International Journal of Applied Engineering Research ISSN 0973-4562 Volume 14, Number 7 (2019) pp. 1633-1634

Properties of AC-Operators on Different Linear Spaces

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Abstract

Berkson and Gillespie[1] introduced the concept of an AC-operator as the natural analogue in the context of well-boundedness of normal operators on Hilbert space. In this paper we explore some properties of these operators and their interpolation properties.

INTRODUCTION

This paper is concerned with presenting new developments in abstract spectral theory, The key notions are well-bounded operator due to D. R. Smart [6] and J. R. Ringrose [5] and AC-operators due to Berkson and Gillespie[1]. An AC-operator is one which possess a functional calculus for the absolutely continuous functions on some rectangle in C. Berkson and Gillespie showed that these operators can be characterized by the fact that they possess a splitting with real and imaginary parts T = A+iB where A and B are commuting well-bounded operators. Berkson and Gillespie showed that if there exists one such representation for which A and B are well-bounded of type (B), then no other representation exists. In particular, AC-operators on reflexive Banach spaces have unique representations. In [2] it is shown that this splitting is not necessarily unique. Furthermore even if T is an AC operator on a Hilbert space H, it does not necessarily follow that αT is an AC operator for all α ∈ C.

Keywords: Banach Space, Well-bounded operators, AC-operators, reflexive spaces, functional calculus.

Subject Code Classification: MSC (2010): 47A60, 47B40.

Well-bounded operators were introduced by D. R. Smart [6].The definition of well-bounded operators on Hilbert space is as follows:

Definition 1: Well-bounded Operator:

A bounded linear operator T on a complex reflexive Banach space is said to be well-bounded if there is a compact interval J = [a, b] and a positive constant M such that

$$\| p(T) \| \leq M\{ \|p\|_J + \operatorname*{var}_J p \} \quad \text{---(I)}$$

for every complex polynomial p, where $\|p\|_J$ denotes sup {| p(t) |: t ∈ J} and

$$\operatorname*{var}_J p = \sup \left\{ \sum_{j=1}^{n} | p(\lambda_j) - p(\lambda_{j+1}) | \,/\, \{\lambda_0 = a, \lambda_1, \ldots, \lambda_n = b \text{ is any partition of } J\} \right\}.$$

Definition 2 : A bounded linear operator T on a Banach space is said to be well-bounded if there exists a compact interval [a, b] ⊆ R such that T admits a bounded AC[a b] functional calculus.

Definition 3 : AC- Operator:

An AC-operator T is one which possesses a functional calculus for the absolutely continuous functions on some rectangle in ℂ. T possess a splitting with real and imaginary parts T = A+iB , where A and B are commuting well-bounded operators.

The following theorem is due to T.A.Gillespie[3].

Theorem 4: For the Hilbert space l_2 which is consisting of square summable sequences of complex numbers ,the well-bounded operators A and B with AB = BA on l_2 , A+B is not well bounded.

Definition 5: Well-bounded operators and Schauder decomposition

Let X be a Banach space. A sequence of projections { P_j}, for n= 0…∞ is said to be a Schauder decomposition for X if

(1) $P_n P_m = P_{\min(m,n)}$, for all integers m, n ≥1.

(2) Pn → I in the strong operator topology;

(3) $P_n \neq P_m$ if m≠n

Example 6 : Let α = (1- i) ,we have αT = (1- i)(A+iB)

= A+ iB- iA+B

= (A+B) + i(B-A).

but the sum of two commuting well-bounded operators is not well-bounded. Hence αT is not an AC-operator.

At least on reflexive spaces such operators posses a type of spectral decomposition theory similar to that for self-adjoint operator but one which allows conditionally rather than unconditionally convergent spectral expansion. Even on general Banach space, every compact well-bounded operator admits a diagonal representation of the form

$$A = \sum_{j=1}^{\infty} \lambda_j P_j$$

where { λ_j} is the set of non-zero eigenvalues of T & { P_j} is the corresponding set of Riesz projection onto the eigenspaces.

Also under suitable conditions on { λ_j} and { P_j} any operator formed in this way is well-bounded. e.g. { Q_j} , for j= 0…∞ are projections associated with schauder decomposition of the space X and $P_j = Q_j - Q_{j-1}$, for j= 0…∞ , then $\sum_{j=1}^{\infty}\lambda_j P_j$ is well-bounded for any decreasing sequence { λ_j} of positive reals converging to zero.

Example 7: Let A=B= $\sum_{j=1}^{\infty}\frac{1}{n}P_j$ are well-bounded operators & Let T= A+ iB = $\sum_{j=1}^{\infty}\frac{1}{n}(1+i)P_j$ which infact forms AC –operator.

Now, α = (1- i) ,we have αT = (1- i)(A+iB)

$$= (1-i)\sum_{j=1}^{\infty}\frac{1}{n}(1+i)P_j$$

$$=\sum_{j=1}^{\infty}\frac{2}{n}(1+i)P_j$$

but the sum of two commuting well-bounded operators is not well-bounded. Hence αT is not an AC-operator.

Example 8: Let X =$\mathbb{C}^3$. Fix x > 0 and define $E_1, E_2 \in B(X)$ by the matrices

$$E_1 = \begin{pmatrix} 1 & 0 & x \\ 0 & 1 & (1+i)\sqrt{\frac{x}{2}} \\ 0 & 0 & 0 \end{pmatrix},\quad E_2 = \begin{pmatrix} 0 & \sqrt{2x} & ix \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Then E_1 and E_2 are commuting projections. In particular they are both well bounded operators.

$$\text{Thus } T = E_1 + iE_2 = \begin{pmatrix} 1 & i\sqrt{2x} & 0 \\ 0 & 1+i & (1+i)\sqrt{\frac{x}{2}} \\ 0 & 0 & i \end{pmatrix}$$

let α = (1- i) ,we have αT = (1- i)($E_1 + iE_2$)

$$= \begin{pmatrix} 1-i & (1+i)\sqrt{2x} & 0 \\ 0 & 2 & 2\sqrt{\frac{x}{2}} \\ 0 & 0 & i+1 \end{pmatrix}$$

$$= (E_1 + E_2) + i(E_2 - E_1)$$

but the sum of two commuting well-bounded operators is not well-bounded. Hence αT is not an AC-operator.

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