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Edge Geodetic Parameters of Some Special Graphs

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Abstract

A set S of vertices of a graph G is an edge geodetic set if every edge of G lies on x-y geodesic for some elements x and y in S. The minimum cardinality of an edge geodetic set of G is the edge geodetic number of G denoted by $g_1(G)$. In this paper, we explore the concept of edge geodetic parameters in the context of various types of special graphs such as Cocktail party graph, Crown graph, Dutch windmill graph, Friendship graph, Shadow graph, Tadpole graph, Windmill graph, Jump graph.

Keywords: Edge geodetic set, Connected edge geodetic set, Restrained edge geodetic set, Split edge geodetic set.

Mathematics Subject Classification 2012: 05C12, 05C76.

1. INTRODUCTION

Let $G = (V, E)$ be a connected graph with node set $V = V(G)$ and the edge set $E = E(G)$. A set S of vertices of a graph G is an edge geodetic set if every edge of G lies on an x-y geodesic for some elements x and y in S. The minimum cardinality of an edge geodetic set of G is the edge geodetic number of G denoted by $g_1(G)$. The edge geodetic number (g_1) was introduced and studied in [9]. The concept of connected edge geodetic number(g_{1c}) was introduced in [8]. A. P. Santhakumaran et al. introduced the concept of restrained edge geodetic number (eg_r) in [6]. The concept of split edge geodetic number (g_{1s}) was introduced in [2]. In [1] Shobha and Venkanagouda M Goudar introduced the concept of total edge geodetic number(g_{1t}). For any undefined terms or notations in this paper can be found in Harary [11].

2. MAIN RESULTS

Crown Graph

Definition 2.1. The Crown graph $H_{n,n}$ is the graph obtained from the complete bipartite graph $K_{n,n}$ by removing a perfect matching. It is defined in [5].

Example: For a Crown graph $H_{4,4}$ given in Figure 1, the darkened vertices is its g_1- set.

The set $S = \{u_1, v_1\}$ is g_1- set so that $g_1(H_{4,4}) = 2$.

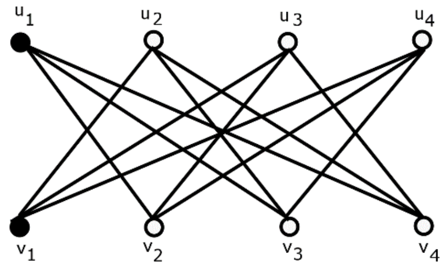

Figure 1: Crown graph

Theorem 2.2. For a Crown graph $H_{n,n}$, $n \geq 3$, $g_1(H_{n,n})$=2.

Proof. Let $G= H_{n,n}$ with vertex set $\{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$ and the edge set $\{(u_i, v_j) : 1 \leq i \neq j \leq n\}$ such that $|V(G)| = 2n$ and $|E(G)| = n$. Let $S = \{u_r, v_s\}$ such that $d(u_r, v_s) = 3$. Clearly, all the edges of G lies in the geodesic joining u_r and $v_s$and hence S is g_1- set. Thus, $g_1(H_{n,n}) = 2$.

Theorem 2.3 If $H_{n,n}$, $n \geq 3$ is any crown graph then $g_{1c}(H_{n,n}) = 4$.

Proof. Let $G = H_{n,n}$. By Theorem 2.2, $S = \{u_r, u_s\}$ is $g_1 -$set of G and g_1 $(G) = 2$. But $< S >$ is disconnected. Let $S' = S \cup \{u_i, u_j\}$ where u_i is adjacent to v_s *and* v_j. Clearly, $< S' >$ is connected. Therefore, S' is $g_{1c} -$set of G. Hence, $g_{1c}(H_{n,n}) = 4$.

Theorem 2.4 For any crown graph $H_{n,n}$, $n \geq 4$ $g_{1t}(H_{n,n}) = \alpha_0(H_{n,n}) + 1$ where α_0 is the vertex covering number of $H_{n,n}$.

Proof. Consider $G = H_{n,n}$. Let α_0 be the vertex covering number of G. We have By the Theorem 2.2 $S = \{u_r, v_s\}$ is the g_1-set of G and g_1 $(G) = 2$. Consider $S_1 = \{\cup u_k, \cup v_l,\ 2 \leq k, l \leq n-2\}$.
Let $S' = S \cup S_1$. Clearly, $< S' >$ has no isolated

vertices. Therefore, S' is g_{1t} − set. Hence $g_{1t}(H_{n,n}) = |S'| = \alpha_0(H_{n,n}) + 1$.

Corollary 2.5 For any crown graph $H_{n,n}, n \geq 4$ $g_{1s}(H_{n,n}) = g_{1t}(H_{n,n})$.

Friendship graph

Definition 2.6 The Friendship graph is a planar undirected graph with 2n+1 vertices and 3n edges. The friendship graph F_3^n can be constructed by joining n copies of the cycle C_3 with a common vertex. It is defined in [3].

Example : For a friendship graph F_3^4 given in Figure 4, the colorless vertices is its edge geodetic set.

The set S = {v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8} is an edge geodetic set of F_3^4 so that $g_1(F_3^4)$ =8.

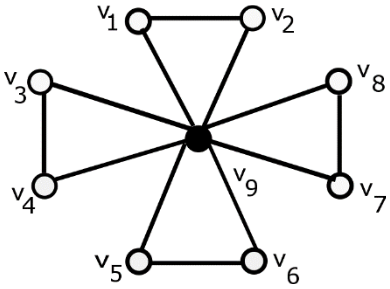

Figure 2: Friendship graph

Theorem 2.7 For any friendship graph F_3^n, n≥ 2 , $g_1(F_3^n)$=2n.

Proof. Let $G = F_3^n$ with $V(G) = \{v_1, v_2, v_3, \ldots, v_k, v_{2n}\}$ such that $|V(G)| = 2n+1$ and $|E(G)| = 3n$ and $\{v_k\}$ be the common vertex . Let $S_1 = \{v_1, v_2, v_3, \ldots, v_{2n}\}$ be g_1- set of G which covers all the edges of G. Therefore, $g_1(G) = 2n$.

Corollary 2.8 For any friendship graph F_3^n, n≥ 2 , $g_{1t}(F_3^n) = g_1(F_3^n)$.

Theorem 2.9 For a friendship graph F_3^n, $g_{1c}(F_3^n)$ = 2n+1.

Proof. By the Theorem 2.7, $S_1 = \{v_1, v_2, v_3, \ldots, v_{2n}\}$ is g_1-set and $g_1(F_3^n)$=2n. Let $S_2 = S_1 \cup \{v_k\}$ forms g_{1c}- set of F_3^n. Therefore, $g_{1c}(F_3^n)$ = 2n+1.

Corollary 2.10. For any friendship graph F_3^n there is no split edge geodetic number and restrained edge geodetic number.

Windmill Graph

Definition 2.11. The Windmill graph $Wd(k,n)$ is an undirected graph obtained by taking k copies of the complete graph K_n with a vertex in common. It is defined in [3].

Example: For a Windmill graph $Wd(3,4)$ given in Figure 3. The colorless vertices is its g_1-set.

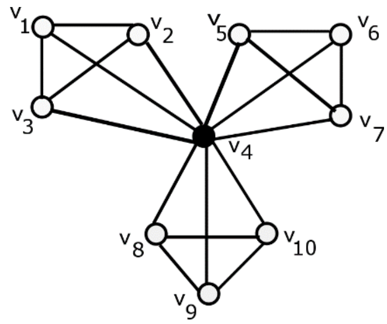

Figure 3: Windmill graph

Let $S = \{v_1, v_2, v_3, v_5, v_6, v_7, v_8, v_9, v_{10}\}$ be g_1 −set so that $g_1(Wd(3,4))$ = 9.

Theorem 2.12 For any Windmill graph $Wd(k,n)$, $g_1(Wd(k,n)) = k(n-1)$.

Proof. et $G = Wd(k,n)$ and $V(G) = \{u_1, u_2, u_3, \ldots, u_{k(n-1)}, x\}$and x is a common vertex. Let $S = \{\{u_1, u_2, u_3, \ldots, u_{(n-1)}\}_{k\ times}\}/x\}$. Clearly, S is g_1-set of G. Therefore, $g_1(Wd(k,n)) = k(n-1)$.

Corollary 2.13 For a Windmill graph , $g_{1t}(Wd(k,n)) = g_1(Wd(k,n))$.

Theorem 2.14 For any Windmill graph $Wd(k,n)$, $g_{1c}(Wd(k,n)) = k(n-1) + 1$.

Proof. Let $G = Wd(k,n)$. By the Theorem 2.12, $S = \{\{u_1, u_2, u_3, \ldots, u_{(n-1)}\}\}_{k\ times}/x\}$ is g_1-set of G and $g_1(G) = k(n-1)$. But $< S >$ is disconnected. Let S_1=$S \cup \{x\}$. Clearly, $< S_1 >$ is connected and hence S_1 is g_{1c} −set. Therefore, $g_{1c}(Wd(k,n)) = k(n-1) + 1$.

Dutch windmill Graph

Definition 2.15 The Dutch windmill graph D_n^k is the graph obtained by taking k copies of the cycle graph C_n with a vertex in common. It is defined in [3].

Example: For a Dutch windmill graph D_4^3 , $S = \{v_1, v_6, v_9\}$ is edge geodetic set of D_4^3 so that $g_1(D_4^3)$ = 3.

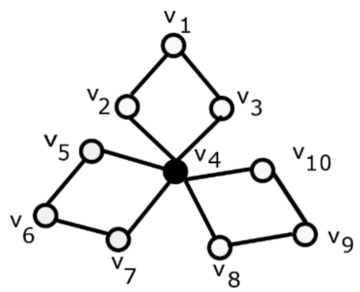

Figure 4: Dutchwind mill graph

Theorem 2.16. Let D_n^k be the Dutch windmill graph then

$$g_1(D_n^k) = \begin{cases} k \ \ if \ \ n \equiv 0 (mod\ 2), \\ 2k \ if \ n \equiv 1\ (mod\ 2). \end{cases}$$

Proof. Let $G = D_n^k$ consisting of k copies of the cycle graph C_n with a vertex in common. Let $V(G) = \{\cup v_i^l\} \cup \{v_j\}$ where $1 \le i \le n-1$, $1 \le l \le k$ and v_j is a common vertex. Then $V(G) = k(n-1) + 1$ and $E(G) = nk$. We discuss the following cases:

Case 1. $Let\ n \equiv 0(mod 2)$. Let $S_1 = \{v_i^1, v_i^2, v_i^3, \dots, v_i^l\}$ such that $d(v_i^1, v_i^2) = d(v_i^2, v_i^3) = d(v_i^3, v_i^4) = \cdots = d(v_i^{l-1}, v_i^l) = diam(G)$ where $\{v_i^1, v_i^2, v_i^3, \dots, v_i^l\}$ are the antipodal vertices of G. Clearly, every edge of G lies in geodesic joining any two vertices of S_1. Therefore, S_1 is g_1-set of G. Hence $g_1(G) = |S_1| = k$.

Case 2. $Let\ n \equiv 1(mod 2)$. Let $S_2 = \{v_i^1, v_{i+1}^1, v_i^2, v_{i+1}^2, v_i^3, v_{i+1}^3 \dots, v_i^l, v_{i+1}^1\}$ such that $d(v_i^1, v_i^2) = d(v_{i+1}^1, v_{i+1}^2) = d(v_i^2, v_i^3) = d(v_{i+1}^2, v_{i+1}^3) = d(v_i^3, v_i^4) = d(v_{i+1}^3, v_{i+1}^4) = \cdots = d(v_i^{l-1}, v_i^l) = d(v_{i+1}^{l-1}, v_{i+1}^l) = diam(G)$ be g_1-set of G. Hence , $g_1(G) = |S_2| = 2k$.

Theorem 2.17. For any Dutch windmill graph D_n^k, $g_{1t}(D_n^k) = 2k$.

Proof. Let $G = D_n^k$. We discuss the following cases:

Case 1. When n is even. By the theorem 2.16, $S_1 = \{v_i^1, v_i^2, v_i^3, \dots, v_i^l\}$ is g_1-set of G. But $< S_1 >$ has isolated vertices. Let $S' = S \cup \{v_{i+1}^1, v_{i+1}^2, v_{i+1}^3, \dots, v_{i+1}^l\}$. Clearly, $< S' >$ has no isolated vertices and hence $g_{1t}(G) = 2k$..

Case 2. When n is odd. By the theorem 2.16, $S_2 = \{v_i^1, v_{i+1}^1, v_i^2, v_{i+1}^2, v_i^3, v_{i+1}^3 \dots, v_i^l, v_{i+1}^1\}$ is g_1-set of G. But $< S_2 >$ has no isolated vertices. Therefore, S_2 is g_{1t}-set and hence$g_{1t}(G) = 2k$.

Theorem 2.18. For any Dutchwind mill graph D_n^k ,

$$g_{1s}(D_n^k) = \begin{cases} k+1 \ \ if \ \ n \equiv 0 (mod\ 2), \\ 2k+1 \ if \ n \equiv 1\ (mod\ 2). \end{cases}$$

Proof. Let $G = D_n^k$. Let $V(G) = \{\cup v_i^l\} \cup \{v_j\}$ where $1 \le i \le n-1$, $1 \le l \le k$ and v_j is a common vertex. We have the following cases:

Case 1. For n is even. By the theorem 2.16, $S_1 = \{v_i^1, v_i^2, v_i^3, \dots, v_i^l\}$ is g_1- set of G. But $< V(G) - S_1 >$ is connected. Let $S' = S_1 \cup \{v_j\}$. Clearly, $< V(G) - S' >$ is disconnected and is g_{1s}-set of G. Hence, $g_{1s}(G) = k + 1$.

Case 2. For n is odd. By the theorem 2.16, $S_2 = \{v_i^1, v_{i+1}^1, v_i^2, v_{i+1}^2, v_i^3, v_{i+1}^3 \dots, v_i^l, v_{i+1}^1\}$ is g_1-set of G. But $< V(G) - S_2 >$ is connected. Let $S'' = S_2 \cup \{v_j\}$ forms g_{1s}-set of G. Therefore, $g_{1s}(G) = 2k + 1$.

Shadow Graph

Definition 2.19. The Shadow graph of G, denoted by $D_2(G)$ is the graph constructed from G by taking two copies of G namely G and G' and by joining each vertex u in G to the neighbours of the corresponding vertex u' in G'. It is defined in [4].

Example: For the shadow graph $D_2(P_5)$ given in Figure 5, $S = \{v_1, v_5, v_1', v_5'\}$ is an edge geodetic set so that $g_1(D_2(P_5))$=4.

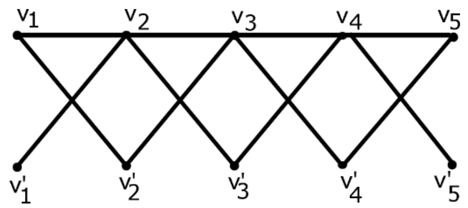

Figure 5: G

Observation 2.20 For any path $P_n, n \ge 4$ the vertex covering number α_0 (P_n) of shadow graph $D_2(P_n)$ is equal to n.

Theorem 2.21. For any path $P_n, n \ge 4$, $g_1(D_2(P_n)) = 4$.

Proof. Let $G = (D_2(P_n))$ and $V(G) = \{v_1, v_2, v_3, \dots, v_n, v_1', v_2', v_3', \dots, v_n'\}$ where $\{v_1, v_2, v_3, \dots, v_n\}$ are the vertices of P_n and $\{v_1', v_2', v_3', \dots, v_n'\}$ are the shadow vertices of P_n respectively. Then $|V(G)| = 2n$ and $|E(G)| = 4n$-6.

For n=3, it is easy to verify that, g_1 (G) = 3.

For $n \ge 4$, let $S = \{v_1, v_n, v_1', v_n'\}$. Since all the edges lie in geodesic joining any two vertices of S, clearly, S is g_1- set. Therefore, $g_1(G)$ = $|S|$ = 4.

Corollary 2.22. Let $C_n, n \ge 4$, be any path then $eg_r(D_2(P_n)) = g_1(D_2(P_n))$.

Theorem 2.23 For any path P_n , $n \ge 4$, $g_{1c}(D_2(P_n)) = \alpha_0(D_2(P_n))$ +2 where α_0 is the vertex covering number of $D_2(P_n)$.

Proof. Let α_0 be the vertex covering number of $G = D_2(P_n)$. By the Theorem, 2.21 $S = \{v_1, v_n, v_1', v_n'\}$ be g_1-set of G and g_1 (G) = 4. But $< S >$ is disconnected. Let S'= S $\cup$ $\{v_2, v_3, \dots, v_{n-1}\}$. Clearly, $< S' >$ is connected. By Observation 2.20, it follows that $g_{1c}(D_2(P_n)) = |S'| = \alpha_0(D_2(P_n))$ +2.

Theorem 2.24 For any path P_n , $n \ge 5$, $g_{1s}(D_2(P_n)) = 5$.

Proof. Let $G = D_2(P_n))$. By the Theorem 2.21, $S = \{v_1, v_n, v_1', v_n'\}$ be g_1-set of G and g_1 (G) = 4. But $<$ V-S $>$ is connected. Let S' = S $\cup$ $\{v_{n-2}\}$. Clearly, $< V - S' >$ is not connected. Hence, S' forms g_{1s} $-$set. Therefore, $g_{1s}(D_2(P_n)) = 5$.

Theorem 2.25 For any star $K_{1,n}$, $n \ge 3$, $g_1[(K_{1,n})] = \Delta(K_{1,n}) + 1$ where Δ is the maximum degree of $D_2(K_{1,n})$.

Proof.. Let G=$D_2(K_{1,n})$ and two copies of star $G = G' \cup G''$. Let $V(G') = \{a_1, a_2, a_3, \dots, a_n, x\}$ and $V(G'') = \{a'_1, a'_2, a'_3, \dots, a'_n, x'\}$ where x and x' are the central vertices of G' and G''. Let $S = V(G'')$. Clearly, S forms g_1 −set. Therefore, $g_1(D_2(K_{1,n})) = |S| = \Delta(K_{1,n}) + 1$.

Corollary 2.26 For any star $K_{1,n}$, $n \geq 3$, $eg_r[D_2(K_{1,n})] = g_1[D_2(K_{1,n})]$.

Theorem 2.27. *For any cycle* C_n, $n \geq 4$, $g_1[D_2(C_n)] = \begin{cases} 4 & for\ n\ is\ even, \\ 6 & for\ n\ is\ odd. \end{cases}$

Proof. Let $G = D_2(C_n)$ and $G = G' \cup G''$ where G' and G'' are the two copies of G such that $V(G') = \{v_1, v_2, \dots, v_n\}$ and $V(G'') = \{v'_1, v'_2, v'_3, \dots, v'_n\}$. We shall discuss the following cases:

Case 1. For n is even. Let $S = \{v_r, v'_r, v_s, v'_s\}$ where v_r is the antipodal vertex of v_s in G' which forms g_1 −set of $D_2(C_n)$. Hence, $g_1(D_2(C_n)) = 4$.

Case 2. For n is odd. Let $S = \{v_p, v'_p, v_r, v'_r, v_s, v'_s\}$ where v_p is the antipodal vertex of v_r and v_s in G' be g_1 −set. Hence, $g_1(D_2(C_n)) = 6$.

Corollary 2.28. . *For any cycle* C_n, $n \geq 4$, $eg_r[D_2(C_n)] = g_1[D_2(C_n)]$.

Corollary 2.29. *For any cycle* C_n, $n \geq 4$, $g_{1s}[D_2(C_n)] = g_1[D_2(C_n)]$.

Cocktail Party Graph

Definition 2.27. The Cocktail Party graph $\overline{L_n}$ is the graph consisting of two rows of paired vertices in which all vertices but the paired ones are connected with a graph edge. It is defined in [5].

Example: For a cocktail party graph $\overline{L_4}$ given in Figure 6, the colorless vertices is its edge geodetic set.. Then $S = \{u_1, u_2, u_3, w_1, w_2, w_3\}$ is $g_1 -\ set$ so that $g_1(\overline{L_n})$= 6.

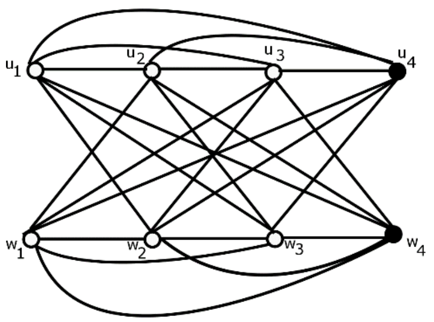

Figure 6: G

Theorem 2.28. The edge geodetic number of a cocktail party graph $\overline{L_n}$ of order $2n$ is equal to $2n - 2$.

Proof. Let $G = \overline{L_n}$. Let $V_1 = \{u_1, u_2, u_3, \dots, u_n\}$ *and* $V_2 = \{w_1, w_2, w_3, \dots, w_n\}$ be the two vertex sets respectively such that every vertex in V_1 has a vertex pair in V_2 except the paired ones (u_i, w_j) for $i = j$. Then $|V(G)| = 2n$. Let $S = \{u_1, u_2, u_3, \dots, u_{n-1}, w_1, w_2, .. w_{n-1}\}$ and every edge of G lies in geodesic joining any pair of vertices of S. Therefore, $g_1(G) = |S| = 2n - 2$.

Corollary 2.29 If $\overline{L_n}$ is any cocktail party graph of order $2n$, then $g_{1c}(\overline{L_n})$= $2n - 2$.

Corollary 2.30 For any cocktail party graph $\overline{L_n}$ of order $2n$, $g_{1s}(\overline{L_n})$= $g_1(\overline{L_n})$.

Corollary 2.31 For any cocktail party graph $\overline{L_n}$ of order $2n$, $g_{1t}(\overline{L_n})$= $g_1(\overline{L_n})$.

Tadpole Graph

Definition 2.32. The $T_{p,n}$ -Tadpole graph, also called a dragon graph, is the graph obtained by joining a cycle graph C_p to a path graph P_n with a bridge. It is defined in [4].

Example: For a Tadpole graph $T(5,2)$ given in Figure 7, The darkened vertices is itsg_1- set.

Let $S = \{x_2, x_4, y_2\}$ is edge geodetic set of $T(5,2)$ so that $T(5,2)$=3.

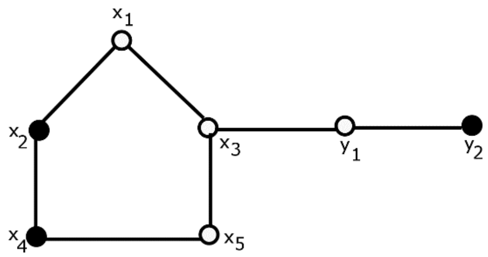

Figure 7: G

Theorem 2.33 For any tadpole graph $T(p,n), n \geq 3$, $g_1(T(p,n)) = \begin{cases} 2 & if\ n \equiv 0(mod 2), \\ 3 & if\ n \equiv 1(mod 2). \end{cases}$

Proof. Let $G = T(p,n)$. Consider $V(G) = \{x_1, x_2, x_3, \dots, x_p, y_1, y_2, y_3, \dots, y_n\}$ where $V(C_p) = \{x_1, x_2, x_3, \dots, x_p\}$ and $V(P_n) = \{y_1, y_2, y_3, \dots, y_n\}$.

We discuss in following two cases:

Case 1: when n is even. Let $S = \{x_i, y_n\}$ where $d(x_i, y_n) = diam(G)$ and . $I[S] = V(G)$. Clearly, S is $g_1 -\ set$ of G . Hence $g_1(G) = 2$.

Case 2. when n is odd. Let $S = \{x_i, x_{i+1}, y_n\}$ where $d(x_i, y_n) = d(x_{i+1}, y_n) = diam(G)$ and S forms $g_1 -\ set$ of G . Hence $g_1(G) = 3$.

Corollary 2.34 For a tadpole graph $T(p,n)$, $eg_r(T(p,n)) = g_1(T(p,n))$.

Theorem 2.35 For a tadpole graph $T(p,n)$, $g_{1t}(T(p,n)) = 4$.

Proof. Let $G = T(p,n)$. We discuss in following two cases:

Case 1. For n is even. By the Theorem 2.33, $S = \{x_i, y_n\}$ is g_1-set of G and $g_1(G) = 2$. But $< S >$ has isolated vertices. Let $S_1 = S \cup \{x_{i+1}, y_{n-1}\}$. Clearly, S_1 is g_{1t} - set of G. Therefore, $g_{1t}(G) = 4$.

Case 2. For n is odd. By the Theorem 2.33, $S = \{x_i, x_{i+1}, y_n\}$ is $g_1 - set$ of G and $g_1(G) = 3$. But $< S >$ has isolated vertices. Let $S_2 = S \cup \{y_{n-1}\}$. Clearly, S_2 is g_{1t} -set of G. Therefore, $g_{1t}(G) = 4$.

Theorem 2.36 For a tadpole graph $T(p,n)$ $g_{1s}(T(p,n)) = \begin{cases} 3 & if\ n \equiv 0(mod 2), \\ 4 & if\ n \equiv 1(mod 2). \end{cases}$

Proof. Let $G = T(p,n)$. We have the following cases:

Case 1. when n is even. By the Theorem 2.33, $S = \{x_i, y_n\}$ is g_1-set of G and $g_1(G) = 2$. But $< V - S >$ is connected. Let $S_1 = S \cup \{x_k\}$ where $\{x_k\}$ be any cut vertex of G. Clearly S_1 is g_{1s}- set of G. Hence $g_{1s}(G) = 3$.

Case 2. when n is odd. By the Theorem 2.33, $S = \{x_i, x_{i+1}, y_n\}$ is $g_1 - set$ of G and $g_1(G) = 3$. Let $S_2 = S \cup \{x_k\}$ where $\{x_k\}$ is the cut vertex of G and $< V - S_2 >$ is disconnected. Hence, $g_{1s}(G) = 4$.

Jump Graph

Definition 2.37 The Jump graph $J(G)$ of a graph G is defined as that graph whose vertices are the edges of G and where two vertices of $J(G)$ are adjacent if and only if the corresponding edges of G are independent. It is defined in [7].

Example: Consider the jump graph of path P_8 given in Figure 8, the set $S = \{u_1, u_2, u_3, u_4, u_5\}$ is an edge geodetic set of $J(P_8)$ so that $g_1[J(P_8)] = 5$.

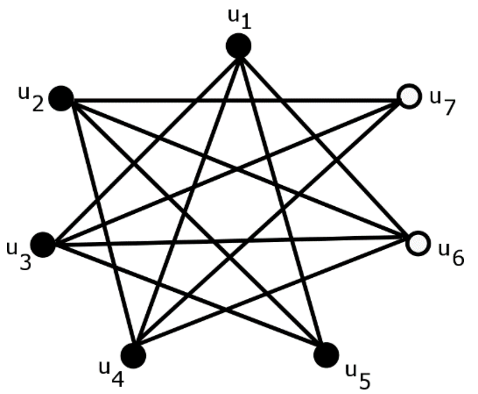

Figure 8: G

Theorem 2.38 For any path $P_n, n \geq 8$,

$$g_1[J(P_n)] = \begin{cases} 2 & for\ n = 5, \\ 3 & for\ n = 6, 7, \\ n - 3 & for\ n \geq 8. \end{cases}$$

Proof. For $n = 5, 6, 7$, proof is obvious.

Further, let $V_1' = \{v_1', v_2', v_3', \dots, v_n'\}$ be the set of vertices in $J(P_n)$ corresponding to the set of independent edges $e_1, e_2, e_3, \dots, e_n$ of P_n. Suppose that $n \geq 8$. Let $S_1 = \{v_1', v_2', v_3', \dots, v_{n-3}'\}$. Clearly, all the edges of $J(P_n)$ lies in geodesic joining any two vertices of S_1. Hence S_1 is $g_1 - set$ of $J(P_n)$. Therefore, $g_1[J(P_n)] = |S_1| = n - 3$.

Corollary2.39 For any path $P_n, n \geq 5$, $g_{1c}[J(P_n)] = g_1[J(P_n)]$.

Corollary2.40 For any path $P_n, n \geq 5$, $g_{1t}[J(P_n)] = g_1[J(P_n)]$.

Theorem 2.41 For any path $P_n, n \geq 8$, $g_{1s}[J(P_n)] = \begin{cases} 2 & for\ n = 6, 7, \\ \alpha_0 + 2 & for\ n \geq 8. \end{cases}$
where α_0 is the vertex covering number of P_n

Proof. Let α_0 be the vertex covering number of P_n. Proof is obvious for $n = 6, 7$. Consider any subgraph P_{n-2} in P_n. Let E be the set of edges in P_{n-2}. By the Theorem 2.38, let V' be the set of vertices in $J(P_{n-2})$ corresponding to the set of edges E of P_n. Clearly, $S_1 = V'$ is $g_1 -$ set of $J(P_n)$. But $< V - S_1 >$ is connected. Let $S_2 = V[J(P_n)] - S_1$ such that $< V - S_2 >$ is disconnected. Hence, $g_{1s}(J(P_n)) = |S_1| = \alpha_0(P_n) + 2$.

Theorem 2.42 For any cycle $C_n, n \geq 5$, then, $g_1[J(C_n)] = \deg([J(v_i)]) + 1$.

Proof. For $n = 4$, $J(C_4)$ is disconnected and hence there is no g_1-set for $J(C_4)$. Let $\{x_1, x_2, x_3, \dots, x_n\}$ be the set of vertices of $J(C_n)$ corresponding to the edges $E = \{e_1, e_2, e_3, \dots, e_n\}$ of C_n and $\deg([J(v_i)]) = n - 3$. Let $S = \{x_1, x_2, x_3, \dots x_{n-2}\}$. Clearly, S covers all the edges of $J(C_n)$. Hence S is g_1-set of $J(C_n)$. Therefore, $= |S| = g_1[J(C_n)) = \deg([J(v_i)]) + 1$.

Corollary 2.43 For any cycle $C_n, n \geq 5$, $g_{1c}(J(C_n)) = g_1[J(C_n)]$.

Corollary 2.44 For any cycle $C_n, n \geq 5$, $g_{1t}(J(C_n)) = g_1[J(C_n)]$.

Corollary 2.45 For any cycle $C_n, n \geq 5$, $g_{1s}(J(C_n)) = n - 2$.

CONCLUSION

In this paper, we obtained the results on edge geodetic parameters of several special graphs.

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