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# Influence of an inclined magnetic field and radiation on unsteady free convective flow of a fluid through a porous medium in a channel with adiabatic

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**Abstract:**

In view of these, we studied the unsteady free convective flow of an incompressible viscous electrically conducting fluid through a porous medium under the action of an inclined magnetic field between two heated vertical plates by keeping one plate is adiabatic. The governing equations of velocity and temperature fields with appropriate boundary conditions are solved by using perturbation technique. The effects of various physical parameters on the velocity and temperature fields are discussed in detail with the help of graphs.

## 1. INTRODUCTION

The interaction between the magnetic field and the conducting fluid drastically modifies the flow, with effects on such significant flow properties as heat transfer, the fact character of which is strongly dependent on the orientation of the magnetic field. When fluid flows through a magnetic field, an electric field or consequently a current may be induced, and in turn the current interacts with the magnetic field to produce a body force on fluid. The production of this current has led to MHD power generators, MHD devices, nuclear engineering and the possibility of thermonuclear power has created a great practical need for understanding the dynamics of conducting. The influence of a magnetic field in viscous incompressible flow of electrically conducting fluid is of use in extrusion of plastics in the manufacture of rayon, nylon etc. The unsteady transient free convection flow of an incompressible dissipative viscous fluid past an infinite vertical plate on taking into account the viscous dissipative heat under the influence of a uniform transverse magnetic field was analyzed by Sreekant et al [1]. (2001). Gourla and Katoch [2] (1991) have studied the unsteady free convection MHD flow between two heated vertical plates. Recently, Bhaskar [3] (2013) have studied the unsteady free convection MHD flow of an incompressible viscous electrically conducting fluid under the action of transverse uniform magnetic field between two heated vertical plates by keeping one plate adiabatic. MHD Poiseuille flow between two infinite parallel porous plates in an inclined magnetic field was analyzed by Manyonge et al [4] (2012). The unsteady MHD poiseuille flow between two infinite parallel plates in an inclined magnetic field with heat transfer has been studied by Idowu et al [5] (2014). Simon [6] (2014) have investigated the effect of heat of transfer on unsteady MHD couette flow between two infinite parallel porous plates in an inclined magnetic field.

However, flow through a porous medium has been of significant interest in recent years particularly among geophysical fluid dynamicists. Examples of natural porous media are beach sand, stand stone, limestone, rye bread, wood, the human lung, bile duct, gall bladder with stones and in small blood vessels. Influence of MHD and radiation effects on oscillatory flow through a porous medium with constant suction velocity has been studied by El-Hakeem [7] (2000). Makinde and Mhone [8] (2005) have investigated the heat transfer and MHD effects on an oscillatory flow in a channel filled with porous medium.

Recently Raghunath and Siva Prasad [ 9] have investigated Heat and mass transfer on MHD flow of Non-Newtonian fluid over an infinite vertical porous plate. Raghunath and Siva [10] Prasad Indigested Heat and mass transfer on Unsteady MHD flow of a second grade fluid through porous medium between two vertical plates.

## 2. MATHEMATICAL FORMULATION

We consider the free convective unsteady MHD flow of a viscous incompressible electrically conducting fluid between two heated vertical parallel plates filled with porous medium. Let $x$ - axis be taken along the vertically upward direction through the central line of the channel and the $y$ - axis is perpendicular to the x-axis. The plates of the channel are kept at $y = \pm h$ distance apart. A uniform magnetic field $B_0$ acts at an angle $\alpha$ $\left(0 \le \alpha \le \frac{\pi}{2}\right)$, to the $y$ -axis $u$ is the velocity in the direction of flow of fluid, along the $x$ -axis and $v$ is the velocity along the $y$ -axis. Consequently, $u$ is a function of $y$ and $t$, but $v$ is independent of $y$. The fluid is assumed to be of low conductivity, such that the induced magnetic field is negligible. In order to derive the equations of the problem, we assume that the fluid is finitely conducting and the viscous dissipation the Joule heats are neglected. The polarization effect is also neglected.

At time $t > 0$, the temperature of the plate at $y = h$ changes according to the temperature function: $T = T_0 + (T_w - T_0)(1 - e^{-n't'})$, where $T_w$ and $T_0$ are the temperature at the plates $y = h$ and at $y = -h$ respectively, and $n(\ge 0)$ is a real number, denoting the decay factor. The equations governing the flow field are given by

$$\frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_0) - \left(\frac{\nu}{k} + \frac{\sigma B_0^2}{\rho}\cos^2\alpha\right)u \qquad (2)$$

$$\frac{\partial T}{\partial t} = \frac{K}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q}{\partial y} \qquad (3)$$

here ρ is the density of the fluid, $B_0$ is the magnetic field strength, σ is the electrical conductivity of the fluid, ν is the co-efficient of kinematic viscosity, $k$ is the permeability of the porous medium. $K$ is the thermal conductivity of the fluid, $c_p$ is the specific heat at constant pressure, β is the co-efficient of thermal expansion, $g$ is the acceleration due to gravity, $T$ is the temperature of the fluid and $q$ is the radiative heat flux. Following Cogley et al. (1968), it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by

$$\frac{\partial q}{\partial y} = 4\alpha_1^2 (T_0 - T) \qquad (4)$$

here $\alpha_1$ is the mean radiation absorption coefficient.

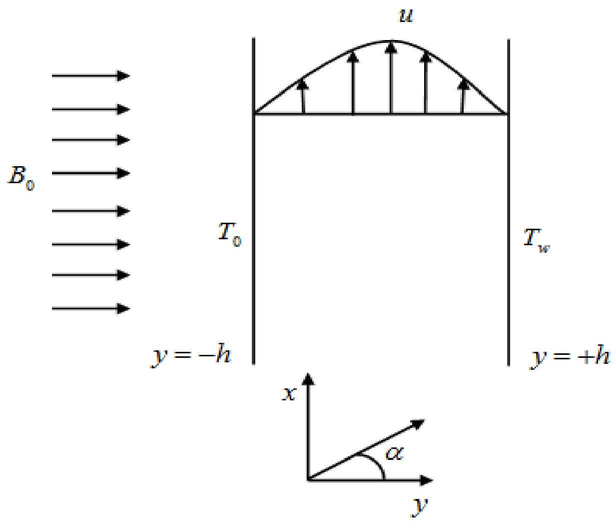


**Fig. 2.1** The physical model

The initial and boundary conditions for the problem are

$t = 0$: $u = 0, T = T_0$ for all $-h \le y \le h$

$t > 0$: $u = 0, T = T_0 + (T_w - T_0)(1 - e^{-nt})$ for $y = h$

$u = 0, \frac{\partial T}{\partial y} = 0$ for $y = -h$ (5)

The non-dimensional variables are

$$\bar{u} = \frac{\nu u}{\beta g h^2 (T_w - T_0)}, \bar{y} = \frac{y}{h}, \bar{T} = \frac{T - T_0}{T_w - T_0},$$

$$\bar{t} = \frac{\nu t}{h^2}, P_r = \frac{\mu C_p}{K}, \bar{n} = \frac{h^2 n}{\nu}, M = B_0 h\sqrt{\frac{\sigma}{\mu}}$$

$$Da = \frac{k}{h^2}\ ,\ R^2 = \frac{4\alpha_1{}^2 h^2}{k} \qquad (6)$$

in which Pr is the Prandtl number, $Da$ is the Darcy number and $M$ is the Hartmann number.

Using the non-dimensional variables (5) in to the equations (2) and (3), we obtain

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - N^2 u + T \qquad (7)$$

$$\Pr\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial y^2} + R^2 T \qquad (8)$$

here $N = \sqrt{\frac{1}{Da} + M^2\cos^2\alpha}$ .

Under the above non-dimensional quantities, the corresponding boundary conditions reduces to

$$t = 0:\ u = 0, T = 0 \quad \text{for} \quad -1 \le y \le 1$$

$$t > 0: u = 0, T = (1 - e^{-nt}) \quad \text{for} \quad y = 1$$

$$u = 0, \frac{\partial T}{\partial y} = 0 \quad \text{for} \quad y = -1$$

(9)

## 3. SOLUTION OF THE PROBLEM

We look for a regular perturbation series solution to solve the Equations (7) and (8) of the form

$$u = u_o(y) + e^{-nt}u_1(y) \qquad (10)$$

$$T = T_0(y) + e^{-nt}T_1(y) \qquad (11)$$

Substituting Equations (10) and (11) into the Equations (6) – (8) and solving the resultant Equations, we obtain

$$u = \begin{bmatrix} c_1\cosh Ny + c_2\sinh Ny \\ +\frac{\cos R(1+y)}{(R^2+N^2)\cos 2R} \end{bmatrix} + \begin{pmatrix} c_3\cosh m_2 y + c_4\sinh m_2 y \\ -\left(\frac{1}{m_1^2+m_2^2}\right)\frac{\cos m_1(1+y)}{\cos 2m_1} \end{pmatrix} e^{-nt} \qquad (12)$$

and

$$T = \frac{\cos R(1+y)}{\cos(2R)} - \frac{\cos m_1(1+y)}{\cos(2m_1)}e^{-nt} \qquad (13)$$

Here $m_1 = \sqrt{R^2 + n\Pr}$ and $m_2 = \sqrt{N^2 - n}$ .

The rate of heat transfer coefficient in terms of Nusselt number $Nu$ at the plate $y = 1$of the channel is given by

$$Nu = -\left.\frac{\partial\theta}{\partial y}\right|_{y=1} = R\tan 2R - \left(m_1\tan(2m_1)\right)e^{-nt} \qquad (14)$$

As $\alpha \to 0$ and $Da \to \infty$ our results coincides with the results of Bhaskar (2013).

## 4. RESULTS AND DISCUSSIONS

In order to see the effect of various physical parameters on the velocity and temperature, we plotted Figs. 2 – 6. The variation of velocity $u$ with inclination angle α for $\Pr = 0.71$ ,$M = 1$, $Da = 0.1$ , $n = 1$ and $t = 1$ is shown in **Fig. 2** It is found that, the velocity increases with an increase in $\alpha$. **Fig. 3** depicts the variation of velocity $u$ with Hartmann number $M$ for $\Pr = 0.71$, $Da = 0.1$, $\alpha = \frac{\pi}{6}$, $n = 1$ and $t = 1$. It is noticed that, the velocity $u$ decreases on increasing $M$ . The variation of velocity $u$ with Darcy number $Da$ for $\Pr = 0.71$ , $\alpha = \frac{\pi}{6}$, $n = 1$ and $t = 1$ is shown in **Fig. 4**. It is noticed that the velocity $u$ increases with increasing $Da$ . **Fig. 5** illustrates the variation of velocity $u$ with decay parameter $n$ for $\Pr = 0.71$ , $Da = 0.01$, $\alpha = \frac{\pi}{6}$, $M = 2$ and $t = 1$. It is observed that, the velocity $u$ decreases with an increase in $n$. The variation of velocity $u$ with Prandtl number Pr for $n = 1$ , $\alpha = \frac{\pi}{6}$, $Da = 0.01$, $M = 2$ and $t = 1$ is presented in **Fig. 6**. It is found that, the velocity $u$ decreases with increasing Pr . **Fig. 7** shows the variation of temperature $T$ with decay parameter $n$ for $\Pr = 0.71$ and $t = 1$. It is found that the temperature $T$ decreases with increasing $n$ . The variation of temperature $T$ with Prandtl number Pr for $n = 1$ and $t = 1$ is shown in

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**Fig.8**. It is observed that the temperature $T$ decreases with increasing Prandtl number $\Pr$.

Table-1 shows the effects of $\Pr$ and $n$ on Nusselt number $Nu$ for $t=1$. It is found that, the $Nu$ decreases with increasing $\Pr$ and $n$.

## 5. CONCLUSIONS

In this chapter, we studied the unsteady free convective flow of an incompressible viscous electrically conducting fluid through a porous medium under the action of an inclined magnetic field between two heated vertical plates by keeping one plate is adiabatic. The expressions for velocity and temperature are obtained by using regular perturbation technique. It is found that the velocity increases with increasing Darcy number $Da$ and $\alpha$, while it decreases with increasing $M, \Pr$ and $n$ and the temperature decreases with increasing $\Pr$ and $n$. It is found that, the $Nu$ decreases with increasing $\Pr$ and $n$.

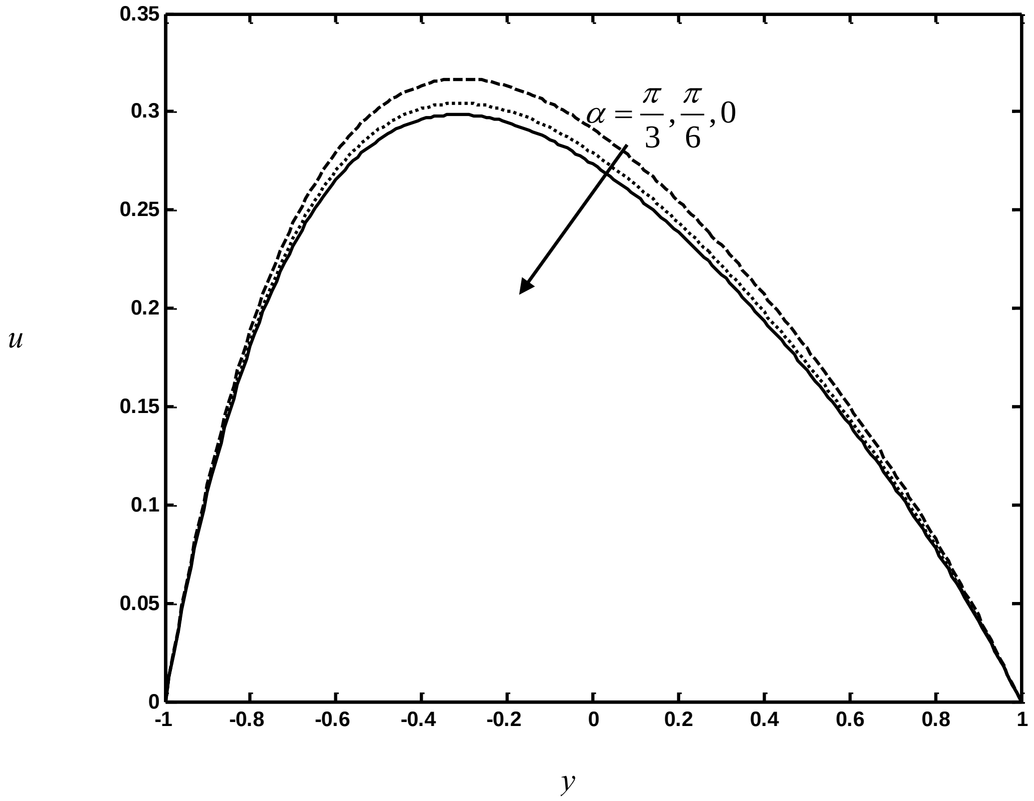


**Fig. 2** The variation of velocity $u$ with inclination angle $\alpha$ for $\Pr = 0.71$, $M = 1$, $Da = 0.1$, $n = 1$ and $t = 1$.

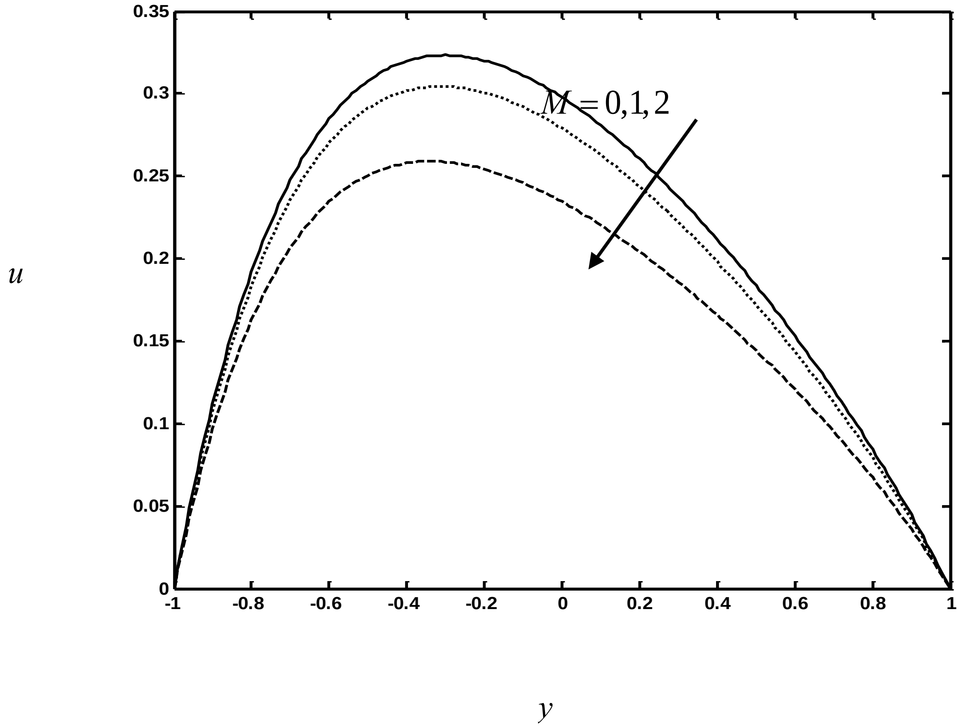


**Fig. 3** The variation of velocity $u$ with Hartmann number $M$ for $\Pr = 0.71$, $Da = 0.1$, $\alpha = \frac{\pi}{6}$, $n = 1$ and $t = 1$.

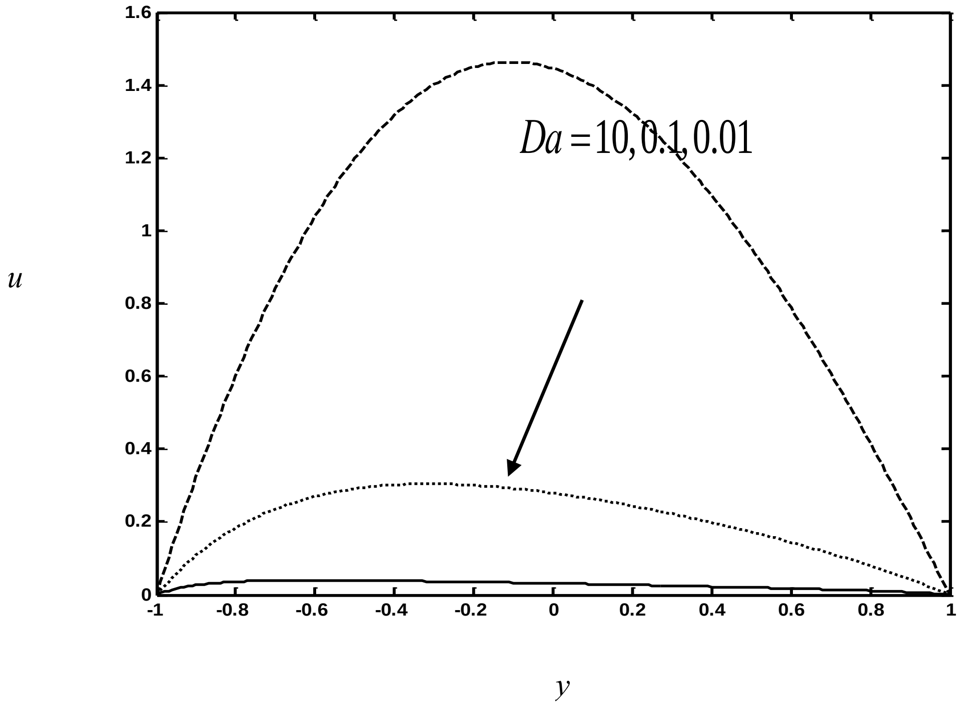


**Fig. 4** The variation of velocity $u$ with Darcy number $Da$ for $\Pr = 0.71$, $\alpha = \frac{\pi}{6}$, $M = 1$, $n = 1$ and $t = 1$.

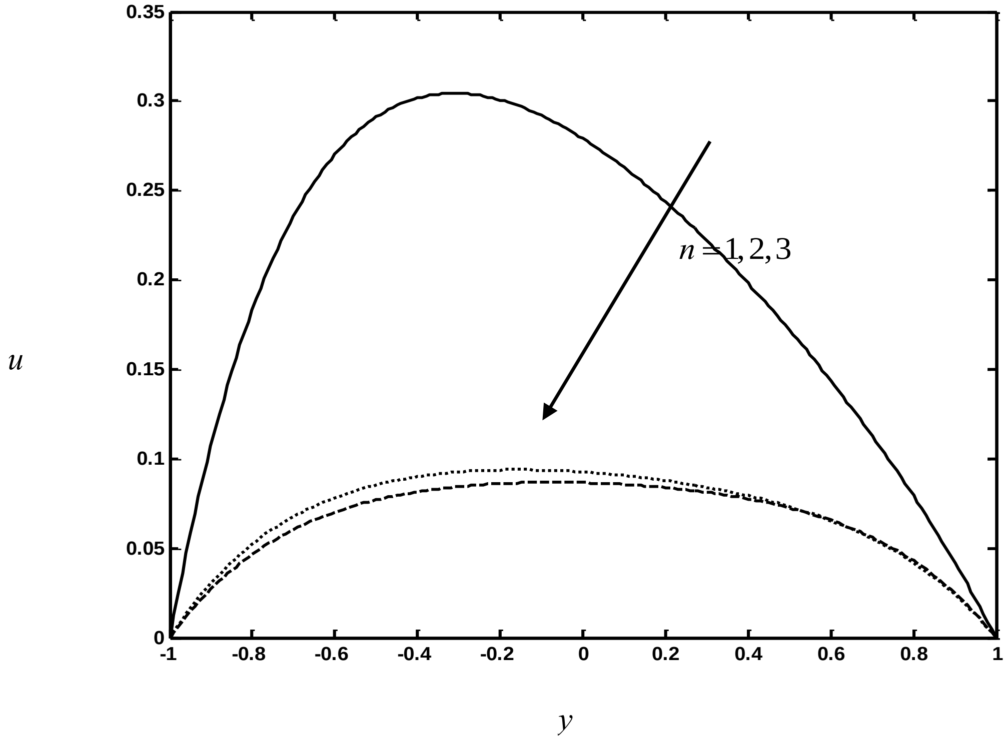


**Fig. 5** The variation of velocity $u$ with $n$ for $\Pr = 0.71$ , $\alpha = \dfrac{\pi}{6}$, $Da = 0.1$, $M = 1$ and $t = 1$.

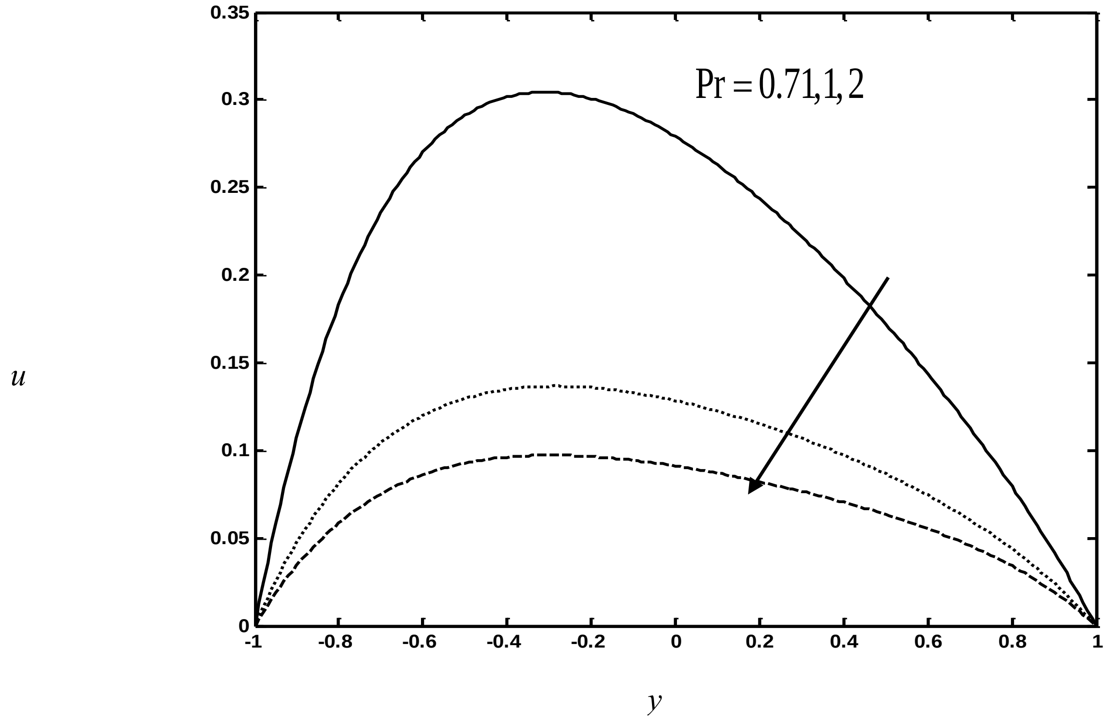


**Fig. 6** The variation of velocity $u$ with Prandtl number Pr for $M = 1$ , $\alpha = \dfrac{\pi}{6}$ , $Da = 0.1$ , $n = 1$ and $t = 1$.

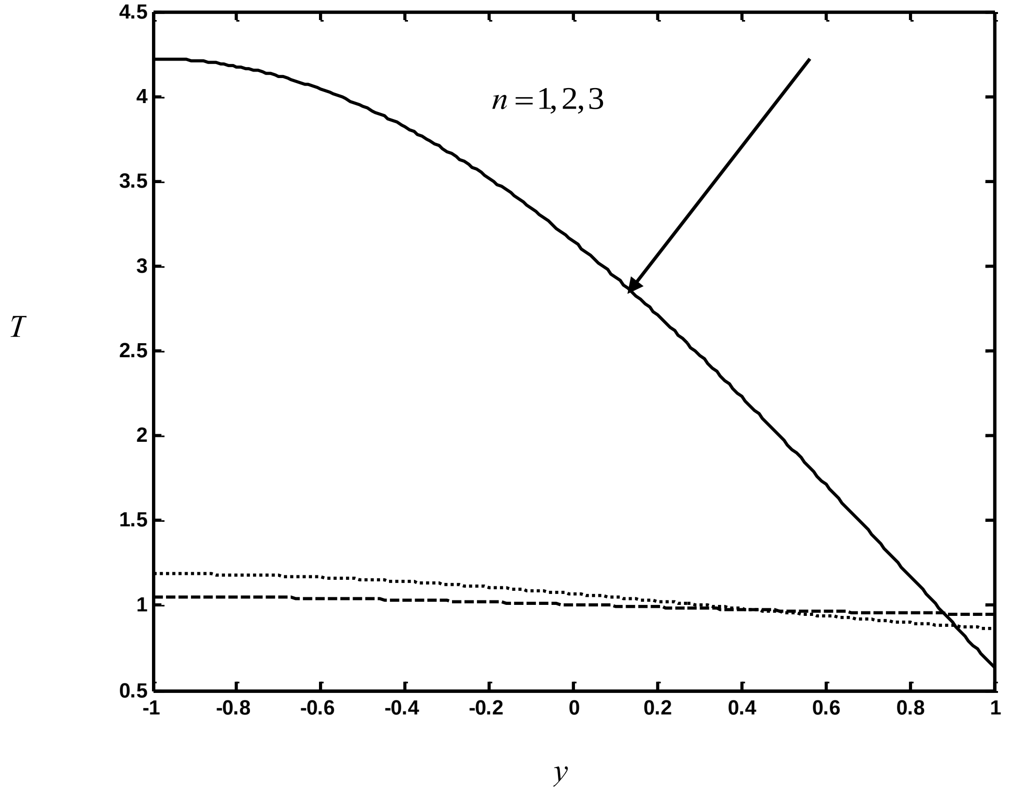


**Fig..7** The variation of temperature $T$ with $n$ for $\Pr = 0.71$ and $t = 1$

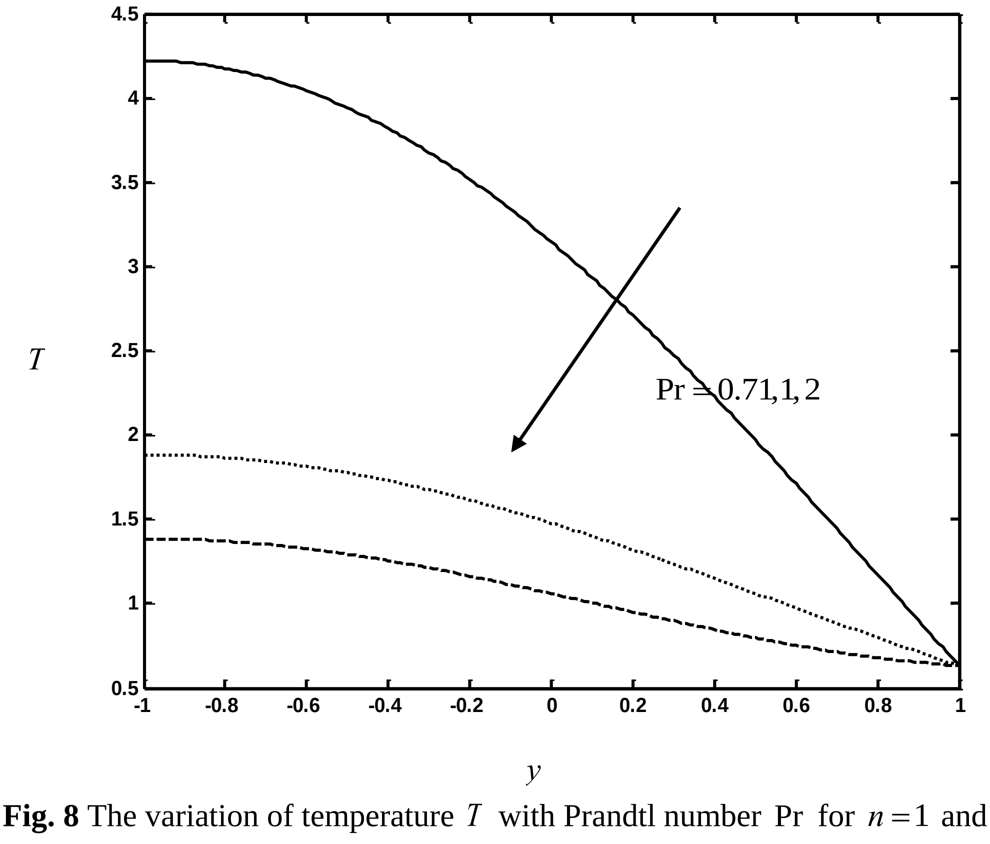


**Fig. 8** The variation of temperature $T$ with Prandtl number $\Pr$ for $n = 1$ and $t = 1$.

**Table-2.1:** Effects of Pr and $n$ on Nusselt number $Nu$ for $t=1$.

| Pr | $n$ | $Nu$ |
|---|---|---|
| **0.71** | **1** | **2.6970** |
| **1** | **1** | **0.8038** |
| **0.71** | **2** | **0.1528** |

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