=1}^{n} \left[\int_0^\infty \left(\frac{p_1 {\lambda_{i1}}^2 (1+x) e^{-\lambda_{i1} x}}{(1+\lambda_{i1})} + \frac{p_2 {\lambda_{i2}}^2 (1+x) e^{-\lambda_{i2} x}}{(1+\lambda_{i2})} + \frac{p_3 {\lambda_{i3}}^2 (1+x) e^{-\lambda_{i3} x}}{(1+\lambda_{i3})} \right) \left(\int_x^\infty \mu_i e^{-\mu_i y} \, dy \right) dx \right]$$

$$R_n = \prod_{i=1}^{n} \left[\int_0^\infty \left(\frac{p_1 {\lambda_{i1}}^2 (1+x) e^{-\lambda_{i1} x}}{(1+\lambda_{i1})} + \frac{p_2 {\lambda_{i2}}^2 (1+x) e^{-\lambda_{i2} x}}{(1+\lambda_{i2})} + \frac{p_3 {\lambda_{i3}}^2 (1+x) e^{-\lambda_{i3} x}}{(1+\lambda_{i3})} \right) (e^{-\mu_i x}) \, dx \right]$$

$$R_n = \prod_{i=1}^{n} \left[\left[\frac{p_1 {\lambda_{i1}}^2}{(1+\lambda_{i1})} \left(\frac{\lambda_{i1} + \mu_{i1} + 1}{(\lambda_{i1} + \mu_i)^2} \right) \right] + \left[\frac{p_2 {\lambda_{i2}}^2}{(1+\lambda_{i2})} \left(\frac{\lambda_{i2} + \mu_i + 1}{(\lambda_{i2} + \mu_i)^2} \right) \right] + \left[\frac{p_3 {\lambda_{i3}}^2}{(1+\lambda_{i3})} \left(\frac{\lambda_{i3} + \mu_i + 1}{(\lambda_{i3} + \mu_i)^2} \right) \right] \right]$$

In general

$$R_n = \prod_{i=1}^{n} \left[\sum_{k=1}^{m} \frac{p_k {\lambda_{ik}}^2}{(1+\lambda_{ik})} \left(\frac{\lambda_{ik} + \mu_i + 1}{(\lambda_{ik} + \mu_i)^2} \right) \right]$$

II: Stress follows exponential distribution and strength follows mixture of three component lindley distribution

$$f_i(x) = \lambda_i e^{-\lambda_i x}$$

$$g_i(y) = \frac{p_1 {\mu_{i1}}^2 (1+y) e^{-\mu_{i1} y}}{(1+\mu_{i1})} + \frac{p_2 {\mu_{i2}}^2 (1+y) e^{-\mu_{i2} y}}{(1+\mu_{i2})} + \frac{p_3 {\mu_{i3}}^2 (1+y) e^{-\mu_{i3} y}}{(1+\mu_{i3})}$$

$$R_n = \prod_{i=1}^{n} \left[\int_0^\infty \lambda_i e^{-\lambda_i x} \left(\int_x^\infty \left(\frac{p_1 {\mu_{i1}}^2 (1+y) e^{-\mu_{i1} y}}{(1+\mu_{i1})} + \frac{p_2 {\mu_{i2}}^2 (1+y) e^{-\mu_{i2} y}}{(1+\mu_{i2})} + \frac{p_3 {\mu_{i3}}^2 (1+y) e^{-\mu_{i3} y}}{(1+\mu_{i3})} \right) dy \right) dx \right]$$

$$R_n = \prod_{i=1}^{n} \left[\int_0^\infty \lambda_i e^{-\lambda_i x} \left[\frac{p_1 {\mu_{i1}}^2}{(1+\mu_{i1})} \left(\frac{(1+x) e^{-\mu_{i1} x}}{\mu_{i1}} + \frac{e^{-\mu_{i1} x}}{{\mu_{i1}}^2} \right) + \frac{p_1 {\mu_{i2}}^2}{(1+\mu_{i2})} \left(\frac{(1+x) e^{-\mu_{i2} x}}{\mu_{i2}} + \frac{e^{-\mu_{i2} x}}{{\mu_{i2}}^2} \right) \right. \right.$$
$$\left. \left. + \frac{p_1 {\mu_{i3}}^2}{(1+\mu_{i3})} \left(\frac{(1+x) e^{-\mu_{i3} x}}{\mu_{i3}} + \frac{e^{-\mu_{i3} x}}{{\mu_{i3}}^2} \right) \right] \right]$$

$$R_n = \prod_{i=1}^{n}\left[\frac{p_1\lambda_i}{(1+\mu_{i1})}\left(\frac{{\mu_{i1}}^2 + 2\mu_{i1} + \lambda_i\mu_{i1} + \lambda_i}{(\lambda_i + \mu_{i1})^2}\right) + \frac{p_2\lambda_i}{(1+\mu_{i2})}\left(\frac{{\mu_{i2}}^2 + 2\mu_{i2} + \lambda_i\mu_{i2} + \lambda_i}{(\lambda_i + \mu_{i2})^2}\right) + \frac{p_3\lambda_i}{(1+\mu_{i3})}\left(\frac{{\mu_{i3}}^2 + 2\mu_{i3} + \lambda_i\mu_{i3} + \lambda_i}{(\lambda_i + \mu_{i3})^2}\right)\right]$$

In general

$$R_n = \prod_{i=1}^{n}\left[\sum_{k=1}^{m}\frac{p_k\lambda_i}{(1+\lambda_{ik})}\left(\frac{{\mu_{ik}}^2 + 2\mu_{ik} + \lambda_i\mu_{ik} + \lambda_i}{(\lambda_i + \mu_{ik})^2}\right)\right]$$

CONCLUSION:

In this the reliability is obtained for time- dependent stress – strength system for finite mixture of lindley distribution. The general expression is obtained for reliability of stress- strength system for finite mixture of distributions for random-independent – stress and random- independent strength when stress follows mixture of lindley distribution and strength follow exponential distribution and vice-versa.

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