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An Algorithmic Approach to Solve Fuzzy Assignment Problem

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Abstract

Assignment Problem, whose objective is to find the optimum assignment of a number of tasks to an equal number of resources at a minimum cost or maximum profit. In this paper, a genetic algorithm approach is proposed to find the fuzzy optimal solution to a fuzzy assignment problem by representing all the parameters as triangular fuzzy numbers. The proposed method is easy to understand and to apply for finding the optimal solution of fuzzy assignment problems occurring in real life situations.

Keywords: Fuzzy assignment problem, Triangular fuzzy number, Graded mean integration method, Genetic Algorithm.

1. INTRODUCTION

An assignment problem is a well-known problem in which our objective is to assign a number of jobs to an equal number of facilities (machines/persons) at a minimum time/cost. In the existing literature, several researchers developed different methodologies for solving generalized assignment problems. Among them, one may refer to the works of Aickelin,U and Dowsland ,K.A[1] Catrysse and Van Wassenhove[2], Chu and Beasley[4], Lorena and Narciso[12,Ross and Soland[15],Wilson[23].Fuzzy assignment problems have received great attention in recent years. For instance, Chen [3] proposed a fuzzy assignment model that did not consider the differences of individuals, and also proved some theorems. Wang [22] solved a similar model by graph theory. Dubois and Fortemps [5] proposed a flexible assignment problem, which combines with fuzzy theory, multiple criteria decision-making and constraint-directed methodology. They also demonstrated and solved an example of fuzzy assignment problem. Muruganandam et.al[14] proposed two objective assignment problem using fuzzy numbers. Lin and Wen [11] proposed an efficient algorithm based on the labelling method for solving fuzzy assignment problems. The algorithm begins with primal feasibility and proceeds to obtain dual feasibility while maintaining complementary slackness until the primal optimal solution is found. Feng and Yang [6] investigated a two objective-cardinality assignment problem. Majumdar and Bhunia [13] proposed an elitist genetic algorithm to solve generalized assignment problem with imprecise cost/time.

In this paper, a solution method is proposed to find the fuzzy assignment problem by genetic algorithm approach. We organize this paper as follows: In section 1, we introduce the fuzzy assignment problem. In section 2, we construct the mathematical model for the problem. In section 3, we give the proposed method for the problem. In section 4, we propose a solution algorithm to solve the problem. In section 5, a numerical example is given to show the efficiency of the algorithm and finally we give a conclusion for our problem.

2. MATHEMATICAL FORMULATION

2.1 The general assignment problem

Suppose there are n people and n jobs. Each job must be done by exactly one person, also each person can do, at most, one job. The problem is to assign the people to the jobs so as to minimize the total cost of completing all of the jobs. The general assignment problem can be mathematically stated as follows:

$$\text{Minimize } Z=\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\ x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1 \text{ for } i=1,2,...,n$$

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ for } j=1,2,...,n$$

$$x_{ij} = 0 \text{ or } 1$$

2.2 Fuzzy assignment problem

$$\text{Minimize } Z=\sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{c}_{ij}\ x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1 \text{ for } i=1,2,...,n$$

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ for } j=1,2,...,n$$

$$x_{ij} = 0 \text{ or } 1$$

3. THE PROPOSED METHOD

3.1 Triangular fuzzy number

A fuzzy number A is a triangular fuzzy number denoted by (a_1, a_2, a_3) and its membership function $\mu_A(x)$ is given below:

$$\mu_A(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & a_1 \le x \le a_2, \\ \dfrac{x - a_3}{a_2 - a_3} & a_2 \le x \le a_3, \\ 0 & \text{otherwise} \end{cases}$$

3.2 Graded mean integration representation method

Chen and Hsieh [19,20,21] proposed graded mean integration representation for representing generalized fuzzy number. Now we describe graded mean integration representation as follows:

Suppose L^{-1} and R^{-1} are inverse functions of functions L and R, respectively, and the graded mean h-level value of generalised fuzzy number $A = (a_1, a_2, a_3, a_4 : w)_{LR}$ is $h[L^{-1}(h) + R^{-1}(h)] / 2$. Then the defuzzified value P(A) by graded mean integration representation of generalised fuzzy number based on the integral value of graded mean h-level is

$$P(A) = \frac{\int_0^w h\left[\frac{L^{-1}(h) + R^{-1}(h)}{2}\right] dh}{\int_0^w hdh}$$

where h is between 0 and w, $0 < w < 1$

If $A = (a_1, a_2, a_3)$ is a triangular fuzzy number. Chen and Hsieh [19,20,21] already find the general formulae of the representation of generalized triangular fuzzy number as follows:

$$P(A) = \frac{1}{2} \frac{\int_0^1 h[a_1 + h(a_2 - a_1) + a_3 - h(a_3 - a_2)]\, dh}{\int_0^1 hdh}$$

$$P(A) = \frac{a_1 + 4a_2 + a_3}{6}$$

3.3 Genetic algorithm approach

Genetic Algorithms (GA) are search based algorithm based on the concepts of genetics and natural selection. GAs is a subset of a much larger branch of computation known as Evolutionary Computation. They were first envisioned by John Holland and were subsequently developed by various researchers. The most important decision to implement a genetic algorithm is the representation of the chromosomes. Most commonly used representation of chromosomes are binary representation in which the bits of chromosomes are represented as 0 or 1, Real valued representation in which the chromosomes are real valued or floating point numbers, Integer representation, in which the chromosomes are integers and Permutation representation, in which the chromosomes are represented by an order of elements. In this paper, binary representation of chromosomes is used to represent the bits of strings.

Each potential solution is encoded in the form of a string and a population of strings is created which is further processed by three operators: Reproduction, Crossover, and Mutation. Reproduction is a process in which individual strings are copied according to their fitness function (Here the fitness function is taken to be the total cost function). Crossover is the process of swapping the content of two strings at some point(s) with a probability. Finally, Mutation is the process of flipping the value at a particular location in a string with a very low probability. A more comprehensive treatment of GA can be found in [7,13]. Shaikh Tajudddin Nizami et al.[17] proposed a new method to find the solution by the use of Genetic Algorithm of artificial Intelligence. Lau.HCW et.al[9] developed a hybrid fuzzy genetic algorithm approach to solve Transportation scheduling problem. Shahram Saeidi[16] et.al proposed a multi-objective genetic algorithm for solving cell formation problem using fuzzy goal programming approach. Srinivas,B et.al[21] proposed a optimal solution to Intuitionistic fuzzy assignment problem using genetic algorithm approach.

4. SOLUTION ALGORITHM

Initialization Algorithm

Step 1: Set the population size as the value of i

Step 2 : Select a matrix R_{ij} where i's row, j's column

Step 3 : Select Ri' from the matrix R_{ij}

Step 4 : Randomly generate chromosome C from 1 to n

Step 5 : Arrange the first row according to C

Step 6 : Change the real values to binary values.

Reproduction Algorithm

Step 1 : Select the lowest chromosome from Ri'

Step 2 : Replace it with best chromosome from Ri'.

Crossover Algorithm

Step 1 : Choose P_{cross} €(0, 1)

Step 2 : Determine parent for crossover operation

Step 3 : Replace the process from i = 1 to n

Step 4 : Randomly generate real number r

Step 5 : Select parent P_i if r <P_{cross}

Step 6 : Chromosome (P_1' , P_2') is selected for crossover process

Mutation Algorithm

Step 1 : Choose P_{mut} €(0, 1) be the mutation probability

Step 2 : Calculate the number of mutation site

M = mutation probability x probability size x Length of string.

Step 3: Randomly select the workers w_1, w_2, . . . w_n using the value of M

Step 4 : Generate random number to choose the bit in the string

Step 5 : Exchange the binary value (0, 1)

Selection Algorithm

Step 1 : Choose the highest values of bits to assign the value in R(i')

Step 2 : If there are two same highest value, choose the lowest cost.

Step 3 : Repeat the process for next generation except the assigned value of the corresponding column of R(i').

5. NUMERICAL EXAMPLE

Four persons are available to do four different jobs. From past records, the cost (in Rupees) that each person takes to do each job is known and is represented by triangular fuzzy numbers and is shown in following Table.

Jobs

Workers				
	(1, 2, 3)	(3, 4, 5)	(5, 6, 7)	(7, 8, 9)
	(2, 3, 4)	(4, 5, 6)	(7, 8, 9)	(1 2, 3,)
	(4, 5, 6)	(6, 7, 8)	(2, 3, 4)	(1, 2, 3)
	(3, 4, 5)	(4, 5, 6)	(5, 6, 7)	(6, 7, 8)

Find the assignment of persons to jobs that will minimize the total cost.

Solution:

Fuzzy assignment problem is converted into crisp assignment problem using graded mean integration method

$$P(A)= \frac{a_1 + 4a_2 + a_3}{6}$$

Defuzzify the above assignment problem we get,

	Jobs			
Workers	2	4	6	8
	3	5	8	2
	5	7	3	2
	4	5	6	7

Row 1	(2)	6	8	4
Gen N	0010	0110	1000	0100
Rep	1000	0110	1000	0100
Crossover	1000	0110	1000	0100
Mutation	1001	0110	1000	0100
Gen N+1	(9)	6	8	4

Row2	8	5	(2)
GenN	1000	0101	0010
Rep	1000	0101	1000
Crossover	1000	0101	1000
Mutation	1000	0101	1001
Gen N+1	8	5	(9)

Row 3 & 4	5	6	(3)	7
Gen *N*	0101	0110	0011	0111
Rep	0101	0110	0111	0111
Crossover	0110	0101	0111	0111
Mutation	0110	0101	0111	0101
Gen N+1	6	5	(7)	5

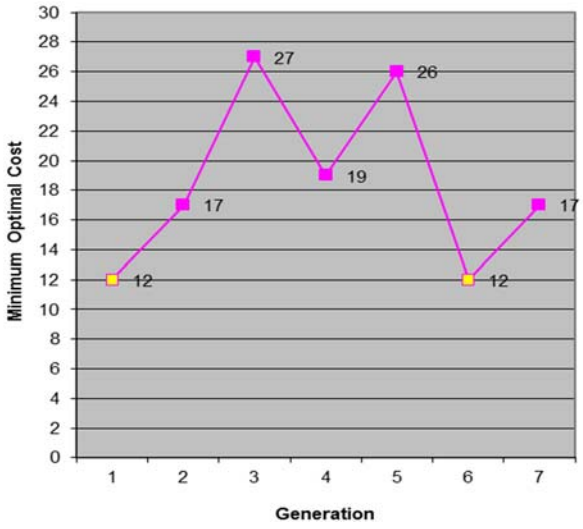
		Jobs			
		1	2	3	4
Workers	1	2*	4	6	8
	2	3	5	8	2*
	3	5	7	3*	2
	4	4	5*	6	7

The optimal assignment

1-1, 2-4, 3-3, 4-2.

Optimal minimum cost = 2+2+3+5=12

Graphical representation of minimum cost at each generation



6. CONCLUSION

Assignment problem is of great use in decision-making. In this paper, a new method is proposed to solve the fuzzy assignment problem, occurring in real life situations. To illustrate the proposed method, a numerical example is solved and obtained results are explained. We have found that in our computational experience the results are as per expectations and satisfaction.

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