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# n-dimensional Time - Fractional Heat equation by Reduced Differential Transform Method

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**Abstract**
In this paper, we apply Reduced Differential Transform Method (RDTM) to solve multi-dimensional time fractional heat equations. It is observed that the proposal technique (RDTM)is highly suitable for such problems. Numerical results re-confirm the efficiency of the suggested algorithm.

**Keywords:** Time fractional heat equations, reduced differential transform method

## 1. INTRODUCTION

We can see the applications of fractional differential equations in all areas of physics, applied and engineering sciences [1]-[8]. Further, the understanding of those physical phenomena in this scientific research is very important to find their exact solutions. Thus the determinations of the exact solutions of these equations are interesting and hence it is important too. Many scientific researchers, mainly had paid an attention for studying the solution of such equations by using various developed method.

Recently, for handling the various kinds of non-linear problems, for example, a fractional differential equations [4], the Variational Iteration Method (VIM) [1]-[3], non-linear differential equations [5], non-linear thermo - elasticity [6], Adomian's Decompositions Method (ADM), non-linear wave equations [7], Homotopy perturbation Method (HPM), Variation of Parameter Method (VPM)and Homotopy Analysis Method(HAM) [8]-[13] successfully applied to find an exact solutions of differential equations.

In this paper we use the Reduced Differential Transform Method (RTDM) [14]-[18], for constructing an appropriate solution to the n-dimensional heat equations of fractional order.This technique is an iterative procedure for finding the Taylor series solution of differential equations. By this method we reduce the size of computational work and which is easily applicable to many physical problems.

## 2. REDUCED DIFFERENTIAL TRANSFORM METHODOLOGY

We discuss the basic definitions of Reduced Differential Transform Method

**Definition 2.1.** *If $u(x,t)$ is an analytic function and differentiated continuously with respect with respect to $'t'$. Then,*
*Let*

$$U_k(x) = \frac{1}{\Gamma(k\alpha+1)}\left[\frac{\partial^{k\alpha}}{\partial t^{k\alpha}}u(x,t)\right]_{t=0} \qquad (1)$$

*where 'α' is a parameter describing the order of the time fractional derivative and the $t$-dimensional spectrum function $U_k(x)$ is the transformed function.*

In this work, the lowercase $u(x,t)$ represents an original function and the $U_k(x)$ upper case represents the transformed function.

**Theorem 2.2.** *If the original function is $w(x,t) = u(x,t) \pm v(x,t)$, then the transformed function is $W_k(x) = U_k(x) \pm V_k(x)$*

**Theorem 2.3.** *If the original function is $w(x,t) = \alpha u(x,t)$, then the transformed function which is of the form $W_k(x) = \alpha U_k(x)$.*

**Theorem 2.4.** *If the original function is $w(x,t) = u(x,t)v(x,t)$, then their transformed function is of the form, $W_k(x) = \sum_{n=0}^{k} U_n V_{k-n}(x) = \sum_{n=0}^{k} V_n U_{k-n}(x)$.*

**Theorem 2.5.** *If the original function is $w(x,t) = \frac{\partial^n}{\partial y^n}u(x,t)$ then*
*$W_k(x) = (k+1)(k+2)...(k+n)U_{k+n}(x)$ is the transformed function.*

**Theorem 2.6.** *If the original function $w(x,t) = x^m y^n u(x,t)$, then*
*$W_k(x) = x^m U_{k-n}(x)$ is the transformed function of the original function.*

**Theorem 2.7.** *If the original function $w(x,t) = \frac{\partial^{N\alpha}}{\partial t^{N\alpha}}u(x,t)$, then*
*$W_k(x) = \frac{\Gamma(k\alpha+N\alpha+1)}{\Gamma(k\alpha+1)}U_{k+N}(x)$ is the transformed function.*

**Definition 2.8.** *The differential inverse transform of $U_k(x)$ is*

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)t^{k\alpha} \qquad (2)$$

*combining the equations (1) and (2), we can obtain the following*

$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{\Gamma(k\alpha+1)}\left[\frac{\partial^{k\alpha}}{\partial t^{k\alpha}}u(x,t)\right]_{t=0} t^{k\alpha}, \qquad (3)$$

By the above definitions, we can understand that the concept of RDTM is derived from the power series expansion of a function.

## 3. NUMERICAL APPLICATIONS OF RDTM

In this section we use the RDIM for solving the $n$- dimensional fractional heat equation. Let us see some examples for the study of reliability and efficiency of the proposed RDTM.

**Example 3.1.** *Consider an $n$- dimensional homogeneous time fractional heat equation,*

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} + \ldots + \frac{\partial^2 u}{\partial x_n^2}, \quad where\ 0 < x_1, \ldots, x_n < \pi, t > 0, 0 < \alpha \leq 1 \tag{4}$$

with an initial condition,

$$u(x_1, x_2, \ldots, x_n, 0) = 2sinx_1\ sinx_2\ \ldots\ sinx_n \tag{5}$$

By applying the RDTM to equation (4), we obtain the following iteration relation (i.e.,)

$$\frac{\Gamma(k\alpha+\alpha+1)}{\Gamma(k\alpha+1)} U_{k+1}(x_1, x_2, \ldots, x_n) = \frac{\partial^2}{\partial x_1^2} U_k(x_1, x_2, \ldots, x_n) + \frac{\partial^2}{\partial x_2^2} U_k(x_1, x_2, \ldots, x_n) + \ldots + \frac{\partial^2}{\partial x_n^2} U_k(x_1, x_2, \ldots, x_n)$$

where $'t'$ is the dimensional spectrum function $U_k(x_1, x_2, \ldots, x_n)$ is the transformed function. We can rewrite the iniial condition as, $U_0(x_1, x_2, \ldots, x_n) = 2sinx_1\ sinx_2\ \ldots\ sinx_n$. By using equation (6), we find the following values of $U_k(x_1, x_2, \ldots, x_n)$ successively.
If $k = 0$ in (6), then we get,

$$U_1(x_1, x_2, \ldots, x_n) = -\frac{2nsinx_1\ sinx_2\ \ldots\ sinx_n}{\Gamma(\alpha+1)}$$

Similarly we obtain

$$\begin{aligned} U_2(x_1, x_2, \ldots, x_n) &= \frac{2n^2 sinx_1\ sinx_2\ \ldots\ sinx_n}{\Gamma(2\alpha+1)} \\ U_3(x_1, x_2, \ldots, x_n) &= -\frac{2n^3 sinx_1\ sinx_2\ \ldots\ sinx_n}{\Gamma(3\alpha+1)} \\ U_4(x_1, x_2, \ldots, x_n) &= \frac{2n^4 sinx_1\ sinx_2\ \ldots\ sinx_n}{\Gamma(4\alpha+1)} \\ &and\ \ so\ on \end{aligned}$$

Finally the differential inverse transform of $U_k(x_1, x_2, \ldots, x_n)$ gives us

$$U(x_1, x_2, \ldots, x_n, t) = \sum_{k=0}^{\infty} U_k(x_1, x_2, \ldots, x_n, t)\, t^{k\alpha} = 2(sinx_1\ sinx_2\ \ldots\ sinx_n) - \frac{2n(sinx_1\ sinx_2\ \ldots\ sinx_n)}{\Gamma(\alpha+1)} t^\alpha + \frac{2n^2(sinx_1\ sinx_2\ \ldots\ sinx_n)}{\Gamma(2\alpha+1)} t^{2\alpha}$$

**Example 3.2.** *Consider the $n$- dimensional in homogeneous time fractional heat equation,*

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} + \ldots + \frac{\partial^2 u}{\partial x_n^2} + 2cosx_1 \ldots cosx_n, 0 < x_1, \ldots, x_n < \pi, t > 0, 0 < \alpha \leq 1 \tag{6}$$

with an initial condition,

$$u(x_1, x_2, \ldots, x_n, 0) = 0 \tag{7}$$

Thus the RDTM of equation (6) is obtained by the following iteration relation

$$\frac{\Gamma(k\alpha+\alpha+1)}{\Gamma(k\alpha+1)} U_{k+1}(x_1, x_2, \ldots, x_n) = \frac{\partial^2}{\partial x_1^2} U_k(x_1, x_2, \ldots, x_n) + \frac{\partial^2}{\partial x_2^2} U_k(x_1, x_2, \ldots, x_n) + \ldots + \frac{\partial^2}{\partial x_n^2} U_k(x_1, x_2, \ldots, x_n) + 2cosx_1 \ldots cosx_n$$

where $'t'$ - dimensional spectrum function $U_k(x_1, x_2, \ldots, x_n)$ is the transformed function.
From the initial condition (7), we can write $U_0(x_1, x_2, \ldots, x_n) = 0$
By the iteration equation (8) we can obtain the following values of $U_k(x_1, x_2, \ldots, x_n)$ successively,

$$\begin{aligned} U_1(x_1, x_2, \ldots, x_n) &= \frac{2cosx_1\ cosx_2\ \ldots\ cosx_n}{\Gamma(\alpha+1)}, \\ U_2(x_1, x_2, \ldots, x_n) &= -\frac{2ncosx_1\ cosx_2\ \ldots\ cosx_n}{\Gamma(2\alpha+1)}, \\ U_3(x_1, x_2, \ldots, x_n) &= \frac{2n^2 cosx_1\ cosx_2\ \ldots\ cosx_n}{\Gamma(3\alpha+1)}, \\ U_4(x_1, x_2, \ldots, x_n) &= -\frac{2n^3 cosx_1\ cosx_2\ \ldots\ cosx_n}{\Gamma(4\alpha+1)} \\ &and\ \ so\ on\ . \end{aligned}$$

Hence the differential inverse transform of $U_k(x_1, x_2, \ldots, x_n)$ is given by

$$\begin{aligned} u(x_1, x_2, \ldots, x_n, t) &= \sum_{k=0}^{\infty} U_k(x_1, x_2, \ldots, x_n, t)\, t^{k\alpha} \\ &= \frac{2(cosx_1\ cosx_2\ \ldots\ cosx_n)}{\Gamma(\alpha+1)} t^\alpha \\ &- \frac{2n(cosx_1\ cosx_2\ \ldots\ cosx_n)}{\Gamma(2\alpha+1)} t^{3\alpha} \\ &+ \frac{2n^2(cosx_1\ cosx_2\ \ldots\ cosx_n)}{\Gamma(3\alpha+1)} t^{3\alpha} \\ &- \frac{2n^3(cosx_1\ cosx_2\ \ldots\ cosx_n)}{\Gamma(4\alpha+1)} t^{4\alpha} + \cdots \end{aligned}$$

## 4. CONCLUSION

Reduced Differential Transformation Method is applied to find an exact solutions of n- dimensional time fractional heat equations. This technique is very efficient and powerful in finding an analytical solutions of a linear partial differential equations of the fractional order. Thus the numerical results reveals the complete reliability and efficiency of the proposed methodology.

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