

# Cost Analysis to a Reliability Model for an Optical Communication Process with Bidirectional and Non-Revertible 1+1 Protection Switching Scheme

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## Abstract

Cost analysis of a system having bidirectional and non-revertible 1+1 protection switching scheme in optical communication process has been analyzed in the present paper. The scheme under consideration comprises four optical lines out of which two are operative (working paths) and the other two are hot-standby (protection paths). Signals are transmitted through working path and protection path from one user to other. We may call the users as user-1 and user-2. User-1 transmits signals through working path as well as protection path which are received by user-2 from working path only, similar transmission take place from user-2 to user-1. Working and protection paths are routed through different optical pipes. If anyone working path gets failed then signals are received from the protection paths by both users. The signals continue to remain receiving from protection paths even if the working paths get repaired before the failure of protection paths and this makes the process non-revertible. After getting the protection path repaired, if the working paths are operative, the protection paths again act as a hot standby.

**Keywords:** Bidirectional protection scheme, optical communication process, working path, protection path, regenerative points, availability, busy period and profit analysis.

## INTRODUCTION

Communication is a process by which information is transmitted and understood between two or more persons/organizations. Effective and fast communication is playing a key role for improving business processing, achieving cost effectiveness, deriving revenue growth and competitive advantages. And hence it is regarded as the foundation of any organization. Optical fiber has played a key role in stimulating the extra ordinary growth in worldwide communication and is vital in enabling the proliferating use of the internet.

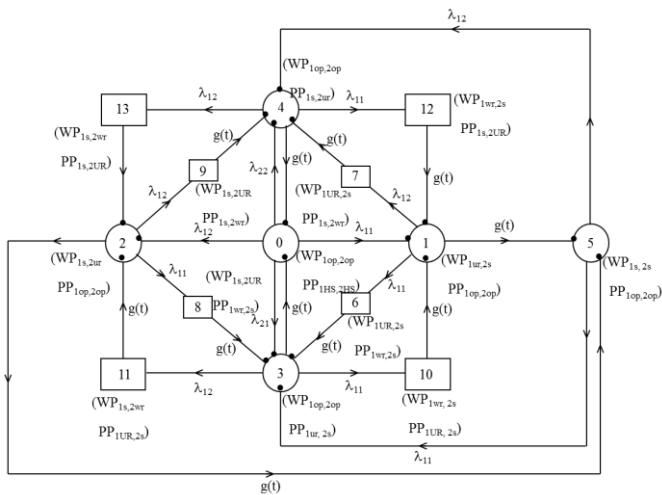
Reliability and availability have widely been studied by various researchers, though the mathematical models on various protection schemes along with making comparisons between various schemes have been discussed, yet models

considering various possibilities depending on failure and repair rates are still need to be developed. Keeping the above in view, cost analysis of a system having bidirectional and non-revertible 1+1 protection switching scheme in optical communication process has been analyzed in the present paper. The scheme under consideration comprises four optical lines out of which two are operative (working paths) and the other two are hot-standby (protection paths). Signals are transmitted through working path and protection path both from one user to other but received by 2<sup>nd</sup> user from working path only. And similar transmission take place from 2<sup>nd</sup> user to 1<sup>st</sup> user. Working and protection paths are routed through different optical pipes. If any one working path gets failed then signals are received from the protection paths by both users. The signals continue to remain receiving from protection paths even if the working paths get repaired before the failure of protection paths and this makes the process non-revertible. After getting the protection path repaired, if the working paths are operative, the protection paths again act as a hot standby.

## UNDERTAKEN ASSUMPTIONS

- Initially working path is taken as operative and the protection path act as hot standby.
- The system becomes inoperable if working path and its corresponding protection path get failed.
- All random variables are independent.
- The failure and repair times are assumed to be exponentially distributed.
- The failures are self-announcing and switching is perfect & instantaneous.
- No more than one unit can repair at a time.
- Repairing of already undertaken unit is not left in between.

**State Transition Diagram (Fig. 1)**



Failed State:  Op. State:  Regenerative Point: ●

**Notations**

- WP<sub>10P</sub> : Operative unit of 1<sup>st</sup> working path
- WP<sub>20P</sub> : Operative unit of 2<sup>nd</sup> working path
- PP<sub>10P</sub> : Operative unit of 1<sup>st</sup> protective path
- PP<sub>20P</sub> : Operative unit of 2<sup>nd</sup> protective path
- WP<sub>1ur</sub> : 1<sup>st</sup> working path is under repair
- WP<sub>2ur</sub> : 2<sup>nd</sup> working path is under repair
- PP<sub>1ur</sub> : 1<sup>st</sup> protective path is under repair
- PP<sub>2ur</sub> : 2<sup>nd</sup> protective path is under repair
- WP<sub>1HS</sub> : 1<sup>st</sup> working path is hot standby
- WP<sub>2HS</sub> : 2<sup>nd</sup> working path is hot standby
- PP<sub>1HS</sub> : 1<sup>st</sup> protective path is hot standby
- PP<sub>2HS</sub> : 2<sup>nd</sup> protective path is hot standby
- WP<sub>1UR</sub> : 1<sup>st</sup> working path is under repair from previous state
- WP<sub>2UR</sub> : 2<sup>nd</sup> working path is under repair from previous state
- PP<sub>1UR</sub> : 1<sup>st</sup> protective path is under repair from previous state
- PP<sub>2UR</sub> : 2<sup>nd</sup> protective path is under repair from previous state
- WP<sub>1wr</sub> : 1<sup>st</sup> working path is waiting for repair
- WP<sub>2wr</sub> : 2<sup>nd</sup> working path is waiting for repair
- PP<sub>1wr</sub> : 1<sup>st</sup> protective path is waiting for repair
- PP<sub>2wr</sub> : 2<sup>nd</sup> protective path is waiting for repair
- λ<sub>11</sub> : Failure rate of operative unit of 1<sup>st</sup> working path
- λ<sub>12</sub> : Failure rate of operative unit of 2<sup>nd</sup> working path
- λ<sub>21</sub> : Failure rate of hot standby unit of 1<sup>st</sup> protective path

λ<sub>22</sub> : Failure rate of hot standby unit of 2<sup>nd</sup> protective path  
 f(t): Repair rate of failed unit of working / protective path

**The Transition State Probabilities**

$$\begin{aligned}
 q_{01}(t) &= \lambda_{11} e^{-(\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22})t} \\
 q_{02}(t) &= \lambda_{12} e^{-(\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22})t} \\
 q_{03}(t) &= \lambda_{21} e^{-(\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22})t} \\
 q_{04}(t) &= \lambda_{22} e^{-(\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22})t} \\
 q_{15}(t) &= e^{-(\lambda_{11} + \lambda_{12})t} \cdot g(t) \\
 q_{16}(t) &= \lambda_{11} e^{-(\lambda_{11} + \lambda_{12})t} \cdot \bar{G}(t) \\
 q_{17}(t) &= \lambda_{12} e^{-(\lambda_{11} + \lambda_{12})t} \cdot \bar{G}(t) \\
 q_{13}^{(6)}(t) &= [\lambda_{11} e^{-(\lambda_{11} + \lambda_{12})t} \odot 1] \cdot g(t) \\
 q_{14}^{(7)}(t) &= [\lambda_{12} e^{-(\lambda_{11} + \lambda_{12})t} \odot 1] \cdot g(t) \\
 q_{25}(t) &= e^{-(\lambda_{11} + \lambda_{12})t} g(t) = q_{15}(t) \\
 q_{28}(t) &= \lambda_{11} e^{-(\lambda_{11} + \lambda_{12})t} \cdot \bar{G}(t) = q_{16}(t) \\
 q_{29}(t) &= \lambda_{12} e^{-(\lambda_{11} + \lambda_{12})t} \cdot \bar{G}(t) = q_{17}(t) \\
 q_{23}^{(8)}(t) &= [\lambda_{11} e^{-(\lambda_{11} + \lambda_{12})t} \odot 1] \cdot g(t) = q_{13}^{(6)}(t) \\
 q_{24}^{(9)}(t) &= [\lambda_{12} e^{-(\lambda_{11} + \lambda_{12})t} \odot 1] \cdot g(t) = q_{14}^{(7)}(t) \\
 q_{30}(t) &= e^{-(\lambda_{11} + \lambda_{12})t} g(t) = q_{15}(t) \\
 q_{3,10}(t) &= \lambda_{11} e^{-(\lambda_{11} + \lambda_{12})t} \cdot \bar{G}(t) = q_{16}(t) \\
 q_{3,11}(t) &= \lambda_{12} e^{-(\lambda_{11} + \lambda_{12})t} \cdot \bar{G}(t) = q_{17}(t) \\
 q_{31}^{(10)}(t) &= [\lambda_{11} e^{-(\lambda_{11} + \lambda_{12})t} \odot 1] \cdot g(t) = q_{13}^{(6)}(t) \\
 q_{32}^{(11)}(t) &= [\lambda_{12} e^{-(\lambda_{11} + \lambda_{12})t} \odot 1] \cdot g(t) = q_{14}^{(7)}(t) \\
 q_{40}(t) &= e^{-(\lambda_{11} + \lambda_{12})t} g(t) = q_{15}(t) \\
 q_{4,12}(t) &= \lambda_{11} e^{-(\lambda_{11} + \lambda_{12})t} \cdot \bar{G}(t) = q_{16}(t) \\
 q_{4,13}(t) &= \lambda_{12} e^{-(\lambda_{11} + \lambda_{12})t} \cdot \bar{G}(t) = q_{17}(t) \\
 q_{41}^{(12)}(t) &= [\lambda_{11} e^{-(\lambda_{11} + \lambda_{12})t} \odot 1] \cdot g(t) = q_{13}^{(6)}(t) \\
 q_{42}^{(13)}(t) &= [\lambda_{12} e^{-(\lambda_{11} + \lambda_{12})t} \odot 1] \cdot g(t) = q_{14}^{(7)}(t) \\
 q_{5,3}(t) &= \lambda_{11} e^{-(\lambda_{11} + \lambda_{12})t} \\
 q_{5,4}(t) &= \lambda_{12} e^{-(\lambda_{11} + \lambda_{12})t} \\
 \text{The nonzero element, } p_{ij} &\text{ is given by} \\
 p_{ij} &= \lim_{s \rightarrow 0} q_{ij}^*(s) \\
 p_{01} + p_{02} + p_{03} + p_{04} &= 1, \quad p_{15} + p_{16} + p_{17} = 1, \\
 p_{15} + p_{13}^{(6)} + p_{14}^{(7)} &= 1, \\
 p_{25} + p_{28} + p_{29} &= 1, \quad p_{25} + p_{23}^{(8)} + p_{24}^{(9)} = 1, \\
 p_{30} + p_{3,10} + p_{3,11} &= 1,
 \end{aligned}$$

$$\begin{aligned}
 p_{30} + p_{31}^{(10)} + p_{32}^{(11)} &= 1, & p_{40} + p_{4,12} + p_{4,13} &= 1, & Q_{3,10}Q_{03} + Q_{3,11}Q_{01}Q_{53}Q_{15} + Q_{3,11}Q_{02}Q_{53}Q_{25} + Q_{3,11}Q_{03} + \\
 p_{40} + p_{41}^{(12)} + p_{42}^{(13)} &= 1, & p_{53} + p_{54} &= 1, & Q_{4,12}Q_{01}Q_{15}Q_{54} + Q_{4,12}Q_{02}Q_{54}Q_{25} + Q_{4,12}Q_{04} + \\
 & & & & Q_{4,13}Q_{01}Q_{54}Q_{15} + Q_{4,13}Q_{02}Q_{54}Q_{25} + Q_{4,13}Q_{04}
 \end{aligned}$$

**Mean Sojourn Times**

The mean sojourn time ( $\mu_i$ ) in the regenerative state 'i' is defined as the time of stay before transition to any other state. If  $\mu_i$  denotes the mean sojourn time in the regenerative state i then

$$\mu_i = E(t)Pr[T > t]$$

$$\mu_0 = \frac{1}{(\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22})}$$

$$\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \frac{1-g^*(\lambda_{11}+\lambda_{12})}{(\lambda_{11}+\lambda_{12})}$$

**Mean Time to System Failure**

Recursive relation for MTSF i.e.  $\phi_i(t)$  are:

$$\phi_0(t) = Q_{01}(t) \otimes \phi_1(t) + Q_{02}(t) \otimes \phi_2(t) + Q_{03}(t) \otimes \phi_3(t) + Q_{04}(t) \otimes \phi_4(t)$$

$$\phi_1(t) = Q_{15}(t) \otimes \phi_5(t) + Q_{16}(t) + Q_{17}(t)$$

$$\phi_2(t) = Q_{25}(t) \otimes \phi_5(t) + Q_{2,8}(t) + Q_{2,9}(t)$$

$$\phi_3(t) = Q_{30}(t) \otimes \phi_0(t) + Q_{3,10}(t) + Q_{3,11}(t)$$

$$\phi_4(t) = Q_{40}(t) \otimes \phi_0(t) + Q_{4,12}(t) + Q_{4,13}(t)$$

$$\phi_5(t) = Q_{53}(t) \otimes \phi_3(t) + Q_{54}(t) \otimes \phi_4(t)$$

Taking Laplace-Stieltjes transforms of equations and solving for  $\phi_0^{**}(s)$ , we get

$$\phi_0^{**}(s) = \frac{N(s)}{D(s)}$$

Where

$$D(s) = \begin{vmatrix} 1 & -Q_{01} & -Q_{02} & -Q_{03} & -Q_{04} & 0 \\ 0 & 1 & 0 & 0 & 0 & -Q_{15} \\ 0 & 0 & 1 & 0 & 0 & -Q_{25} \\ -Q_{30} & 0 & 0 & 1 & 0 & 0 \\ -Q_{40} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -Q_{53} & -Q_{54} & 1 \end{vmatrix}$$

$$D(s) = 1 - Q_{53}Q_{15}Q_{01}Q_{30} - Q_{53}Q_{25}Q_{02}Q_{30} - Q_{30}Q_{03} - Q_{15}Q_{01}Q_{40}Q_{54} - Q_{40}Q_{04} - Q_{25}Q_{02}Q_{40}Q_{54}$$

$$N(s) = \begin{vmatrix} 1 & -Q_{01} & -Q_{02} & -Q_{03} & -Q_{04} & 0 \\ Q_{16} + Q_{17} & 1 & 0 & 0 & 0 & -Q_{15} \\ Q_{28} + Q_{29} & 0 & 1 & 0 & 0 & -Q_{25} \\ Q_{3,10} + Q_{3,11} & 0 & 0 & 1 & 0 & 0 \\ Q_{4,12} + Q_{4,13} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -Q_{53} & -Q_{54} & 1 \end{vmatrix}$$

$$N(s) = 1 + Q_{01}Q_{16} + Q_{01}Q_{17} + Q_{02}Q_{28} + Q_{02}Q_{29} + Q_{3,10}Q_{01}Q_{53}Q_{15} + Q_{3,10}Q_{02}Q_{53}Q_{25} +$$

The Mean Time to System Failure (MTSF) when the system starts from the state '0' is,

$$MTSF = T_0 = \lim_{s \rightarrow 0} \frac{1 - \phi_0^{**}(s)}{s}$$

Using L' Hospital rule and putting the value of  $\phi_0^{**}(s)$ , we have

$$T_0 = \frac{N}{D}$$

$$\begin{aligned}
 N &= \mu_0 + \mu_1 p_{01} + \mu_2 p_{02} \\
 &\quad + \mu_3 (p_{01} p_{15} p_{53} + p_{02} p_{25} p_{53} + p_{03}) \\
 &\quad + \mu_4 (p_{01} p_{15} p_{54} + p_{02} p_{25} p_{54} + p_{04}) \\
 &\quad + \mu_5 (p_{01} p_{15} + p_{02} p_{25})
 \end{aligned}$$

$$D = 1 - p_{53} p_{01} p_{15} p_{30} - p_{53} p_{02} p_{25} p_{30} - p_{30} p_{03} - p_{01} p_{15} p_{54} p_{40} - p_{40} p_{04} - p_{02} p_{25} p_{54} p_{40}$$

**Availability Analysis**

Recursive relations for  $A_i(t)$  are:

$$A_0(t) = M_0(t) + q_{01}(t) \otimes A_1(t) + q_{02}(t) \otimes A_2(t) + q_{03}(t) \otimes A_3(t) + q_{04}(t) \otimes A_4(t)$$

$$A_1(t) = M_1(t) + q_{15}(t) \otimes A_5(t) + q_{13}^{(6)}(t) \otimes A_3(t) + q_{14}^{(7)}(t) \otimes A_4(t)$$

$$A_2(t) = M_2(t) + q_{25}(t) \otimes A_5(t) + q_{23}^{(8)}(t) \otimes A_3(t) + q_{24}^{(9)}(t) \otimes A_4(t)$$

$$A_3(t) = M_3(t) + q_{30}(t) \otimes A_0(t) + q_{31}^{(10)}(t) \otimes A_1(t) + q_{32}^{(11)}(t) \otimes A_2(t)$$

$$A_4(t) = M_4(t) + q_{40}(t) \otimes A_0(t) + q_{41}^{(12)}(t) \otimes A_1(t) + q_{42}^{(13)}(t) \otimes A_2(t)$$

$$A_5(t) = M_5(t) + q_{53}(t) \otimes A_3(t) + q_{54}(t) \otimes A_4(t)$$

Where

$$\begin{aligned}
 M_0(t) &= e^{-(\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22})t} \\
 M_1(t) &= e^{-(\lambda_{12} + \lambda_{21})t} \overline{G}(t)
 \end{aligned}$$

$$\begin{aligned}
 M_2(t) &= e^{-(\lambda_{12} + \lambda_{21})t} \overline{G}(t) \\
 M_3(t) &= e^{-(\lambda_{12} + \lambda_{21})t} \overline{G}(t)
 \end{aligned}$$

$$\begin{aligned}
 M_4(t) &= e^{-(\lambda_{12} + \lambda_{21})t} \overline{G}(t) \\
 M_5(t) &= e^{-(\lambda_{12} + \lambda_{21})t} \overline{G}(t)
 \end{aligned}$$

The Availability of the system, in steady state is given by

$$A_0 = \lim_{s \rightarrow 0} sA_0^*(s) = \frac{N_1}{D_1},$$

$$D_1 = \mu_0 p_{15} + K_1(p_{13}^{(6)} + p_{01} p_{15}) + K_2(p_{14}^{(7)} + p_{02} p_{15}) + K_3(p_{13}^{(6)} - p_{04}(p_{15})^2 p_{53}) + p_{03}(p_{15})^2 p_{54} + p_{15} p_{53} - p_{04} p_{15} p_{13}^{(6)} + p_{03} p_{15} p_{14}^{(7)} + K_4(p_{14}^{(7)} - p_{03}(p_{15})^2 p_{54}) + p_{04}(p_{15})^2 p_{53} + p_{15} p_{54} - p_{03} p_{15} p_{14}^{(7)} + p_{04} p_{15} p_{13}^{(6)} + \mu_5 [p_{15}(p_{13}^{(6)} + p_{14}^{(7)}) + p_{01} p_{15} + p_{02} p_{15}]$$

$$N_1 = \mu_4 + p_{01} p_{15} \mu_1 + p_{01}(p_{15})^2 \mu_5 + p_{02}(p_{15})^2 \mu_5 + p_{02} p_{15} \mu_2 + p_{13}^{(6)} \mu_1 + p_{53}(p_{15})^2 \mu_4 - p_{53}(p_{15})^2 \mu_3 + p_{13}^{(6)} \mu_4 p_{15} - p_{13}^{(6)} \mu_3 p_{15} - (p_{15})^2 \mu_5 + p_{15} \mu_0 - p_{15} \mu_2 - p_{13}^{(6)} \mu_2 + \mu_2 - p_{53} p_{15} \mu_4 + p_{53} p_{15} \mu_3 - p_{03} p_{15} \mu_4 + p_{03} p_{15} \mu_3 + p_{13}^{(6)} \mu_3 + p_{13}^{(6)} \mu_3 p_{01} p_{15} + p_{13}^{(6)} \mu_3 p_{02} p_{15} - p_{53} p_{01} \mu_4 (p_{15})^2 - p_{53} p_{02} \mu_4 (p_{15})^2 + p_{53} p_{01} \mu_3 (p_{15})^2 + p_{53} p_{02} \mu_3 (p_{15})^2 - p_{13}^{(6)} \mu_4 - p_{13}^{(6)} \mu_4 p_{01} p_{15} - p_{13}^{(6)} \mu_4 p_{02} p_{15} + p_{15} \mu_5$$

### Busy Period Analysis of Ordinary Repairman

Recursive relations for  $B_i(t)$  are:

$$B_0(t) = q_{01}(t) \odot B_1(t) + q_{02}(t) \odot B_2(t) + q_{03}(t) \odot B_3(t) + q_{04}(t) \odot B_4(t)$$

$$B_1(t) = W_1(t) + q_{15}(t) \odot B_5(t) + q_{13}^{(6)}(t) \odot B_3(t) + q_{14}^{(7)}(t) \odot B_4(t)$$

$$B_2(t) = W_2(t) + q_{25}(t) \odot B_5(t) + q_{23}^{(8)}(t) \odot B_3(t) + q_{24}^{(9)}(t) \odot B_4(t)$$

$$B_3(t) = W_3(t) + q_{30}(t) \odot B_0(t) + q_{31}^{(10)}(t) \odot B_1(t) + q_{32}^{(11)}(t) \odot B_2(t)$$

$$B_4(t) = W_4(t) + q_{40}(t) \odot B_0(t) + q_{41}^{(12)}(t) \odot B_1(t) + q_{42}^{(13)}(t) \odot B_2(t)$$

$$B_5(t) = W_5(t) + q_{53}(t) \odot B_3(t) + q_{54}(t) \odot B_4(t)$$

Where

$$W_1(t) = W_2(t) = W_3(t) = W_4(t) = W_5(t) = \overline{G}(t)$$

Taking Laplace transform of equations and solving them for  $B_0^*(s)$ . In steady-state the total fraction of the time for which the system is under repair of the repairman is given by

$$B_0 = \lim_{s \rightarrow 0} sB_0^*(s) = \frac{N_2}{D_1}$$

$$\text{Where } N_2 = \mu_2 - p_{13}^{(6)} \mu_4 - p_{13}^{(6)} \mu_4 p_{01} p_{15} - p_{13}^{(6)} \mu_4 p_{02} p_{15} + p_{13}^{(6)} \mu_3 + p_{13}^{(6)} \mu_3 p_{01} p_{15} + p_{13}^{(6)} \mu_3 p_{02} p_{15} + p_{15} \mu_5 - (p_{15})^2 \mu_4 p_{01} p_{53} - (p_{15})^2 \mu_4 p_{02} p_{53} + (p_{15})^2 \mu_3 p_{01} p_{53} + (p_{15})^2 \mu_3 p_{02} p_{53} + \mu_4 + p_{13}^{(6)} \mu_4 p_{15} - p_{13}^{(6)} \mu_3 p_{15} + p_{01} p_{15} \mu_1 + p_{01}(p_{15})^2 \mu_5 + p_{02}(p_{15})^2 \mu_5 + p_{03} p_{15} \mu_3 - (p_{15})^2 \mu_5 - p_{03} p_{15} \mu_4 + (p_{15})^2 \mu_4 p_{53} + p_{13}^{(6)} \mu_1 + p_{02} p_{15} \mu_2 - (p_{15})^2 \mu_3 p_{53} - p_{15} \mu_2 - p_{13}^{(6)} \mu_2 - p_{53} p_{15} \mu_4 + p_{53} p_{15} \mu_3 + p_{15}$$

and  $D_1$  is already specified

### Expected Number of Visits

Recursive relations for  $V_i(t)$ :

$$V_0(t) = Q_{01}(t) \odot [V_1(t) + 1] + Q_{02}(t) \odot [V_2(t) + 1] + Q_{03}(t) \odot [V_3(t) + 1] + Q_{04}(t) \odot [V_4(t) + 1]$$

$$V_1(t) = Q_{15}(t) \odot V_5(t) + Q_{13}^{(6)}(t) \odot V_3(t) + Q_{14}^{(7)}(t) \odot V_4(t)$$

$$V_2(t) = Q_{25}(t) \odot V_5(t) + Q_{23}^{(8)}(t) \odot V_3(t) + Q_{24}^{(9)}(t) \odot V_4(t)$$

$$V_3(t) = Q_{30}(t) \odot V_0(t) + Q_{31}^{(10)}(t) \odot V_1(t) + Q_{32}^{(11)}(t) \odot V_2(t)$$

$$V_4(t) = Q_{40}(t) \odot V_0(t) + Q_{41}^{(12)}(t) \odot V_1(t) + Q_{42}^{(13)}(t) \odot V_2(t)$$

$$V_5(t) = Q_{53}(t) \odot V_3(t) + Q_{54}(t) \odot V_4(t)$$

Taking Laplace transforms of equations and solving them for  $V_0^*(s)$ . In study-state the expected no. of visits of expert repairman is given by  $V_0 = \lim_{s \rightarrow 0} sV_0^*(s) = \frac{N_3}{D_1}$ ,  
 Where  $N_3 = p_{15}$  and  $D_1$  is already specified.

### Profit Analysis

Expected profit incurred to system is given by

$$\text{Profit } (P) = C_0 A_0 - C_1 B_0 - C_2 V_0$$

Where

$C_0$  = Revenue per unit up time

$C_1$  = Cost per unit up time for which the repairman is busy for repair

$C_2$  = Cost per visit of repairman

### Numerical Results and Discussion

Considering  $g(t) = \alpha e^{-\alpha t}$

$$\text{We have } p_{01} = \frac{\lambda_{11}}{\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22}}$$

$$p_{02} = \frac{\lambda_{12}}{\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22}}$$

$$p_{03} = \frac{\lambda_{21}}{\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22}}$$

$$p_{04} = \frac{\lambda_{22}}{\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22}}$$

$$p_{15} = p_{25} = p_{30} = p_{40} = \frac{\alpha}{\lambda_{11} + \lambda_{12} + \alpha},$$

$$p_{16} = p_{28} = p_{3,10} = p_{4,12} = \frac{\lambda_{11}}{\lambda_{11} + \lambda_{12} + \alpha},$$

$$p_{17} = p_{29} = p_{3,11} = p_{4,13} = \frac{\lambda_{12}}{\lambda_{11} + \lambda_{12} + \alpha},$$

$$p_{53} = \frac{\lambda_{11}}{\lambda_{11} + \lambda_{12}}, p_{54} = \frac{\lambda_{12}}{\lambda_{11} + \lambda_{12}}$$

$$p_{13}^{(6)} = p_{23}^{(8)} = p_{31}^{(10)} = p_{41}^{(12)} = \frac{\lambda_{11}}{\lambda_{11} + \lambda_{12} + \alpha},$$

$$p_{14}^{(7)} = p_{24}^{(9)} = p_{32}^{(11)} = p_{42}^{(13)} = \frac{\lambda_{12}}{\lambda_{11} + \lambda_{12} + \alpha}$$

Fig. 2 shows the behavior of MTSF with respect to Repair Rate ( $\alpha$ ) for different value of failure rate ( $\lambda_{11}$ ). The graph reveals that MTSF get increased with the increase in the values of repair rate ( $\alpha$ ) but have lower values for higher values of  $\lambda_{11}$ .

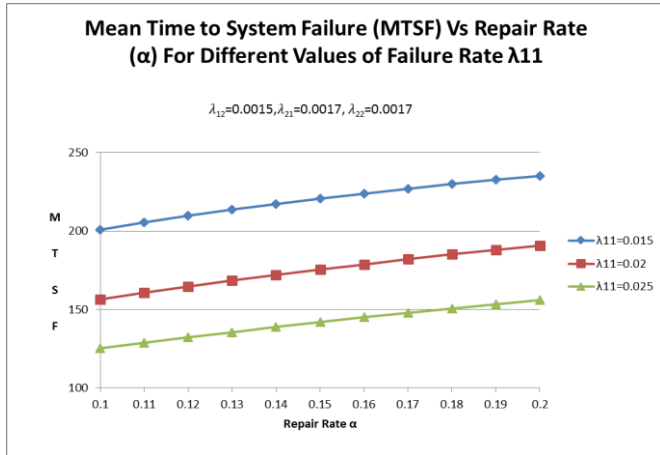


Fig. 2

Fig. 3 shows the behavior of Availability ( $A_0$ ) with respect to repair rate ( $\alpha$ ) for different value of Failure Rate,  $\lambda_{11}$ . The graph reveals that Availability ( $A_0$ ) get increased with the increase in the values of repair rate ( $\alpha$ ) but have lower values for higher values of  $\lambda_{11}$ .

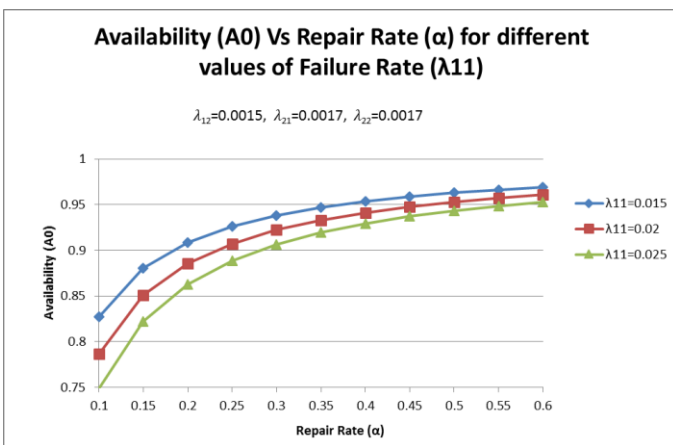


Fig. 3

Fig. 4 represents the behavior of profit (P) with respect to Failure Rate ( $\lambda_{11}$ ) for different values Failure Rate ( $\lambda_{21}$ ). It is observed from the graphs:

The profit decreases with the increase in the values of Failure Rate ( $\lambda_{11}$ ) and has higher for lower values of  $\lambda_{21}$ .

- i. For  $\lambda_{21} = 0.0010$ , the profit is positive or zero or negative according as Failure Rate ( $\lambda_{11}$ ) < or = or > 0.0030 and hence failure rate should not be greater than 0.00295.
- ii. For  $\lambda_{21} = 0.0015$ , the profit is positive or zero or negative according as Failure Rate ( $\lambda_{11}$ ) < or = or > 0.0028 and hence failure rate should not be greater than 0.00275.

- iii. For  $\lambda_{21} = 0.0020$ , the profit is positive or zero or negative according as Failure Rate ( $\lambda_{11}$ ) < or = or > 0.0025 and hence failure rate should not be greater than 0.00245

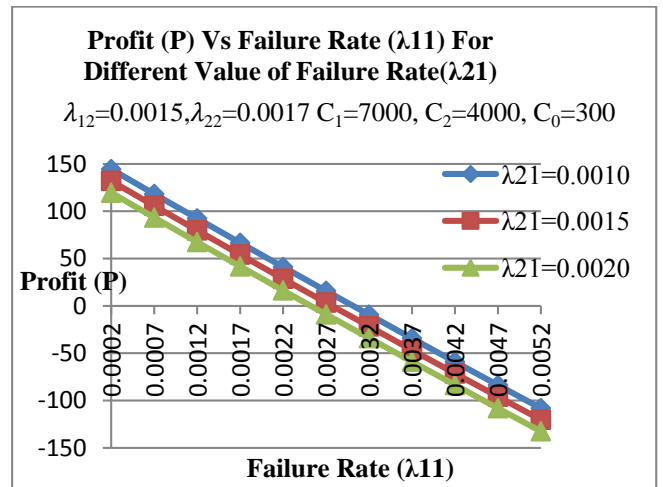


Fig. 4

Fig.5 depicts the behavior of profit (P) with respect to revenue per unit up time ( $C_0$ ) for different values Failure Rate ( $\lambda_{11}$ ). It is observed from the graphs:

The profit increases with the increase in the values of  $C_0$  and has higher for lower values of  $\lambda_{11}$

- i. For  $\lambda_{11} = 0.0010$ , the profit is positive or zero or negative according as  $C_0 >$  or = or < 214.3333 and hence revenue per unit up time should be fixed not less than 214.3333.
- ii. For  $\lambda_{11} = 0.0015$ , the profit is positive or zero or negative according as  $C_0 >$  or = or < 240.133 and hence revenue per unit up time should be fixed not less than 240.133.
- iii. For  $\lambda_{11} = 0.0020$ , the profit is positive or zero or negative according as  $C_0 >$  or = or < 265.8123 and hence revenue per unit up time should be fixed not less than 265.8123.

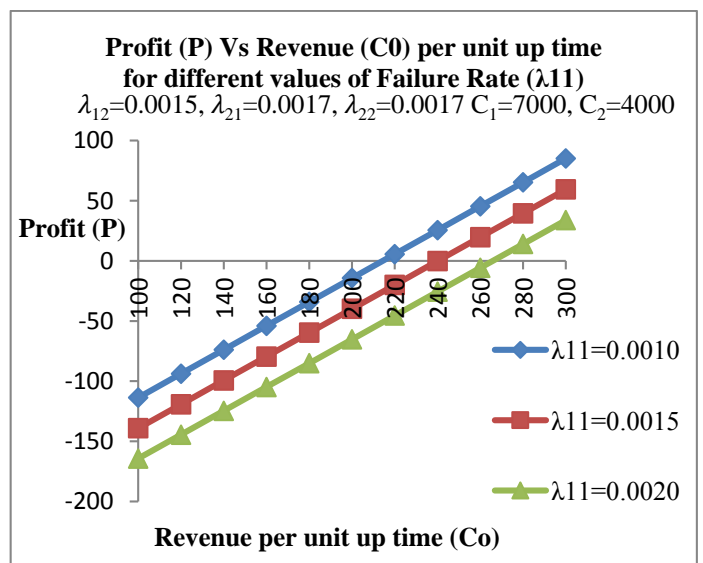


Fig. 5

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