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Secure Set Domination in Graphs

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Abstract

Let the graph G = (V_G, E_G) be a connected graph. A set D of VG is said to be a set dominating set if for each I ⊆ V – D, there exists a non- empty set S of D such that the induced sub graph < I ∪ S > is connected. A subset D⊆V_G is called a secure dominating set of a graph G if for every vertex v∈ V-D there exists u∈D such that v is adjacent to u and D_1= (D-{u}) ∪ {v} is a dominating set. A set dominating set D is an SSD – dominating set of G if D is also a set dominating set of G. The secure set dominating number $\gamma_s^s(G)$ is the minimum cardinality of a secure set dominating set, in this paper we introduce properties of this parameter and we give some relation with another parameters.

Keywords: Domination, Set domination, Secure domination, SSD-domination.

INTRODUCTION

All graphs taken here are finite, undirected with neither loops nor multiple edges. Any undefined term in this paper may be found in Haynes T. W., Hedetniemi S. T., Slater P. J. [3].

Let G = (V_G, E_G) be a graph with 'n' vertices and 'm' edges, thus $|V_G|$ = n and $|E_G|$ = m. The graph $\overline{G}$ is the complement of graph g having n vertices of G and two vertices are linked if these are not linked in G. The open neighborhood of a vertex v of G defined as the set N(v) = { u ∈ V_G ; uv ∈ E_G}. The degree of vertex v is the cardinality of open neighborhood denoted as $d_G(v)$ [3]. The maximum cardinality of minimal dominating set is called upper dominating set, denoted by Γ(G)[1].

The path with n vertices is denoted as P_n and the cycle with n vertices is denoted by C_n. The wheel on n vertices is a graph which formed by connecting a vertex to remained vertices of a cycle C_{n-1} and is denoted as W_n, n≥4. We denote a complete bipartite graph with two partite sets of cardinality m and n by $K_{m,n}$. The graph $K_{1,n-1}$ denotes a star [3].

A subset D ⊆ V_G is a dominating set of G if for each vertex of V–D has a neighbor in D. The minimum cardinality of a dominating set is the domination number is of G. For deep analysis of domination in graph, see [3,4].

Let G = (V_G, E_G) be a connected graph and a set D is subset of V is a set – dominating set if for each set V_1 ⊆ V– D, there exists an D_1 ⊆ D which is non- empty such that the subgraph < D_1∪V_1> induced by D_1∪V_1 is connected and the minimum cardinality of a set dominating set of G is the set domination number denoted as $\gamma_s(G)$ [6].

A subset S ⊆ V_G is called a secure dominating set of G if for each vertex v ∈ V \D there exists u∈S such that uv∈ E and S_1 = (S –{u}) ∪{v} is a dominating set and the minimum cardinality of secure dominating set is the secure dominating number which is denoted by $\gamma^s(G)$ [2].

The theory of secure domination in graphs was presented by Cockayne E. J. Grobler P. J. P., Gründlingh W. R., Munganga J., and Vuuren J. H. [2] by considering the following situation, we want to place guards at selected positions of a museum in such a way that whole positions of museum are covered or adjacent. If one guard out of guards of selected positions changes his position to unselected position, then the new configuration of guards is also covers or adjacent each position of a museum.

We present a fresh variant of secure domination namely the secure set domination (SSD –domination), and we start the analysis of this type of parameter. A secure dominating set D is an SSD –dominating set of G if D is a set dominating set of G. The minimum cardinality of this set is SSD –dominating number and is denoted by $\gamma_s^s(G)$ .we consider all graphs with non –isolated vertices.

MAIN Results

In this section, we study the secure regular set domination in various graphs i.e., complete graph, cyclic graph, path graph, wheel graph, complete bipartite graph. We initiate with the following straightforward observations.

Definition 2.1 Let G= (V_G,E_G) be a graph. A secure dominating set D of G is called a secure set dominating set if for every set V_1 ⊆ V\ D there exists a non empty set D_1 ⊆ D such that induced sub graph < V_1∪D_1> induced by V_1∪D_1 is connected. The minimum cardinality of secure set dominating set is called the secure set domination number of G and is denoted by $\gamma_s^s(G)$.

Definition 2.2 The maximum cardinality of a minimal secure set dominating set of G is upper secure set dominating number, denoted by $\Gamma_s^s(G)$.

Clearly, a dominating set D is secure set dominating set if and only if the set D is secure as well as set dominating set.

Observation 2.2 Let G be a connected graph then

γ (G) $\leq \gamma_s$ (G) $\leq \gamma_s^s$ (G).

Observation 2.3 For any graph G without isolated vertices,

γ (G) $\leq \gamma^s$ (G) $\leq \gamma_s^s$ (G).

Observation 2.4: For any complete graph G with n vertices, γ (K_n)= 1.

Observation 2.5: For a cycle graph G with n vertices,

$$\gamma_s^s(C_n)=\begin{cases} n-2, & n\neq 1,2 \\ 1, & n=1,2 \end{cases}$$

Observation 2.6: For a path graph G with n vertices,

$$\gamma_s^s(P_n)=\begin{cases} 1, & n=1,2 \\ 2, & n=3 \\ n-2, & n\geq 4 \end{cases}$$

Observation 2.7: For a wheel graph G = W_n with n vertices,

$\gamma_s^s(W_n)$ = n -3

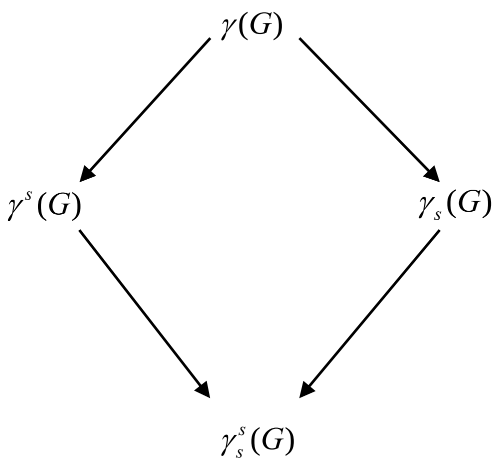

Fig. 1. Relationship of Domination Parameters

We have the following properties for $\gamma_s(G)$, $\gamma^s(G)$ and $\gamma_s^s(G)$

Theorem 2.8: Let G be a complete bipartite graph then $\gamma_s^s(K_{m,n})=\left\lceil \frac{m+n}{2} \right\rceil$.

Proof: Let A and B are defining partite sets of $K_{m,n}$ with cardinality m and n respectively. [5]Assume that m ≤ n. Now, it is sufficient to find D ⊆A ∪ B such that D is secure set dominating set with cardinality $\left\lceil \frac{m+n}{2} \right\rceil$.

We shall take half vertices of A and half vertices of B.

Case 1: If m and n are divisible by 2, D takes m/2 vertices of A and n/2 of B.

Case 2: If m is divisible by 2 but not n, D takes m/2 vertices of A and $\lceil n/2 \rceil$ vertices of B.

Case 3: If n is divisible by 2 but not m, D takes $\lceil m/2 \rceil$ vertices of A and n/2 vertices of B.

Case 4: if neither m nor n is divisible by 2, D takes $\lceil m/2 \rceil$ vertices of A and $\lceil n/2 \rceil$ vertices of B.

Each case gives that the set D is connected and dominating. Let D_1 = D∩A and D_2 = D∩B. For any non-empty set X of D and let X_1 = X∩A, X_2 = X∩B.

If $X_1 \neq \phi$, $X_2 \neq \phi$ then N(X)-D = (A∪B)\D therefore,

$$|N(X)-D|=\left\lceil \frac{m+n}{2} \right\rceil,$$

$$|N[X]-D|=|D|=\left\lceil \frac{m+n}{2} \right\rceil \geq |N(X)-D|.$$

If X_1 = X, $|D_2|\geq|B-D_2|-1$, while $|N[X]-D|=|X|+|D_2|$.

In each case $|D_2|\geq|B-D_2|-1$, $X_1 = X \neq \phi$ therefore $|N[X]-D|\geq|N(X)-D|$. If X_2 = X, we can prove $|N[X]-D|\geq|N(X)-D|$. Hence $\gamma_s^s(K_{m,n})=\left\lceil \frac{m+n}{2} \right\rceil$ since $\gamma_s(K_{m,n})=2$.

Theorem 2.9: Let G = $K_{m,n}$ be a complete bipartite graph. Then $\Gamma_s^s(G)$ = max(m,n).

Proof: Let A and B be the partite sets of G such that $|A|=m, |B|=n$ and let m ≥ n. Clearly, A is a minimal secure set dominating set of G. Thus $\Gamma_s^s(G)\geq m$.

Let us assume that the set D be any minimal secure set dominating set of G. If D has more than 3 vertices of A and B, then D\{u} is a secure set dominating set of G for all u ∈ D. This is a contradiction for minimality of D. So, the set D contains less than 3 vertices of the set A. If $|A\cap D|=2$ and $|B\cap D|\leq m-2$, then $\Gamma_s^s(G)=|D|\leq m$. If $|A\cap D|=2$ and $|B\cap D|\geq m-1$, then S is not γ_s^s (G)-set. If D has exactly one vertex of A, the set D has at least m-1 vertices of B. If D contains m vertices of B, then D is not γ_s^s (G)-set. Hence $|B\cap D|=m-1$ and this implies that $\Gamma_s^s(G)=|D|=m$. If the set D and A has no common vertex then

D=B, this implies that $\Gamma_s^s(G)=|D|=m$. Thus we observe that $\Gamma_s^s(G)=|D|\leq m$ for all cases. So, $\Gamma_s^s(G)$ = max(m,n).

Theorem 2.10: A secure set dominating set D of a graph G is minimal if and only if the one condition hold for each vertex x $\in$D

i. Either x is not adjacent to D or the set $D_1 = (D\setminus\{x\})\cup\{y\}$ is not dominating set for all $y \in N(x)\cap D$.

ii. There exists a set $V_1 \subseteq V\setminus D$ such that each connected subgraph induced by V_1 and vertices of D contains x and there exists a vertex x of D for each $y \in V\setminus D$ such that the set $D_1 = (D\setminus\{x\})\cup\{y\}$ is dominating set.

iii. There exists a set $V_1 \subseteq V\setminus D$ such that each connected subgraph induced by V_1 and vertices of D does not contain x and there exists a vertex x of D for each $y \in V\setminus D$ such that the set $D_1 = (D\setminus\{x\})\cup\{y\}$ is dominating set.

Proof: Let the set D be a γ_s^s (G)-set of G. Consider that the property (i) is not true, then there exists a vertex $y \in D \cap N(x)$ for some $x\in D$ such that D_1 is dominating set and for each vertex y of V\D either $N(y)\cap D \neq \{x\}$ or D_1 is dominating set. This implies that D\{x} is minimal SSD-set of G, which is a contradiction for D. Thus the property (i) is true.

Also, for each set $D_1^{'}\subseteq D^{'}\subseteq V\setminus D^{'}$ by (ii) and (iii) there exists a set $D_1^{'}\subseteq D^{'}$ such that the induced subgraph < $V_1^{'}\cup D_1^{'}$> is connected and there exists a vertex of D for each vertex y of V\D such that D_1 dominates V_G. We observe that the set $D^{'}$ is a SSD-set, which is a contradiction.

Converse is trivial.

CONCLUSION AND SCOPE

We have defined a new variant of the parameter of domination namely secure set domination number of various graphs and we found the equality relationship of the selected number of domination parameters in case of complete bipartite graph $K_{m,n}$. Also, we have observed that the secure set domination number is always less than of its order.

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