Efficient Recovery of Structural Health Monitoring Signal based on Kronecker Compressive Sensing

Sandeep Reddy Surakanti
Department of Electrical Engineering,
University of South Florida, Tampa, Florida, USA

Seyed Alireza Khoshnevis
Department of Electrical Engineering,
University of South Florida, Tampa, Florida, USA

Hamed Ahani
Department of Mechanical Engineering,
UNC Charlotte, Charlotte, North Carolina, USA

Vahid Izadi
Department of Mechanical Engineering,
UNC Charlotte, Charlotte, North Carolina, USA

Abstract
In structural health monitoring (SHM), sensors intermittently monitor the structure and send the data to a remote server for further processing. Data compression can be used to reduce the required storage, and for efficient use of communication bandwidth, because of the huge volumes of sensor data produced from the monitoring sensors. Recently, compressive sampling (CS) had been introduced as an efficient, fast, and linear method of sampling data. The length of the signal for compression has a direct relation to the complexity of the compression system and the quality of the recovered system. In traditional CS approaches, the length of the signal was experimentally chosen. If we compress the signal in a smaller size, the compression system would be more efficient in terms of required computational complexity and time for compression. On the other hand, if we decrease the length of the signal too much, the quality of the reconstructed signal would be degraded. Very recently, the Kronecker technique in CS recovery has been introduced in order to compensate for the loss of accuracy. In this work, we investigate the applicability of Kronecker-based CS recovery for Seismic signals. The simulation results show that this technique in recovery can highly improve the quality, while sensors can compress the signal in minimal size. Applying the Kronecker technique in recovery enabled us to recover the original seismic signal with high accuracy up to 7 dB.

Keywords: Vibration data, compressive sensing (CS), Kronecker-based recovery

I. INTRODUCTION
Structural health monitoring (SHM) systems have been widely used in civil structures because they enable us to control and monitor the damage evolution and as a result, enhance safety. A whole SHM system contains sensors (wireless or wired), data acquisition system (Analogue to digital converters), data transfer unit, storage, data management unit, and a damage detection unit. In general, due to the large scale and complexity of the civil structures, numerous sensors are needed in SHM systems. These sensors intermittently monitor the structure and send it to a remote server for the process. Limited available power and computational resources of such sensors, and for efficient use of communication channel, some fast and simple compression can be used. There are numerous compression methods for seismic signal, one of the fastest and simplest is based on CS [1, 2]. In [3, 4], CS has been used to compress the SHM data in order to reduce the amount of data which will decrease the power consumption of the sensor and the bandwidth of the wireless transmitters. Compression phase of CS can be assumed as a linear operation, as it can be achieved after multiplying a sensing matrix by a vector which contains the uncompressed samples. CS can compress the signals which are potentially sparse. Although some recent recovery techniques have been introduced that can do the recovery without having the prior knowledge about the sparsity, their works are limited to certain conditions or for a particular class of measurement matrices [5]. CS has been extensively used in different areas of signal processing such as biomedical [6-8], image, video, and even radar to reduce power consumption, transmission bandwidth, and noise. Khoshnevis et al. have used this method on radar signals and biomedical signals. In general, signals must have a sparse representation under a predefined dictionary. Sparse vector has lots of zero (or small) coefficients and only a few large coefficients. In practice, most signals are either sparse in the temporal domain or can be mapped into a sparse domain. The seismic signal is one of the signals that can be sparsely presented via a discrete cosine transform (DCT). CS has been widely applied on seismic signal and its pros and cons have been investigated by previous work [9, 10]. Basically, the compression phase of CS is very fast, energy-efficient and requires less computational resources. This property of CS makes it suitable for applications that sensors have limited resources. On the other hand, the compression ratio of CS is not as much as other nonlinear compression techniques. But for a given compression ratio, CS provides a very fast and efficient compression system. In the system identification methods to design an implementable algorithm with high accuracy, the CS-based moving average and moving windows are known in a black box sysID for the reaction wheel [11, 12]. In addition, the CS-based moving windows method are used in the FDIR and shared control frameworks [13, 14]. It intends to increase the speed and simplicity of the compressors, in [15-17] proposed a circuit architecture that can generate compressed ECG samples, in a linear method, and with CR 75%. The sparsity of the ECG signal and
proposed a circuit based on the CS method that can compress ECG samples, almost in real-time and based on compressed sensing (CS). They applied CS in a minimum size in order to accelerate the compression phase and accordingly reducing the power consumption. The recovery phase of CS is rather complex, however, since compressed samples can be sent to a remote server (cloud), this issue can be overlooked. In addition, CS is used in heterogeneous traffic network which there are autonomous vehicles in the system that need to send their data to a cloud [16]. There are various recovery algorithms for CS. Recently Zanddizari et al. proposed a preprocessing technique called, Kronecker-based CS recovery, which aims to accelerate the compression phase of CS while providing higher accuracy in the recovery phase [18-20]. In this study, we investigate how the Kronecker technique can be employed in the recovery process of CS when we are compressing the seismic signal. Our simulation results show this approach can simplify the compression phase, and at the recovery, it can improve the quality of the reconstructed signal.

II. REVIEW OF COMPRESSIVE SENSING

The main purpose of CS is to reconstruct a signal from a few numbers of measurements which is several folds smaller than the Nyquist-Shannon rate. One prerequisite for almost all CS-based compression systems is the sparsity of the subject signal. A seismic signal $x \in \mathbb{R}^N$ can be linearly transferred into the sparse domain via a dictionary $D \in \mathbb{R}^{N \times N}$ as follow

$$s = D^{-1} x = D^{-1} x$$  \hspace{1cm} (1)

where $s \in \mathbb{R}^N$ is the sparse representation of $x$. Discrete cosine transform (DCT) is one of the most common dictionaries that can present very sparse representation for a seismic signal. Having this prior knowledge that objective signal is potentially sparse can enable us to use CS compression technique. CS measurements can be obtained as follow

$$y = \Phi x = \Phi D s = \Phi y = \Phi x = \Phi D$$  \hspace{1cm} (2)

Where $\Phi \in \mathbb{R}^{M \times N}$ is the projection (sensing) matrix $(M \ll N)$, and $y \in \mathbb{R}^M$ is the compressed (measurement) vector. Since we transfer a signal from $N$ dimension into a smaller $M$ dimension, compression can be achieved. It is evident that this compression is linear because only one matrix is multiplied to a vector. Sensing matrix $\Phi$ can be chosen from the random class of matrices, or deterministic ones. Restricted Isometry Property (RIP) is one of the conditions that enable us to use random sensing matrices,

$$\left(1 - \delta_2\right) ||s||_2^2 \leq ||\Phi D s||_2^2 \leq \left(1 + \delta_2\right) ||s||_2^2$$  \hspace{1cm} (3)

Based on RIP condition, random matrices such as independent and identically distributed (i.i.d) Gaussian distribution with zero mean and variance of $1/M$ can preserve the RIP condition, and as a result, they can be used for compression phase. Besides, deterministic matrices can also be used in CS systems where the mutual coherence is the defining factor to evaluate their performance [21, 22]. The mutual coherence between $\Phi$ and $D$ can directly affect the quality of reconstruction in CS. By definition, mutual coherence can be obtained as follow:

$$\mu(\Phi, D) = \sqrt{M \cdot \max_j | \langle \Phi_j, d_j \rangle |} \cdot |\Phi_j \in \Phi, d_j \in D$$  \hspace{1cm} (3)

Where $\Phi_j$ is the ith row of the sensing matrix, and $\psi_j$ denotes the jth column of sparsifying basis. It has been proven that lower mutual coherence can bring a higher quality of reconstruction. Also, deterministic matrices have structure, require less storage, and their results are reproducible. Recently a deterministic binary block diagonal (DBBD) matrix has been proposed which has lower mutual coherence with DCT dictionary [21]. In comparison with random matrices, it can provide higher accuracy in the recovery [23, 24]. Our simulation results also show that this matrix can provide higher accuracy than that of the Gaussian matrix when we compress the seismic signal. After designing the sensing matrix, and having prior knowledge about sparsifying basis, linear compression can be applied and compressed samples will be sent to a remote server for recovery. Recovery of CS requires solving a $l_0$-minimization problem as follows

$$\min ||s||_0 \text{ subject to } \Phi D s = y$$  \hspace{1cm} (4)

But, the $l_0$-minimization is a non-deterministic polynomial-time hard (NP-hard) problem. Lots of work have been done in CS in order to find a solution for (4), and they finally could approximate this problem by using basis pursuit (BP), which relaxes the $l_0$-minimization to a $l_2$-minimization problem. Also, some greedy algorithms such as orthogonal matching pursuit (OMP), compressive sampling matching pursuit (CoSaMP) [25-27], normalized iterative hard thresholding (NIHT) [28], and smoothed L0 norm (SL0) [29], can be used. In [4], the concept of multi-dimensional sparsity is explained and the CoSaMP method has been used for enhancement of three-dimensional radar models. Also, in [30], the combination of system response processing and Hilbert-Huang transform method which is an important signal processing method was used to specify the determination of the system.

III. KROENCKER-BASED CS RECOVERY

The size of the measurement matrix $\Phi \in \mathbb{R}^{M \times N}$ has a direct effect on the compression ratio $(N/M)$, number of multiplication and addition operations, and the elapsed time for generating compressed samples. Regarding compression phase in CS, $\hat{y}_{N \times 1} = \Phi \Phi_{MN} \Phi_{MN} \hat{x}_{MN \times 1}$ the product needs $(M \times N)$ multiplication and also $M \times (N - 1)$ addition operations. As an example, if $(m,n) = (128,256)$, 32768 multiplication operations are needed, but if we divide it into two smaller segments of $n = 128$ samples, then we need $8192 \times 2 = 16384$ multiplication operations (half of the
larger size). Therefore, it is obvious that sensing signal in smaller segment size would require much less computational resources, and as a result, less power consumption. However, when we decrease the length of segments, in recovery we may lose some accuracy. To compensate the recovery quality, Kronecker technique can be used [18, 19]. Kronecker product of matrix $\mathbf{G} \in \mathbb{R}^{p \times q}$ and arbitrary matrix $\mathbf{Q} \in \mathbb{R}^{m \times n}$ can be obtained as follow,

$$
\mathbf{G}_{p \times q} \otimes \mathbf{Q}_{m \times n} = 
\begin{bmatrix}
G_{11}Q & G_{12}Q & \ldots & G_{1q}Q \\
G_{21}Q & G_{22}Q & \ldots & G_{2q}Q \\
\vdots & \vdots & \ddots & \vdots \\
G_{p1}Q & G_{p2}Q & \ldots & G_{pq}Q
\end{bmatrix}
$$

(5)

In ordinary CS recovery algorithm both measurement vector $\mathbf{y}$ and projection matrix $(\Phi \mathbf{D})$ must be provided to a recovery algorithm $\mathbf{G}$ in order to reconstruct the sparse vector $\mathbf{s}$ as follow,

$$
\mathbf{s}_{m \times 1} = \mathbf{G}(\mathbf{y}_{m \times 1}, (\Phi \mathbf{D})_{m \times n}) 
$$

(6)

If we have $t$ measurement vectors, we required to run $t$ times recovery algorithm to find corresponding $t$ sparse vectors. But based on Kronecker approach, we concatenate $t$ measurement vectors into a one augmented measurement vector $\mathbf{y}_{m \times 1 \times t}$. Then, the recovery algorithm $\mathbf{G}$ is run one time by the following input arguments:

$$
\mathbf{s}_{m \times 1} = \mathbf{G}(\mathbf{y}_{m \times 1 \times t}, (I_{mk} \otimes \mathbf{D}_{m \times n})D_{m \times n})
$$

(7)

where $I$ is the common identity matrix. It is proved in [18] that this approach can decrease the mutual coherence between sensing matrix and sparsifying bases, as a result, it can increase the quality of the reconstructed signal. The Kronecker enhanced CS has previously been used on several types of signals such as biomedical by Khoshnevis et al. and where they achieved great improvement over the conventional CS [31].

IV. SIMULATION RESULTS

MIT Green building is located in the campus of Massachusetts Institute of Technology (MIT) in Cambridge and was constructed during the period of 1962-1964 was chosen. This building has 83.7 m tall with 21 stories. This building is constructed of cast-in-place reinforced concrete with two shear walls in its western and eastern sides. United States Geological Survey (USGS) equipped the Green building via 36 uniaxial EpiSensor ES-US2 force balance accelerometers. These sensors are build by Kinemetrics Inc., CA, USA. Sampling rate is set to 200 Hz with recording ranges of $\pm 4g$. The MIT Green building has been considered to a number of large environmental loads since its construction in 2010. The vibration data of this building have been stored via the sensing system and they are available for researchers. In this paper, they are used for compression purpose.

We chose 2048 samples of one of the sensors for compression. We applied CS compression method with and without Kronecker technique in the recovery of this signal. Two different types of measurement matrices, Gaussian and DBBD matrices and also two different sparsifying bases, Discrete Cosine transform (DCT) and Wavelet Dictionary was investigated. For sensing in very small size and observing the effect of Kronecker technique, we selected $n = 8, 16, 32, 64, \text{ and } 128$ which is the length of the window for applying CS, and the Kronecker factor that has been used was $t = 8$. In other words, for ordinary CS recovery we segment each measurement vector separately, but for Kronecker-based method, 8 measurement vectors are concatenated, then one shot recovery is applied to reconstruct 8 sparse vectors (discussed section IV). We used SL0 as CS recovery algorithm [32] We observed that Kronecker-based CS recovery drastically improve the recovery of vibration signal up to 6 dB. Fig. 1 shows the effect of Kronecker technique in the improvement of the signal quality. It is obvious that for very small length of sampling, the quality of ordinary recovery is very low, but via Kronecker technique, quality can be improved.

![Fig. 1. Applying Kronecker-based CS recovery for CR=50%. signal. Gaussian measurement matrix and wavelet sparsifying basis.](image-url)
Also, we tested the DBBD matrix on the same signals. Figure 3 and 4 show the results of recovery for this measurement matrix with Wavelet Dictionary and DCT dictionary, respectively.

Our simulation results verified that Kronecker technique can improve the quality of CS recovery when sensing is done in very small size. DBBD matrix outperforms the Gaussian and for DBBD matrices, DCT matrix showed higher quality. In Fig. 5 and 6, we increased the compression ratio to 75% which means (n/m = 4). It is shown that for higher compression ratio, DBBD matrix can also improve the quality of the reconstructed signal.

To visually show the effect of CS compression technique in vibration signals, the performed simulation for CR=75% is presented in Fig. 7; this figure shows that CS can be effectively applied for this class of biomedical signals and if the Kronecker-based approach is used in the recovery the output error would decrease drastically. It can be observed that this simple preprocessing technique can improve the quality of reconstruction. If we take into account the effect of Kronecker-technique in recovery phase, we would be capable of sensing the vibration signal at very small size. Sensing in smaller size directly lead to the simpler compression system.

Fig. 2. Applying Kronecker-based CS recovery for CR=50%. signal. Gaussian measurement matrix and DCT sparsifying basis.

Fig. 3. Applying Kronecker-based CS recovery for CR=50%. signal. DBBD and Wavelet sparsifying basis.

Fig. 4. Applying Kronecker-based CS recovery for CR=50%. signal. DBBD and DCT sparsifying basis.

Fig. 5. Applying Kronecker-based CS recovery for CR=75%. signal. DBBD and Wavelet sparsifying basis.

Fig. 6. Applying Kronecker-based CS recovery for CR=75%. signal. DBBD and DCT sparsifying basis.
V. CONCLUSIONS

The vibration data of MIT green building was considered for this study. Simulation results show that CS compression method can be used for compressing vibration data. Two types of measurement matrices were considered for sensing process. Two different sparsifying bases were experimented with each of those measurement matrices. Two compression ratio, 50% and 75% were tested. Results show the simple deterministic matrix DBBD outperform the Gaussian measurement matrix. For the DBBD matrix, DCT dictionary provides higher quality. The Kronecker-based technique was used to improve the quality of reconstruction. All simulations showed that CS can be applied in very small size and in the recovery via Kronecker technique quality of recovery can be highly improved. Sensing in a smaller size is useful in terms of power consumption and the elapsed time for the sensing process, particularly for the building sensors that intermittently sense the behavior of the construction.

REFERENCES


