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On b-Colouring of Central Graph of Cubic Graph and Sunlet Graph

S.Karthikeyan[#1], U.Mary[*2]

[#]Department of Mathematics, KPR Institute of Engineering and Technology, Coimbatore, India.

[*]Department of Mathematics, Nirmala College for Women, Coimbatore, India.

Abstract

A proper (vertex) k -colouring of a simple graph $G=(V,E)$ is defined as a vertex colouring from a set of k -colours such that no two adjacent vertices share a common colour. The b-chromatic number $\varphi(G)$ parameter of a graph G is the maximum number of colours for which G has a proper colouring such that every colour class contain a vertex adjacent to every other colour. The central graph $C(G)$ of a graph G is obtained by sub-dividing each edge of G by exactly once and joining all the non adjacent vertices of G . In this paper we examine the b -chromatic number of central graph of cubic graph and sunlet graph.

Keywords: Proper colouring, b -colouring, b -chromatic number, Cycle graph, Cubic graph, Sunlet graph

I. INTRODUCTION

In this paper, we consider a simple, finite, connected and undirected graph with vertex set $V(G)$ and edge set $E(G)$. A b-coloring by k -colors is a proper coloring of the vertices of a graph G such that in each color class there exists a vertex which has neighbors in all the other $k-1$ color classes. The b-chromatic number $\varphi(G)$ is the largest integer k such that G admits a b-coloring with k -colors. The concept of b-coloring was introduced by Irving and Manlove [3] in 1999 and showed that the problem of determining b-chromatic number is NP-hard for general graphs but it is polynomial for trees. The upper bounds for the b-chromatic number were investigated in the work of Kouider and Maheo [5]. The b-chromatic number for Peterson graph and power of a cycle was discussed by Chandrakumar S [7] and Nicholous T. Balakrishnan R [8], Francis Raj and Kavaskar T were find out the b-chromatic number of Cartesian product of some families of graph. Vivin and Venkatachalam [9,13] investigated the b-chromatic number of corona graphs and the b-chromatic number of line graph, middle graph and total graph of sunlet graph and wheel graph families. The b-chromatic number of helm and closed helm graph were examined by Vaidya and Shukla [11]. Ansari, Chandel and Jamal [15] find out the b-chromatic number of Prism graph families. In this work we examine the the b-chromatic number of Central Graph of cubic graph $C(G_n)$ and sunlet graph $C(S_n)$.

II. PRELIMINARIES

2.1 Cycle graph

A cycle graph or circular graph is a graph that consists of a single cycle, or in other words, some number of vertices connected in a closed chain.

2.2 Central graph

The central graph [13] $C(G)$ of a graph G is obtained by subdividing each edge of G exactly once and joining all the non adjacent vertices of G .

2.3 Cubic graph

Cubic graph G_n is a graph in which all vertices have degree three.

2.4 Sunlet graph

The sunlet graph S_n is the graph on $2n$ vertices obtained by attaching n pendant edges to a cycle graph C_n .

III. MAIN RESULT

3.1 Theorem

For the central graph of cubic graph (G_n), $\varphi[C(G_n)] = n + \lfloor n/2 \rfloor, n > 2.$

Proof:

Consider the cubic graph G_n , with $2n$ vertices and each vertices having maximum degree 3. The vertex set of central graph of the cubic graph $C(G_n)$ can be partitioned as follows, the outer cycle graph C_n with vertex set is $\{v_1, v_2, ..., v_n\}$ and the inner cycle graph C'_n with vertex set is $\{u_1, u_2, ..., u_n\}$. The vertex $u_{i(i+1)}$ is introduced in each edge of u_i and u_{i+1} where $i = 1,2,3,..., n-1$. The vertex $v_{i(i+1)}$ is introduced in each edge of v_i and v_{i+1} where $i = 1,2,3,..., n-1$, and the vertex introduced in each edge u_i and v_i is w_i for $i = 1,2,3,..., n$. Assigning the colours to $C(G_n)$ as follows, first we assign the colours $\{c_1, c_2, ..., c_n\}$ to the outer cycle vertices $\{v_1, v_2, ..., v_n\}$ with n colours.

Suppose we assign the colour c_{n+1} to the vertex u_1, then the vertex v_1 will not realize the new colour c_{n+1}. Therefore, in order to obtain a proper colouring, we also assign the colour c_{n+1} to the vertex u_2. Procedure to colour the remaining inner cycle vertices is described in the following:

Case-1:

When n is even, colour the vertices u_{2i-1}, u_{2i} as c_{n+i} for $i = 1,2,3,...,n/2$, in this case the maximum colouring possibility is $n/2$ colours.

Case-2:

When n is odd, colour the vertices u_{2i-1}, u_{2i} as c_{n+i} for $i = 1,2,3,...,(n-1)/2$, in this case the maximum colouring possibility is $(n-1)/2$ colours. And the remaining uncoloured vertex is u_n, suppose if we introduce a new colour to this vertex it cannot harmonize the colour c_n.

Also the vertices $u_{i(i+1)}, v_{i(i+1)}$ and w_i having maximum degree two, furthermore $n + \lfloor n/2 \rfloor$ is a maximum possible colouring for $C(G_n)$. Therefore the b-chromatic number of central graph of cubic graph is $\varphi[C(G_n)] = n + \lfloor n/2 \rfloor, n > 2$.

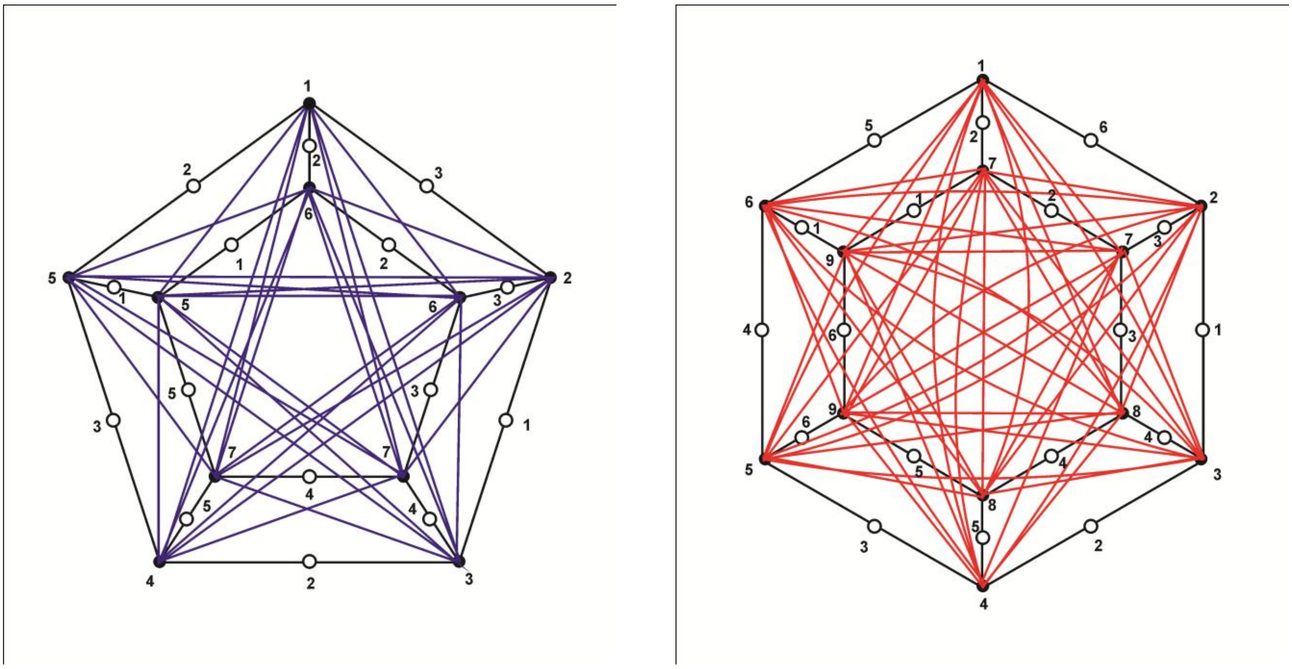

Fig 1: b-colouring of Central graph of Cubic graph G_5 & G_6

3.2Theorem

For the central graph of sunlet graph (S_n), $\varphi[C(S_n)] = \begin{cases} n + n/2, \textit{if } n \textit{ is even} \\ n + \lfloor (n-1)/2 \rfloor, \textit{if } n \textit{ is odd} \end{cases}$

Proof:

Consider the sunlet graph S_n, with $2n$ vertices formed by attaching n pendent edge to the cycle graph C_n. The vertex set of central graph of sunlet graph $C(S_n)$ can be partitioned as, the cycle graph C_n with vertex set is $\{v_1, v_2, ...v_n\}$. Furthermore each pendent vertex added in to cycle graph and it is denoted by $\{u_1, u_2, ...u_n\}$, and it should form a complete graph with n vertices. The vertex introduced in each edge v_i and v_{i+1} is $v_{i(i+1)}$ for $i = 1,2,3,...,n-1$, the vertex introduced in each edge u_i and v_i be w_i for $i = 1,2,3,...,n$. Assigning the colours to $C(S_n)$ as follows, first we should assign the colours $\{c_1, c_2, ...c_n\}$ to the complete graph with n colours.

Suppose we assign the colour c_{n+1} to the vertex v_1, then the vertex u_1 will not realize the new colour c_{n+1}. Therefore, in order to obtain a proper colouring, we also assign the colour c_{n+1} to the vertex v_2 . Procedure to colour the remaining inner cycle vertices is described in the following:

Case-1:

When n is even, colour the vertices v_{2i-1}, v_{2i} as c_{n+i} for $i = 1,2,3,...,n/2$, in this case the maximum colouring possibility is $n/2$ colours.

Case-2:

When n is odd, colour the vertices v_{2i-1}, v_{2i} as c_{n+i} for $i = 1,2,3,...,(n-1)/2$, in this case the maximum colouring possibility is $(n-1)/2$ colours. And the remaining uncoloured vertex is v_n , suppose if we introduce a new colour to this vertex it cannot harmonize the colour c_n .

Furthermore new colours cannot be introduced in this graph because the vertices $v_{i(i+1)}$ and w_i having maximum degree two.. Therefore the b-chromatic number of central graph of sunlet graph is $\varphi[C(S_n)] = \begin{cases} n + n/2, if\ n\ is\ even \\ n + \lfloor (n-1)/2 \rfloor, if\ n\ is\ odd \end{cases}$.

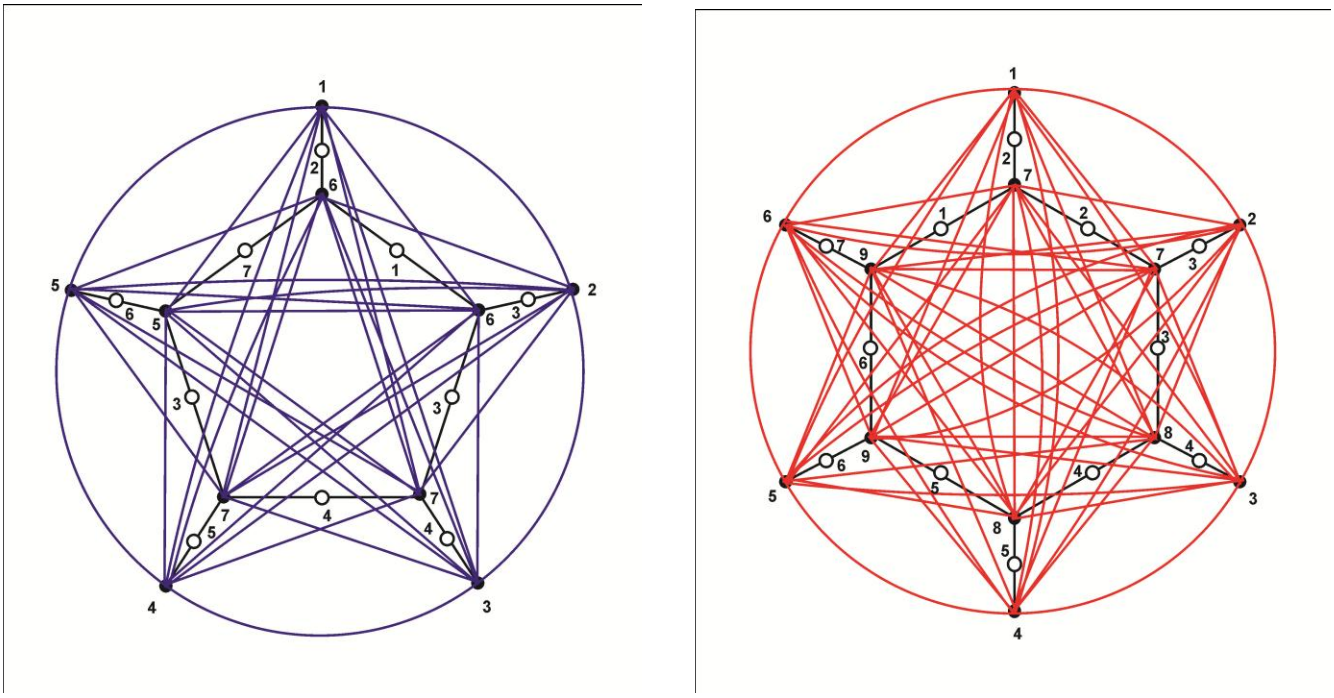

Fig 2: b-colouring of Central graph of Sunlet graph S_5 & S_6

IV. CONCLUSIONS

In this paper, we examine the b-chromatic number of

- Central graph of cubic graph, $\varphi[C(G_n)] = n + n/2.$
- Central graph of sunlet graph, $\varphi[C(S_n)] = \begin{cases} n + n/2, if\ n\ is\ even \\ n + \lfloor (n-1)/2 \rfloor, if\ n\ is\ odd \end{cases}$

REFERENCES

[1] J. Clark and D. A. Holton (1991) , *A First Look at Graph Theory*, World Scientific, New Zealand.

[2] L. Hemminger and L. W. Beineke (1978), "*Line graphs and line digraphs*", Selected Topics in Graph theory, Academic Press, 271– 305.

[3] R.W. Irving and D.F. Manlove (1999), "*The b chromatic number of a graph*", Discrete Applied Mathematics, 91(1-3), 127-141.

[4] F.Harary (1997), Graph Theory, Narosha Publishing House, Calcutta.

[5] Mekkia Kouider and Maheo (2002), "Some *bounds for the b-chromatic number some families of graphs*," Discrete Mathematics, 256, 267-277.

[6] Sandi Klavzar and Marko Jakovac (2010), "*The b-chromatic number cubic graphs*," Graphs and Combinatorics, 26, 107-118.

[7] S. Chandra Kumar and T. Nicholas (2012), "*b-coloring in Square of Cartesian Product of Two Cycles* ", Annals of Pure and Applied Mathematics 1(2), 131-137.

[8] R. Balakrishnan and K. Ranganathan (2012), *A textbook of Graph Theory,* Springer, New York.

[9] J.Vernold Vivin, M. Venkatachalam (2012), "*The b-*

chromatic number of corona graphs", Utilitas Mathematica, 88, 299-307.

[10] M. Alkhateeb (2012), *On b-colourings and b-continuity of graphs*, Ph.D Thesis, Technische Universitt Bergakademie, Freiberg, Germany.

[11] S. K. Vaidya, M. S. Shukla (2014), "*b-Chromatic number of helm and closed helm*", International Journal of Mathematics and Scientific Computing, 4(2), 43-47.

[12] R. Balakrishnan, S. Francis Raj and T. Kavaskar (2014), " *b-Chromatic Number of Cartesian Product of Some Families of Graphs*", Graphs and Combinatorics, 30, 511–520.

[13] J.Vernold Vivin, M. Venkatachalam (2015), " On *b-Chromatic Number of sunlet graph and wheel graph families*", Journal of Egyptian Mathematical Soceity, 23, 215–218.

[14] R. Arundhadhi, V.Ilyarani (2017), "Total colouring of Closed Helm, Flower and Bistar Graph Family", International Journal of Scientific and Research Publications, 616-622.

[15] Nadeem Ansari and Chandel R S and Rizwana Jamal (2018), "*b-chromatic number of Prism graph families*", An International Journal of Applications of Applied Mathematics, 13, 961-964.