International Journal of Applied Engineering Research ISSN 0973-4562 Volume 14, Number 7 (2019) pp. 1566-1569

Movable Outer-connected Domination in Graphs

Jocecar Lomarda-Hinampas

College of Teacher Education
Bohol Island State University-Main Campus
CPG North Avenue, 6300 Tagbilaran City, Bohol, Philippines

Abstract
A nonempty set $S \subseteq V(G)$ is a 1-*movable outer-connected dominating set* of G if $S = V(G)$ and for every $v \in S$, $S \setminus \{v\}$ is an outer-connected dominating set of G or $S \subset V(G)$ is an outer-connected dominating set of G and for every $v \in S$, either $S \setminus \{v\}$ is an outer-connected dominating set of G or there exists $u \in (V(G) \setminus S) \cap N_G(v)$ such that $(S \setminus \{v\}) \cup \{u\}$ is an outer-connected dominating set of G. The cardinality of the smallest 1-movable outer-connected dominating set of G is called 1-*movable outer-connected domination number* of G, denoted by $\widetilde{\gamma}^1_{mc}(G)$. A 1-movable outer-connected domination number of G with cardinality equal to $\widetilde{\gamma}^1_{mc}(G)$ is called $\widetilde{\gamma}^1_{mc}$-*set* of G. This paper presents characterizations of 1-movable outer-connected dominating sets in the join of two graphs and the corresponding values or bounds of the parameter are determined.

Mathematics Subject Classification: 05C69

Keywords: Domination, outer-connected domination, 1-movable domination 1-movable outer-connected domination

1. INTRODUCTION

Let $G = (V(G), E(G))$ be a graph and $v \in V(G)$. The *open neighborhood* of v is the set $N_G(v) = N(v) = \{u \in V(G) : uv \in E(G)\}$ and the *closed neighborhood* of v is the set $N_G[v] = N[v] = N(v) \cup \{v\}$. If $S \subseteq V(G)$, then the *open neighborhood* of S is the set $N_G(S) = N(S) = \cup_{v \in S} N_G(v)$ and the *closed neighborhood* of S is the set $N_G[S] = N[S] = S \cup N(S)$.

A subset S of $V(G)$ is a *dominating set* of G if for every $v \in V(G) \backslash S$, there exists $u \in S$ such that $uv \in E(G)$. A dominating set S is an *outer-connected dominating set* of G if the subgraph $\langle V(G) \setminus S \rangle$ is connected. The *outer-connected domination number* of G denoted by $\widetilde{\gamma}_c(G)$, is the cardinality of the smallest outer-connected dominating set of G. Outer-connected dominating sets in graphs and some of its variants were investigated in [1] [3], [5], [6], [7] and [8].

A nonempty set $S \subseteq V(G)$ is a 1-*movable dominating set* of G if S is a dominating set of G and for every $v \in S$, $S \setminus \{v\}$ is a dominating set of G or there exists a vertex $u \in (V(G) \backslash S) \cap N_G(v)$ such that $(S \setminus \{v\}) \cup \{u\}$ is a dominating set of G. The 1-*movable domination number* of a graph G, denoted by $\gamma^1_m(G)$, is the cardinality of the smallest 1-movable dominating set of G. A 1-movable dominating set of G with cardinality equal to $\gamma^1_m(G)$ is called γ^1_m-*set* of G. This concept was investigated in [2] and [4].

A nonempty set $S \subseteq V(G)$ is a 1-*movable outer-connected dominating set* of G if $S = V(G)$ and for every $v \in S$, $S \setminus \{v\}$ is an outer-connected dominating set of G or $S \subset V(G)$ is an outer-connected dominating set of G and for every $v \in S$, either $S \setminus \{v\}$ is an outer-connected dominating set of G or there exists $u \in (V(G) \setminus S) \cap N_G(v)$ such that $(S \setminus \{v\}) \cup \{u\}$ is an outer-connected dominating set of G. The cardinality of the smallest 1-movable outer-connected dominating set of G is called 1-*movable outer-connected domination number* of G, denoted by $\widetilde{\gamma}^1_{mc}(G)$. A 1-movable outer-connected domination number of G with cardinality equal to $\widetilde{\gamma}^1_{mc}(G)$ is called $\widetilde{\gamma}^1_{mc}$-*set* of G.

2. RESULTS

Remark 2.1. *For every connected nontrivial graph G, $1 \leq \widetilde{\gamma}^1_{mc}(G) \leq n$ and the bounds are sharp.*

Consider the complete graph K_8 and C_8 in Figure 1. These graphs have 1-movable outer-connected dominating set and it can be verified that $\widetilde{\gamma}^1_{mc}(K_8) = 1$ and $\widetilde{\gamma}^1_{mc}(C_8) = 8$.

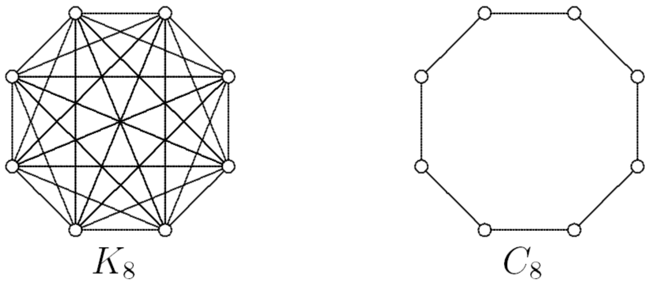

Figure 1: Graphs K_8 and C_8

Theorem 2.2. *Let G and H be connected nontrivial graphs. A nonempty set $S \subseteq V(G+H)$ is a 1-movable outer-connected dominating set of $G+H$ if and only if one of the following holds:*

(i) $S = V(G+H)$

(ii) *S is a dominating set of G such that if $|S| = 1$, then either S is 1-movable dominating set of G or there exists $y \in V(H)$ such that $\{y\}$ dominates H;*

(iii) *S is a dominating set of H such that if $|S| = 1$, then either S is 1-movable dominating set of H or there exists $a \in V(G)$ such that $\{a\}$ dominates G;*

(iv) *$S = D_1 \cup D_2 \subset V(G+H)$ where $D_1 \subseteq V(G)$ and $D_2 \subseteq V(H)$ such that*
(a) if $D_1 = V(G)$, then $\langle V(H) \setminus D_2 \rangle$ is connected and for every $v \in D_2$, $\langle V(H) \setminus (D_2 \setminus \{v\}) \rangle$ or $\langle V(H) \setminus [(D_2 \setminus \{v\}) \cup \{u\}] \rangle$ is connected for some $u \in (V(H) \setminus D_2) \cap N_H(v)$ and
(b) if $D_2 = V(H)$, then $\langle V(G) \setminus D_1 \rangle$ is connected and for every $v \in D_1$, $\langle V(G) \setminus (D_1 \setminus \{v\}) \rangle$ or $\langle V(G) \setminus [(D_1 \setminus \{v\}) \cup \{u\}] \rangle$ is connected for some $u \in (V(G) \setminus D_1) \cap N_G(v)$.

Proof: Assume that S is a 1-movable outer-connected dominating set of $G+H$. If $S = V(G+H)$, then (i) holds. Suppose that $S \subset V(G+H)$. Consider when $S \subseteq V(G)$. Since S is a dominating set of $G+H$, it is also a dominating set of G. Suppose $|S| = 1$. Let $S = \{a\}$ for some $a \in V(G)$. Since S is a 1-movable outer-connected dominating set of $G+H$, there exist $y \in (V(G+H) \setminus S) \cap N_{G+H}(a)$ such that $S \setminus \{a\} \cup \{y\} = \{y\}$ is an outer-connected dominating set of $G+H$. Suppose $y \in (V(G) \setminus S) \cap N_G(a)$ then $S \setminus \{a\} \cup \{y\} = \{y\}$ is a dominating set of G. Hence, S is a 1-movable dominating set of G. Suppose $y \notin (V(G) \setminus S) \cap N_G(a)$. Then $y \in V(H) \setminus S$. This means that $S \setminus \{a\} \cup \{y\} = \{y\}$ is a dominating set of H. Hence (ii) holds. Similarly, (iii) holds if $S \subseteq V(H)$. Suppose $S = D_1 \cup D_2 \subset V(G+H)$ where $D_1 \subseteq V(G)$ and $D_2 \subseteq V((H)$. Now consider first the case when $D_1 = V(G)$. Suppose $S \setminus \{v\}$ is an outer-connected dominating set of $G+H$. Then $\langle V(G+H) \setminus (S \setminus \{v\}) \rangle = \langle V(H) \setminus (D_2 \setminus \{v\}) \rangle$ is connected. Suppose $S \setminus \{v\}$ is not an outer-connected dominating set of $G+H$. Then there exist $u \in V(G+H) \setminus S) \cap N_{G+H}(v)$ such that $S \setminus \{v\} \cup \{u\}$ is an outer-connected dominating set of $G+H$. Hence,$\langle V(G+H) \setminus [S \setminus \{v\} \cup \{u\}] \rangle = \langle V(H) \setminus [D_2 \setminus \{v\} \cup \{u\}] \rangle$ is connected. Thus, (iva) holds. Simlarly (ivb) holds if $D_2 = V((H)$.

For the converse suppose that (i) holds. Let $v \in S = V(G+H)$. Then clearly, $S \setminus \{v\}$ is a dominating set of $G+H$ and $\langle V(G+H) \setminus S \setminus \{v\} \rangle = \langle \{v\} \rangle$ is connected. Hence, S is a 1-movable outer-connected dominating set of $G+H$. Suppose (ii) holds. Since S is a dominating set of G, it follows that S is a dominating set of $G+H$. Suppose $2 \leq |S| = |V(G)|$. Then $\langle V(G+H) \setminus S \rangle$ is connected. Hence, S is an outer-connected dominating set of $G+H$. Let $v \in S$. Then there exist $u \in V(H)$ such that $S \setminus \{v\} \cup \{u\}$ is a dominating set of $G+H$ and $\langle V(G+H) \setminus [S \setminus \{v\} \cup \{u\}] \rangle = \langle V(G) \setminus (S \setminus \{v\}) \rangle + \langle V(H) \setminus \{u\} \rangle$ is connected. Hence, $S \setminus \{v\} \cup \{u\}$ is an outer-connected dominating set of $G+H$. Suppose $|S| = 1$ and S is a 1-movable dominating set of G. Let $S = \{a\}$ for some $a \in V(G)$. Then $\langle V(G+H)) \setminus S \rangle = \langle V(G) \setminus S \rangle + H$ is connected. Hence, S is an outer-connected dominating set of $G+H$. Since S is a 1-movable dominating set of G, there exist $u \in V(G)$ such that $S \setminus \{a\} \cup \{u\} = \{u\}$ is a dominating set of G and hence of $G+H$ and $\langle V(G+H) \setminus [S \setminus \{a\} \cup \{u\}] \rangle = \langle V(G) \setminus \{u\} \rangle + H$ is connected. Hence, $S \setminus \{a\} \cup \{u\} = \{u\}$ is an outer-connected dominating set of $G+H$. Suppose that S is not a 1-movable dominating set of G. By assumption, S is a dominating set of G and hence of $G+H$ and there exist $y \in V(H)$ such that $\{y\}$ dominates H. Clearly, S is an outer-connected dominating set of $G+H$ and $S \setminus \{a\} \cup \{y\} = \{y\}$ is a dominating set of $G+H$. Further, $\langle V(G+H) \setminus [S \setminus \{a\} \cup \{y\}] \rangle = G + \langle V(H) \setminus \{y\} \rangle$ is connected. Thus $S \setminus \{a\} \cup \{y\}$ is an outer-connected dominating set of $G+H$. Thus, S is 1-movable outer-connected dominating set of $G+H$. Similarly, if (iii) holds, then S is a 1-movable outer-connected dominating set of $G+H$. Suppose (iv) holds. Clearly, S is a dominating set of $G+H$. Suppose first that $|D_1| = |V(G)|$ and $1 \leq |D_2| < n$. Then by (iva), $\langle V(G+H) \setminus S\rangle = \langle V(H) \setminus D_2\rangle$ is connected. Hence, S is an outer-connected dominating set of $G + H$. Let $v \in S$ and suppose $v \in D_1$. Then $S \setminus \{v\} = D_1 \setminus \{v\} \cup D_2$ is a dominating set of $G + H$ and $\langle V(G + H) \setminus S \setminus \{v\}\rangle = \langle V(G) \setminus D_1 \setminus \{v\}\rangle + \langle V(H) \setminus D_2\rangle$ is connected. Hence, $S \setminus \{v\}$ is an outer-connected dominating set of $G + H$. Suppose $v \in D_2$. Since $S \setminus \{v\}$ is a dominating set of $G + H$ and $\langle V(H) \setminus D_2 \setminus \{v\}\rangle$ is connected, $S \setminus \{v\} = D_1 \cup (D_2 \setminus \{v\})$ is an outer-connected dominating set of $G + H$. Suppose $\langle V(H) \setminus D_2 \setminus \{v\}\rangle$ is not connected. By assumption, there exist $u \in (V(H) \setminus D_2) \cap N_H(v)$ such that $\langle V(H) \setminus D_2 \setminus \{v\} \cup \{u\}\rangle$ is connected. This implies that $S \setminus \{v\} \cup \{u\} = D_1 \cup (D_2 \setminus \{v\}) \cup \{u\}$ is a dominating set of $G + H$ and $\langle V(G + H) \setminus [S \setminus \{v\} \cup \{u\}]\rangle = \langle V(H) \setminus [D_2 \setminus \{v\} \cup \{u\}]\rangle$ is connected. Hence,$S \setminus \{v\} \cup \{u\}$ is an outer-connected dominating set of $G + H$. Suppose that $1 \leq |D_1| < m$ and $1 \leq |D_2| < n$. Then $S = D_1 \cup D_2$ is a dominating set of $G + H$ and $\langle V(G + H) \setminus S\rangle = \langle V(G) \setminus D_1\rangle + \langle V(H) \setminus D_2\rangle$ is connected. Hence, S is an outer-connected dominating set of $G + H$. Let $v \in S$ and suppose $v \in D_1$. If $2 \leq |D_1| < m$ then $D_1 \setminus \{v\} \neq \varnothing$. Hence, $S \setminus \{v\} = D_1 \setminus \{v\} \cup D_2$ is a dominating set of $G + H$ and $\langle V(G + H) \setminus S \setminus \{v\}\rangle = \langle V(G) \setminus D_1 \setminus \{v\}\rangle + \langle V(H) \setminus D_2\rangle$ is connected. Hence, $S \setminus \{v\}$ is an outer-connected dominating set of $G + H$. Similarly, if $v \in D_2$, then $S \setminus \{v\}$ is an outer-connected dominating set of $G + H$. Finally suppose $1 \leq |D_1| < m$ and $|D_2| = |V(H)|$. This case follows the same argument when $|D_1| = 1$ and $1 \leq |D_2| < n$. Therefore, S is a 1-movable outer-connected dominating set of $G + H$. □

Corollary 2.3. *For every connected nontrivial graphs G and H,*

$$\widetilde{\gamma}^1_{mc}(G+H) \begin{cases} 1, \text{if } \gamma(G) = 1 = \gamma(H) \text{ or } \gamma^1_m(G) = 1 \text{ or } \gamma^1_m(H) = 1 \\ 2, \text{otherwise} \end{cases}$$

Theorem 2.4. *Let H be a connected nontrivial graph. A nonempty set $S \subseteq V(K_1 + H)$ is a 1-movable outer-connected dominating set of $K_1 + H$ if and only if one of the following holds:*

(i) $S = V(K_1 + H)$

(ii) *S is a dominating set of H and for every $v \in S$, either $S \setminus \{v\}$ or $S \setminus \{v\} \cup \{u\}$ is a dominating set of H or $\langle V(H) \setminus (S \setminus \{v\})\rangle$ is connected or*

(iii) *$S = V(K_1) \cup S_H$ where $S_H \subset V(H)$ and*
(a) $\langle V(H) \setminus S_H\rangle$ is connected
(b) either S_H or $S_H \cup \{z\}$ is a dominating set of H and
(c) for every $v \in S_H$, $\langle V(H) \setminus (S_H \setminus \{v\})\rangle$ or $\langle V(H) \setminus S_H \setminus \{v\} \cup \{u\}\rangle$ is connected for some $u \in (V(H) \setminus S_H) \cap N_H(v)$.

Proof: Let H be a connected nontrivial graph and $V(K_1) = \{x\}$. Suppose that S is a 1-movable outer-connected dominating set of $K_1 + H$. If $S = V(K_1 + H)$, then (i) holds. Suppose $S \subset V(K_1 + H)$. Consider first the when $S \subseteq V(H)$. Then S is a dominating set of H. Let $v \in S$. Since S is a 1-movable outer-connected dominating set of $K_1 + H$, $S \setminus \{v\}$ is an outer-connected dominating set of $K_1 + H$ and hence a dominating set of H. Suppose that $S \setminus \{v\}$ is not an outer-connected dominating set of K_1+H. By assumption, there exist $u \in V(K_1+H) \setminus S) \cap N_{K_1+H}(v)$ such that $S \setminus \{v\} \cup \{u\}$ is an outer-connected dominating set of $K_1 + H$. If $u \neq x$, then $u \in V(H) \setminus S$ and $S \setminus \{v\} \cup \{u\}$ is a dominating set of H. Suppose $u = x$. Then $\langle V(K_1+H) \setminus [S \setminus \{v\} \cup \{u\}]\rangle = \langle V(H) \setminus S \setminus \{v\}\rangle$ is connected. Suppose $S = V(K_1) \cup S_H$ where $S_H \subset V(H)$. By assumption, $\langle V(K_1 + H) \setminus S_H\rangle = \langle V(H) \setminus S_H\rangle$ is connected. Let $v \in S$ and suppose $v = x$. Since S is a 1-movable outer-connected dominating set of $K_1 + H$, $S \setminus \{v\} = S_H$ is an outer-connected dominating set of H. Hence, S_H is a dominating set of H. Suppose $S \setminus \{v\}$ is not an outer-connected dominating set of H. By assumption, there exist $u \in V(H)$ such that $S \setminus \{v\} \cup \{u\}$ is an outer-connected dominating set of $K_1 + H$. This means that $S \setminus \{v\} \cup \{u\}$ is a dominating set of H. Suppose $v \neq x$. Then $v \in S_H$. Consider first when $S \setminus \{v\}$ is an outer-connected dominating set of $K_1 + H$. Then, $\langle V(K_1 + H) \setminus S \setminus \{v\}\rangle = \langle V(H) \setminus S_H \setminus \{v\}\rangle$ is connected. Suppose $S \setminus \{v\}$ is not an outer-connected dominating set of $K_1 + H$. Then there exists $u \in (V(H) \setminus S_H) \cap N_H(v)$ such that $S \setminus \{v\} \cup \{u\}$ is an outer-connected dominating set of $K_1 + H$. This implies that $\langle V(K_1 + H) \setminus S \setminus \{v\} \cup \{u\}\rangle = \langle V(H) \setminus [S_H \setminus \{v\} \cup \{u\}]\rangle$ is connected.

For the converse, suppose that (i) holds. Then S is a dominating set of $K_1 + H$. Let $v \in S$. Then $S \setminus \{v\}$ is a dominating set of K_1+H and $\langle V(K_1+H) \setminus S \setminus \{v\}\rangle = \langle \{v\}\rangle$ is connected. Thus, $S \setminus \{v\}$ is an outer-connected dominating set of K_1+H. Hence, S is a 1-movable outer-connected dominating set of $K_1 + H$. Suppose (ii) holds. Then $\langle V(K_1+H) \setminus S\rangle = K_1 + \langle V(H) \setminus S\rangle$ is connected and the set S is clearly a dominating set of K_1+H. Hence, S is an outer-connected dominating set of $K_1 + H$. Let $v \in S$. Suppose first that $S \setminus \{v\}$ is a dominating set of H. Then $S \setminus \{v\}$ is a dominating set of $K_1 + H$ and $\langle V(K_1+H) \setminus S \setminus \{v\}\rangle = K_1 + \langle V(H) \setminus S \setminus \{v\}\rangle$ is connected. Hence, $S \setminus \{v\}$ is an outer-connected dominating set of K_1+H. Suppose $S \setminus \{v\}$ is not an outer-connected dominating set of $K_1 + H$. By assumption, there exist $u \in (V(H) \setminus S) \cap N_H(v)$ such that $S \setminus \{v\} \cup \{u\}$ is a dominating set of H and hence of $K_1 + H$. Moreover, $\langle V(K_1 + H) \setminus [S \setminus \{v\} \cup \{u\}]\rangle = K_1 + \langle V(H) \setminus [S \setminus \{v\} \cup \{u\}]\rangle$ is connected. Hence $S \setminus \{v\} \cup \{u\}$ is an outer-connected dominating set of $K_1 + H$. Suppose $S \setminus \{v\} \cup \{u\}$ is not a dominating set of H. By assumption $\langle V(H) \setminus S \setminus \{v\}\rangle = \langle V(K_1 + H) \setminus S \setminus \{v\} \cup \{x\}\rangle$ is connected and $S \setminus \{v\} \cup \{x\}$ is a dominating set of $K_1 + H$. Hence,$S \setminus \{v\} \cup \{x\}$ is an outer-connected dominating set of $K_1 + H$. Therefore, S is a 1-movable outer-connected dominating set of $K_1 + H$. Suppose (iii) holds. By $(iiia)$, S is an outer-connected dominating set of $K_1 + H$. Let $v \in S$. Suppose $v = x$. By $(iiib)$, it follows $S_H \setminus \{v\}$ or

$S_H \cup \{u\} = S \setminus \{v\} \cup \{u\}$ is an outer-connected dominating set of $K_1 + H$ for some $u \in V(K_1 + H) \setminus S \cap N_{K_1+H}((v)$. Suppose $v \neq x$. Then $v \in S_H$. By $(iiic)$, it follows that $S \setminus \{v\}$ or $S \setminus \{v\} \cup \{u\}$ is an outer-connected dominating set of $K_1 + H$ for some $u \in V(K_1 + H) \setminus S \cap N_{K_1+H}(v)$. Therefore, S is a 1-movable outer-connected dominating set of $K_1 + H$. □

Corollary 2.5. *For every connected nontrivial graph H,*

$$\widetilde{\gamma}^1_{mc}(K_1 + H) = \begin{cases} 1, \text{if } \gamma(H) = 1 \\ \gamma(H), \text{if } \gamma(H) \neq 1 \end{cases}$$

Theorem 2.6. *Let m and n be positive integers. A subset S of $V(K_{m,n})$ is a 1-movable outer-connected dominating set of $K_{m,n}$ if and only one of the following holds:*

(i) $S = V(K_{m,n})$

(ii) $S = S_G \cup S_H$ *where* $2 \leq |S_G| < m$ *and* $2 \leq |S_H| < n$.

Proof: Let m and n be positive integers. Suppose S is a 1-movable outer-connected dominating set of $K_{m,n}$. If $S = V(K_{m,n})$ then (i) holds. Suppose $S \subset V(K_{m,n})$. Since $\overline{K}_m$ and $\overline{K}_n$ are empty graphs, $S = S_G \cup S_H$ where $S_G \subseteq V\left(\overline{K}_m\right)$ and $S_H \subseteq V\left(\overline{K}_n\right)$. Assume that $|S_1| = 1$, say, $S_G = \{a\}$ for some $a \in V\left(\overline{K}_m\right)$. If $|S_H| = n$, then $S \setminus \{a\} = S_H = V\left(\overline{K}_n\right)$ is a dominating set of $V(K_{m,n})$ but $\langle V(K_{m,n}) \setminus (S \setminus \{a\})\rangle = \langle V(K_{m,n}) \setminus S_H\rangle = \overline{K}_m$ is disconnected. This contradics the assumption. Hence,$|S_G| \geq 2$. Suppose $|S_G| = m$. Then $\langle V(K_{m,n}) \setminus S_G\rangle = \overline{K}_n$ is disconnected which contradicts the assumption. Hence, $2 \leq |S_G| < m$. Similarly, $2 \leq |S_H| < n$.

For the converse, suppose that (i) holds. Clearly, $S = V\left(K_{m,n}\right)$ is a 1-movable outer-connected dominating set of $K_{m,n}$. Suppose that (ii) holds. Then S is a dominating set of $K_{m,n}$ and $\langle V\left(K_{m,n}\right) \setminus S\rangle$ is connected. Hence, S is an outer-connected dominating set of $K_{m,n}$. Let $v \in S$. Suppose $v \in S_G$. Since $|S_G| \geq 2$, $S_G \setminus \{v\} \neq \varnothing$. Hence, $S \setminus \{v\} = (S_G \setminus \{v\}) \cup S_H$ is an outer-connected dominating set of $K_{m,n}$. Similarly, if $v \in S_H$, then $S \setminus \{v\} = S_G \cup (S_H \setminus \{v\})$ is an outer-connected dominating set of $K_{m,n}$. Therefore, S is a 1-movable outer-connected dominating set of $K_{m,n}$. □

Corollary 2.7. *Let $m \geq 2$ and $n \geq 2$ be integers. Then,* $\widetilde{\gamma}^1_{mc}(K_{m,n}) = 4$.

REFERENCES

[1] M.H. Akhbari, R. Hasni, O. Favaron and S.M. Sheikholeslami. On the outer-connected domination in graphs. Journal of Combinatorial Optimization 26(1). July 2013

[2] J. Blair, R. Gera and S. Horton. Movable dominating sensor sets in networks. Journal of Combinatorial Mathematics and Combinatorial Computing, 77 (2011) ,103-123.

[3] J. Cyman and J. Raczek. Total Outer-connected domination numbers of tress. Discrete Applied Mathematics, 157 (2009), Issue 15, 3198-3202.

[4] R.G. Hinampas,Jr., and S.R. Canoy, Jr. 1-movable domination in graphs. Applied Mathematical Sciences, 8 (2014), no. 172, 8565-8571.

[5] R.G. Hinampas,Jr., J.Lomarda-Hinampas and A.R.Dahunan. 1-movable Independent Outer-connected Domination in Graphs. Global Journal of Pure and Applied Mathematics, 13(2017),no.1,41-49.

[6] R.G. Hinampas, Jr., J Lomarda-Hinampas and A Dahunan. Independent Outer-connected Domination in Graphs. Global Journal of Pure and Applied Mathematics, 13(2017),no.1,1-7.

[7] H. Jiang and E. Shan. Outer-connected domination in graphs. Utilitas Mathematica 81 (2010)

[8] J.L. Hinampas, R. G. Hinampas, Jr and L.C. Canada. Movable total outer-connected dominating sets in graphs . Far East Journal of Mathematical Sciences, 102(2017),no.11,2713-2725.