International Journal of Applied Engineering Research ISSN 0973-4562 Volume 14, Number 7 (2019) pp. 1750-1757

Geodetic Parameters on Switching of a Vertex

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Abstract

In this paper, we examine the results on some geodetic parameters on switching of pendant and central vertex to path P_n, arbitrary vertex to cycle C_n, rim vertex to wheel W_n and apex vertex to helm graph H_n.

Keywords: Geodetic number, maximum independent number, vertex covering number.

Mathematics Subject Classification: 05C12.

1. INTRODUCTION

As usual, all graphs are considered here simple, undirected, finite and connected. Here elements of sets $V(G)$ and $E(G)$ are the vertices and edges respectively. We defined $I[u, v]$ to the set of all vertices lying on some $u-v$ geodesic of G and for a non-empty subset S of $V(G)$, $I[S] = \cup_{u,v \in S} I[u, v]$. A set S of vertices of G is called a geodetic set in G if $I[S] = V(G)$ and a geodetic set of minimum cardinality is the geodetic number $g(G)$ and it was introduced in [13]. The concept of non-split, restrained, connected, doubly connected, split, total geodetic and total outer independent geodetic number of a graph are introduced in [5], [7], [10], [3], [9], [4] and [2]. In the present work we prove that the graph obtained by switching of pendant and central vertex for path P_n, any arbitrary vertex in cycle C_n, rim vertex of wheel W_n and apex vertex of helm graph H_n for the different parameters. For any undefined terms or notations in this paper can be found in Harary [14] and Chartrand [12].

2. SWITCHING OF A PENDANT VERTEX OF P_n

Definition 2.1. The switching of a vertex v of G means removing all the edges incident to v and adding edges joining to every vertex which are not adjacent to v in G. We denote the resultant graph by $\tilde{G}$.

Example 2.2. Consider a graph $\widetilde{P_8^p}$ is obtained by switching of a pendant vertex v_1 is shown in figure 1.

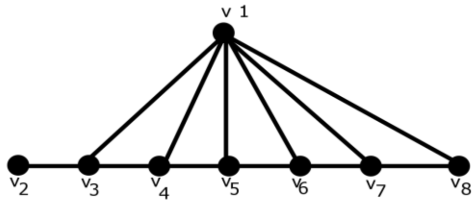

Figure 1: $\widetilde{P_8^p}$

Observation 2.3. For path P_n, v_1 be the switching vertex, v_2 be the support vertex of v_1 and v_n be the another pendant vertex, the vertices v_2 and v_n are always lies in the geodetic set.

Let $\widetilde{P_n^p}$ be the graph obtained by switching of pendant vertex p of path P_n. Without loss of generality, let the switched vertex be v_1. Let $V(P_n) = \{v_1, v_2, ..., v_n\}$, $V\left(\widetilde{P_n^p}\right) = \{v_1, v_2, ..., v_n\}$, $\Delta\left(\widetilde{P_n^p}\right) = n-2 = deg_{\widetilde{P_n^p}}(v_1)$, $deg_{\widetilde{P_n^p}}(v_2) = 1$ $deg_{\widetilde{P_n^p}}(v_n) = 2$ and $deg_{\widetilde{P_n^p}}(v_i) = 3, \forall\, i \in \{3,4, \ldots\ldots, n-1\}$.

Theorem 2.4. For any path $P_n = \{v_1, v_2, ..., v_n\}$ and v_1 be the switching vertex, then

$$g\left(\widetilde{P_n^p}\right) = \begin{cases} n-1 & for\ n = 3,4, \\ \dfrac{n}{2} & for\ n > 5, n \equiv 0(mod 2), \\ \dfrac{n-1}{2} & for\ n \geq 5, n \equiv 1(mod 2). \end{cases}$$

Proof. Let $\widetilde{P_n^p}$ be the graph obtained by switching of a pendant vertex v_1 of path P_n. Let $V\left(\widetilde{P_n^p}\right) = \{v_1, v_2, ..., v_n\}$ and $deg_{\widetilde{P_n^p}}(v_i) = 3, \quad \forall\, i \in \{3,4, \ldots\ldots, n-1\}$ are the internal vertices. To prove this result, we consider the following cases.

Case (i) For n = 3,4 the results is obvious.

Case (ii) Suppose $n \equiv 0(mod2)$ and $n > 5$. Consider the vertex set $S = \{v_2, v_n\} \cup \{\cup_{i=1}^{\frac{n-4}{2}} v_{2i+3}\}$, where v_{2i+3} is the internal non-adjacent vertices and $v_{2i+3} \notin N_{\widetilde{P_n^p}}(v_2)$ such

International Journal of Applied Engineering Research ISSN 0973-4562 Volume 14, Number 7 (2019) pp. 1750-1757
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that $I[S] = V\left(\widetilde{P_n^p}\right)$. Clearly S is the geodetic set of $\widetilde{P_n^p}$. Therefore, $g\left(\widetilde{P_n^p}\right) = |S| = |\{v_2, v_n\} \cup \{\cup_{i=1}^{\frac{n-4}{2}} v_{2i+3}\}| = \frac{n}{2}$.

Case (iii) Suppose n ≡1(mod2) and $n \geq 5$. Consider the vertex set $S = \{v_2, v_n\} \cup \{\cup_{i=1}^{\frac{n-5}{2}} v_{2i+3}\}$, where $d_{\widetilde{P_n^p}}(v_2, v_{2i+3}) = 3 = d_{\widetilde{P_n^p}}(v_2, v_n)$, v_{2i+3} is the internal non-adjacent vertices and $v_{2i+3} \notin N_{\widetilde{P_n^p}}(v_2)$ such that $I[S] = V\left(\widetilde{P_n^p}\right)$. Clearly S is the geodetic set of $\widetilde{P_n^p}$. Therefore, $g\left(\widetilde{P_n^p}\right) = |S| = |\{v_2, v_n\} \cup \{\cup_{i=1}^{\frac{n-5}{2}} v_{2i+3}\}| = \frac{n-1}{2}$.

Theorem 2.5. For any path P_n of order $n \geq 5$, $g\left(\widetilde{P_n^p}\right) = g_r\left(\widetilde{P_n^p}\right)$.

Proof. Let

$$S = \begin{cases} \{v_2, v_n\} \cup \{\cup_{i=1}^{\frac{n-4}{2}} v_{2i+3}\} \ for\ n \equiv 0(mod2), \\ \{v_2, v_n\} \cup \{\cup_{i=1}^{\frac{n-5}{2}} v_{2i+3}\} \ for\ n \equiv 1(mod2). \end{cases}$$

be the geodetic set and $< V\left(\widetilde{P_n^p}\right) - S >$ has no isolated vertex. Then the set S itself forms a $g_r - set$ of $\widetilde{P_n^p}$ such that $g\left(\widetilde{P_n^p}\right) = g_r\left(\widetilde{P_n^p}\right)$. We have by Theorem 2.4,

$$|S| = g\left(\widetilde{P_n^p}\right) = \begin{cases} \frac{n}{2} & for\ n \equiv 0(mod2), \\ \frac{n-1}{2} & for\ n \equiv 1(mod2). \end{cases}$$

Hence $g\left(\widetilde{P_n^p}\right) = g_r\left(\widetilde{P_n^p}\right)$.

Corollary 2.6. For any path P_n of order $n \geq 5$, $g\left(\widetilde{P_n^p}\right) = g_{ns}(\widetilde{P_n^p})$.

Corollary 2.7. For any path P_n of order $n \geq 5$,

$$ig\left(\widetilde{P_n^p}\right) = \begin{cases} \frac{n}{2} & for\ n \equiv 0(mod2), \\ \frac{n-1}{2} & for\ n \equiv 1(mod2). \end{cases}$$

Theorem 2.8. For any path P_n of order $n \geq 6$,

$$g_s\left(\widetilde{P_n^p}\right) = \begin{cases} \frac{n+2}{2} & for\ n\ is\ even, \\ \frac{n+1}{2} & for\ n\ is\ odd. \end{cases}$$

Proof. Let $V\left(\widetilde{P_n^p}\right) = \{v_1, v_2, ..., v_n\}$. We have two cases to prove this result.

Case (i) Suppose n is even . By Theorem 2.4, S= $\{v_2, v_n\} \cup \{\cup_{i=1}^{\frac{n-4}{2}} v_{2i+3}\}$ be the geodetic set of $\widetilde{P_n^p}$ and $< V\left(\widetilde{P_n^p}\right) - S >$ is connected. Consider $S_1 = S \cup \{v_1\}$ which makes $< V\left(\widetilde{P_n^p}\right) - S_1 >$ is disconnected. Clearly S_1 forms a $g_s - set$. Therefore, $g_s\left(\widetilde{P_n^p}\right) = |S_1| = \frac{n+2}{2}$.

Case (i) Suppose n is odd. By Theorem 2.4, S= $\{v_2, v_n\} \cup \{\cup_{i=1}^{\frac{n-5}{2}} v_{2i+3}\}$ be the geodetic set of $\widetilde{P_n^p}$ and $< V\left(\widetilde{P_n^p}\right) - S >$ is connected. Consider $S_1 = S \cup \{v_1\}$ which makes $< V\left(\widetilde{P_n^p}\right) - S_1 >$ is disconnected. Clearly S_1 forms a $g_s - set$. Therefore, $g_s\left(\widetilde{P_n^p}\right) = |S_1| = \frac{n+1}{2}$.

Theorem 2.9. For any path P_n of order $n \geq 5$,

$$g_t\left(\widetilde{P_n^p}\right) = \begin{cases} \alpha_0\left(\widetilde{P_n^p}\right) + 2 & for\ n\ is\ even, \\ \alpha_0\left(\widetilde{P_n^p}\right) + 1 & for\ n\ is\ odd. \end{cases}$$

Proof. Let $V(\widetilde{P_n^p}) = \{v_1, v_2, ..., v_n\}$. We have two cases to prove this results.

Case(i) Let n be even. From case(i) of Theorem 2.8, S= $\{v_1, v_2, v_n\} \cup \{\cup_{i=1}^{\frac{n-4}{2}} v_{2i+3}\}$ be the g_s-set of $\widetilde{P_n^p}$ with $|S| = \frac{n+2}{2} = \alpha_0\left(\widetilde{P_n^p}\right) + 1$ and $< S >$ has isolated vertex. Consider $A = \{v_3\}$ such that $v_3 \in V(\widetilde{P_n^p})$. Thus $S_1 = S \cup A$ be the vertex set which makes $< S_1 >$ has no isolated vertex. Therefore, $g_t\left(\widetilde{P_n^p}\right) = |S_1| = \alpha_0\left(\widetilde{P_n^p}\right) + 2$.

Case(ii) Let n be odd. From case(ii) of Theorem 2.8, S= $\{v_1, v_2, v_n\} \cup \{\cup_{i=1}^{\frac{n-5}{2}} v_{2i+3}\}$ be the g_s-set of $\widetilde{P_n^p}$ with $|S| = \frac{n+1}{2} = \alpha_0\left(\widetilde{P_n^p}\right)$ and $< S >$ has isolated vertex. Consider $A = \{v_3\}$ such that $v_3 \in V(\widetilde{P_n^p})$. Thus $S_1 = S \cup A$, be the vertex set which makes $< S_1 >$ has no isolated vertex. Therefore, $g_t\left(\widetilde{P_n^p}\right) = |S_1| = \alpha_0\left(\widetilde{P_n^p}\right) + 1$.

Theorem 2.10. For any path P_n of order n ≥ 5, $g_t\left(\widetilde{P_n^p}\right) = g_c\left(\widetilde{P_n^p}\right)$.

Proof. Let S be the total geodetic set of $\widetilde{P_n^p}$ and $< S >$ is connected. Thus S itself forms a $g_c - set$. Clearly $g_t\left(\widetilde{P_n^p}\right) = g_c\left(\widetilde{P_n^p}\right)$.

Corollary 2.11. For any path P_n of order n ≥ 5, $g_{toi}\left(\widetilde{P_n^p}\right) = g_c\left(\widetilde{P_n^p}\right)$.

Corollary 2.12. For any path P_n of order $n \geq 5$,

$$g_{ss}\left(\widetilde{P_n^p}\right) = \begin{cases} \alpha_0\left(\widetilde{P_n^p}\right) + 2 & for\ n\ is\ even, \\ \alpha_0\left(\widetilde{P_n^p}\right) + 1 & for\ n\ is\ odd. \end{cases}$$

3. SWITCHING OF A CENTRAL VERTEX OF P_n

Observation 3.1. Switching of the central vertex of P_n is denoted by $\widetilde{P_n^c}$, where c is a central vertex, $\widetilde{P_n^c}$ be the graph

International Journal of Applied Engineering Research ISSN 0973-4562 Volume 14, Number 7 (2019) pp. 1750-1757

obtained by switching of central vertex $v_{\frac{n}{2}}$ or $v_{\frac{n+1}{2}}$ of P_n. Without loss of generality, let the switched vertex be $v_{\frac{n}{2}}$. Let V $(P_n) = \{v_1, v_2, \ldots, v_n\}$, V $(\widetilde{P_n^c}) = \{v_1, v_2, \ldots, v_n\}$, $deg_{\widetilde{P_n^c}}\left(v_{\frac{n}{2}}\right) = n-3 = \Delta(\widetilde{P_n^c})$, $deg_{\widetilde{P_n^c}}(v_1) = 2 = deg_{\widetilde{P_n^c}}(v_n)$ and $deg_{\widetilde{P_n^c}}(v_2) = deg_{\widetilde{P_n^c}}(v_3) = \cdots deg_{\widetilde{P_n^c}}\left(v_{\frac{n}{2}-2}\right) = deg_{\widetilde{P_n^c}}\left(v_{\frac{n}{2}+2}\right) = deg_{\widetilde{P_n^c}}(v_{n-1}) = 3$.

Example 3.2. Let $\widetilde{P_9^c}$ be the graph obtained by switching of the central vertex v_5 is shown in figure 2. The darkened vertices is its geodetic set.

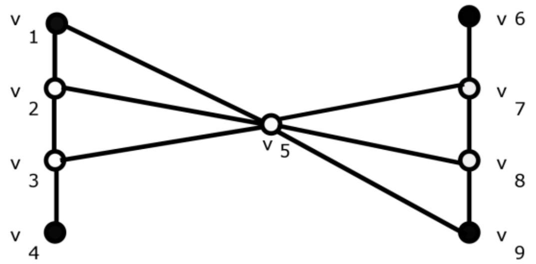

Figure 2: $\widetilde{P_9^c}$

Theorem 3.3. For any path $P_n = \{v_1, v_2, v_3, \ldots, v_n\}$ and $v_{\frac{n}{2}}$ be the switching vertex of order $n \geq 5$, then

$$g(\widetilde{P_n^c}) = \begin{cases} \frac{n}{2} & for\ n \equiv 0\ or\ 2(mod 4), \\ \frac{n+1}{2} & for\ n \equiv 1(mod4), \\ \frac{n-1}{2} & for\ n \equiv 4(mod4). \end{cases}$$

Proof. Let G = $\widetilde{P_n^c}$. Switching vertex be $v_{\frac{n}{2}}$ and V(G) = $\{v_1, v_2, v_3, \ldots, v_n\}$. Suppose S is any geodetic set of G, then it is obvious that $v_{\frac{n}{2}-1}$, $v_{\frac{n}{2}+1}$ must belong to S as $deg_G\left(v_{\frac{n}{2}-1}\right) = deg_G\left(v_{\frac{n}{2}+1}\right) = 1$. We consider three cases to prove the result.

Case(i) Let n be even and n ≡ 0 or 2(mod4) and n ≥ 8. We construct a vertex set S as follows:

S=

$$\begin{cases} \left\{v_{2i+1}, v_{2i+\frac{n}{2}+2} / 1 \leq i \leq \frac{n-8}{4}\right\} \\ \cup \left\{v_1, v_{\frac{n}{2}-1}, \quad v_{\frac{n}{2}+1}, v_n\right\} \quad for\ n \equiv 0(mod4) \\ \left\{v_{2i+1} / 1 \leq i \leq \frac{n-10}{4}\right\} \cup \\ \left\{v_{2i+\frac{n}{2}+2} / 1 \leq i \leq \frac{n-6}{4}\right\} \cup \\ \cup \left\{v_1, v_{\frac{n}{2}-1}, \quad v_{\frac{n}{2}+1}, v_n\right\} \ for\ n \equiv 2(mod4) \end{cases}$$

be the non-adjacent vertices of G with $|S| = \frac{n}{2}$ for $n \equiv 0 or 2(mod4)$,where $v_{2i+1}, v_{2i+\frac{n}{2}+2}$ are the vertices with distance 2 or 3 for $n \equiv 0$ *or* $2(mod4)$. Clearly S is a geodetic set of G. Suppose $v_k \in S$, The set $S - \{v_k\}$ is not a geodetic set of G for all vertex $v_l \in V(G) - (S - \{v_k\})$ there exists $v_j \in S - \{v_k\}$ such that $|d_G(v_l) - d_G(v_j)| \leq 1$. Therefore S is a minimum geodetic set. Clearly $g(\widetilde{P_n^c}) = \frac{n}{2}$.

Case(ii) Suppose n is odd and $n \equiv 1(mod4)$ and $n > 8$. Consider $S = \left\{v_{2i+1}, v_{\frac{n+4i+5}{2}} / 1 \leq i \leq \frac{n-9}{4}\right\} \cup \{v_1, v_{\frac{n}{2}-1}, v_{\frac{n}{2}+1}, v_n\}$ be the set of non-adjacent vertices of G with distance equal to 2 or 3 of G, $d(v_{2i+1}, , v_{\frac{n}{2}-1}) = 3$ $d(v_{\frac{n+4i+5}{2}}, v_{\frac{n}{2}+1})$ and $I[S] = V(G)$. Hence S is the geodetic set with minimum. Clearly $g(\widetilde{P_n^c}) = \frac{n-1}{2}$.

Case(iii) Suppose n is odd and $n \equiv 3(mod4)$ and $n > 8$. Let $S = \left\{v_{2i}, v_{\frac{n+4i+5}{2}} / 1 \leq i \leq \frac{n-7}{4}\right\} \cup \{v_1, v_{\frac{n}{2}-1}, v_{\frac{n}{2}+1}, v_n\}$ be the set of vertices with $V(G) - S$ is adjacent to atleast one vertex of S such that $I[S] = V(G)$. Therefore $g(\widetilde{P_n^c}) = \frac{n+1}{2}$.

Corollary 3.4. For any path $P_n = \{v_1, v_2, v_3, \ldots, v_n\}$ and $v_{\frac{n}{2}}$ be the switching vertex of order $n \geq 5$, then

$$g_{ns}(\widetilde{P_n^c}) = g(\widetilde{P_n^c})$$

Corollary 3.5. For any path P_n of order $n \geq 5$, then

$$g_r(\widetilde{P_n^c}) = g_{ns}(\widetilde{P_n^c})$$

Theorem 3.6. If the graph $\widetilde{P_n^c}$ is obtained by switching of central vertex of path P_n, $n \geq 8$ then $g_s(\widetilde{P_n^c}) = g(\widetilde{P_n^c}) + 1$.

Proof. Consider $V(\widetilde{P_n^c}) = \{v_1, v_2, \ldots, v_n\}$ and switching vertex be $v_{\frac{n}{2}}$ we have by Theorem 3.2,

S=

$$\begin{cases} \left\{v_{2i+1}, v_{2i+\frac{n}{2}+2} / 1 \le i \le \frac{n-8}{4}\right\} \\ \cup \left\{v_1, v_{\frac{n}{2}-1}, \quad v_{\frac{n}{2}+1}, v_n\right\} \quad for\ n \equiv 0(mod 4) \\ \left\{v_{2i+1} \le i \le \frac{n-10}{4}\right\} \cup \\ \left\{v_{2i+\frac{n}{2}+2} \le i \le \frac{n-6}{4}\right\} \cup \\ \cup \left\{v_1, v_{\frac{n}{2}-1}, \quad v_{\frac{n}{2}+1}, v_n\right\} \quad for\ n \equiv 2(mod 4) \\ \left\{v_{2i+1}, v_{\frac{n+4i+5}{2}} / 1 \le i \le \frac{n-9}{4}\right\} \\ \cup \left\{v_1, v_{\frac{n}{2}-1}, \quad v_{\frac{n}{2}+1}, v_n\right\} \quad for n \equiv 1(mod 4) \\ \left\{v_{2i}, v_{\frac{n+4i+5}{2}} / 1 \le i \le \frac{n-7}{4}\right\} \\ \cup \left\{v_1, v_{\frac{n}{2}-1}, \quad v_{\frac{n}{2}+1}, v_n\right\} \quad for\ n \equiv 3(mod 4) \end{cases}$$

be the geodetic set, but $< V(\widetilde{P_n^c}) - S >$ is connected. Consider $S \cup \{\frac{n}{2}\}$ be the g_s-set which makes $< V(\widetilde{P_n^c}) - (S \cup \{\frac{n}{2}\}) >$ is disconnected. Therefore $g_s(\widetilde{P_n^c}) = \left|S \cup \{\frac{n}{2}\}\right| = g(\widetilde{P_n^c}) + 1$.

Theorem 3.7. For any path P_n of order $n \ge 8$, $g_t(\widetilde{P_n^c}) = g_c(\widetilde{P_n^c}) = g_s(\widetilde{P_n^c}) + 2$.

Proof. Let $V(\widetilde{P_n^c}) = \{v_1, v_2, \ldots, v_n\}$ without loss of generality let the switching vertex be $v_{\frac{n}{2}}$.

Let

S=

$$\begin{cases} \left\{v_{2i+1}, v_{2i+\frac{n}{2}+2} / 1 \le i \le \frac{n-8}{4}\right\} \\ \cup \left\{v_1, v_{\frac{n}{2}-1}, v_{\frac{n}{2}}, v_{\frac{n}{2}+1}, v_n\right\} \quad for\ n \equiv 0(mod 4) \\ \left\{v_{2i+1} \le i \le \frac{n-10}{4}\right\} \cup \\ \left\{v_{2i+\frac{n}{2}+2} \le i \le \frac{n-6}{4}\right\} \cup \\ \cup \left\{v_1, v_{\frac{n}{2}-1}, v_{\frac{n}{2}}, \quad v_{\frac{n}{2}+1}, v_n\right\} \quad for\ n \equiv 2(mod 4) \\ \left\{v_{2i+1}, v_{\frac{n+4i+5}{2}} / 1 \le i \le \frac{n-9}{4}\right\} \\ \cup \left\{v_1, v_{\frac{n}{2}-1}, v_{\frac{n}{2}}, v_{\frac{n}{2}+1}, v_n\right\} \quad for n \equiv 1(mod 4) \\ \left\{v_{2i}, v_{\frac{n+4i+5}{2}} / 1 \le i \le \frac{n-7}{4}\right\} \\ \cup \left\{v_1, v_{\frac{n}{2}-1}, v_{\frac{n}{2}}, v_{\frac{n}{2}+1}, v_n\right\} \quad for\ n \equiv 3(mod 4) \end{cases}$$

be the g_s− set of $\widetilde{P_n^c}$ such that $< S >$ has isolated vertex. Now consider $S' = \{v_{\frac{n}{2}-2}, v_{\frac{n}{2}+2}\} \subset V(\widetilde{P_n^c}) - S$, where $v_{\frac{n}{2}-2}, v_{\frac{n}{2}+2}$ is adjacent to $v_{\frac{n}{2}-1}, v_{\frac{n}{2}+1}$, then $S \cup S'$ which makes $< S \cup S' >$ has no isolated vertex and it is connected. Clearly $S \cup S'$ forms both $g_t - set\ and\ g_c - set$. By Theorem 3.6 we have $|S| = g_s(\widetilde{P_n^c})$. Therefore $g_t(\widetilde{P_n^c}) = g_c(\widetilde{P_n^c}) = |S \cup S'| = |S| + |S'| = g_s(\widetilde{P_n^c}) + 2$.

Corollary 3.7. For any path P_n of order $n \ge 8$, $g_{ss}(\widetilde{P_n^c}) = g(\widetilde{P_n^c}) + 3$.

Corollary 3.8. For any path P_n of order $n \ge 8$, $g_{toi}(\widetilde{P_n^c}) = g(\widetilde{P_n^c}) + 3$.

4. SWITCHING OF ARBITRARY VERTEX OF C_n

Observation 4.1. *Switching of an arbitrary vertex of C_n is denoted by $\widetilde{C_n^a}$, where a is an arbitrary vertex. Without loss of generality let the switching vertex be v_1. The graph $\widetilde{C_n^a}$ posses the following types of vertices.*

$$\Delta(\widetilde{C_n^a}) = n - 3 = deg_{\widetilde{C_n^a}}(v_1), deg_{\widetilde{C_n^a}}(v_2) = 1 = deg_{\widetilde{C_n^a}}(v_n), deg_{\widetilde{C_n^a}}(v_i) = 3, for\ all\ i \in \{3,4,\ldots,n-1\}$$

Example 4.2 Let $\widetilde{C_8^a}$ be the graph obtained by switching of an arbitrary vertex of cycle C_8 is shown in figure 3. . The darkened vertices is its geodetic set.

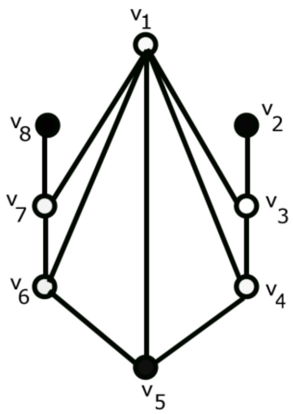

Figure 3: $\widetilde{C_8^a}$

The geodetic set S = {v_2 , v_5 , v_8 } and the geodetic number is g($\widetilde{C_8^a}$) = 3.

Theorem 4.3. *If the graph $\widetilde{C_n^a}$ is obtained by switching of an arbitrary vertex of C_n, $n \ge 6$,* then

International Journal of Applied Engineering Research ISSN 0973-4562 Volume 14, Number 7 (2019) pp. 1750-1757
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$$g(\widetilde{C_n^a}) = \begin{cases} \frac{n-2}{2} & for\ n\ is\ even, \\ \frac{n-1}{2} & for\ n\ is\ odd. \end{cases}$$

Proof. Let V ($\widetilde{C_n^a}$) = {v_1 , v_2 , ..., v_n }. Suppose S is any geodetic set of $\widetilde{C_n^a}$ then it is obvious that v_2 and v_n must belong to S as $deg_{\widetilde{C_n^a}}(v_2) = 1 = deg_{\widetilde{C_n^a}}(v_n)$. To prove this result we have two cases.

Case(i) Let n be even. For n=6, result is obvious.

For $n \geq 8$, consider a set $S = \{v_2, v_n, v_{2i+3}/1 \leq i \leq \frac{n-6}{2}\}$ be the non-adjacent vertex set, where $d_{\widetilde{C_n^a}}(v_2, v_{2i+3}) = 3 = d_{\widetilde{C_n^a}}(v_{2i+3}, v_n)$. Thus $I[S]$ = V ($\widetilde{C_n^a}$). Suppose $v_k \in \cup_{i=1}^{\frac{n-6}{2}} v_{2i+3}$. Consider $S_1 \subset S$ and $v_k \notin S_1$, then $|S_1| < |S|$, for any $v_k \in$ V$(\widetilde{C_n^a})$, $v_k \notin I[S_1]$, then $I[S_1] \neq$ V ($\widetilde{C_n^a}$). Hence S_1 is not a geodetic set of $\widetilde{C_n^a}$. Clearly S is the geodetic set of $\widetilde{C_n^a}$. Therefore, $g(\widetilde{C_n^a}) = |S| = \frac{n-2}{2}$.

Case(ii) Let n be odd. Consider a set $S = \{v_2, v_n, v_{2i+3}/ 1 \leq i \leq \frac{n-5}{2}\}$ be the non-adjacent vertex set, where $d_{\widetilde{C_n^a}}(v_2, v_{2i+3}) = 3 = d_{\widetilde{C_n^a}}(v_{2i+3}, v_n)$. Thus $I[S]$ = V ($\widetilde{C_n^a}$). Suppose $v_k \in \cup_{i=1}^{\frac{n-5}{2}} v_{2i+3}$. Consider $S_1 \subset S$ and $v_k \notin S_1$, then $|S_1| < |S|$, for any $v_k \in$ V$(\widetilde{C_n^a})$, $v_k \notin I[S_1]$, then $I[S_1] \neq$ V ($\widetilde{C_n^a}$). Hence S_1 is not a geodetic set of $\widetilde{C_n^a}$. Clearly S is the geodetic set of $\widetilde{C_n^a}$. Therefore, $g(\widetilde{C_n^a}) = |S| = \frac{n-1}{2}$.

Corollary 4.4. If C_n of order n ≥ 6, then

$$ig(\widetilde{C_n^a}) = \begin{cases} \frac{n-2}{2} & for\ n\ is\ even, \\ \frac{n-1}{2} & for\ n\ is\ odd. \end{cases}$$

Theorem 4.5. For any cycle C_n of order n ≥ 6, $g_{ns}(\widetilde{C_n^a}) = \alpha_0(\widetilde{C_n^a}) - 1$.

Proof. Let V ($\widetilde{C_n^a}$) = {v_1 , v_2 , v_3 , ..., v_n }. We can prove this result by two subcases.

Subcase(i) Suppose n be even. It is clear that $\alpha_0(\widetilde{C_n^a}) - 1 = \frac{n-2}{2}$. Let $S = \{v_2, v_n, v_{2i+3}/1 \leq i \leq \frac{n-6}{2}\}$ be the geodetic set and $<$ V ($\widetilde{C_n^a}$) $- S >$ is connected. Clearly S forms the g_{ns} -set. We have case (i) of Theorem 4.3, |S| $=\frac{n-2}{2}$. Therefore, $g_{ns}(\widetilde{C_n^a}) = \frac{n-2}{2} = \alpha_0(\widetilde{C_n^a}) - 1$.

Subcase(ii) Suppose n be odd. It is clear that $\alpha_0(\widetilde{C_n^a}) - 1 = \frac{n-1}{2}$. Let $S = \{v_2, v_n, v_{2i+3}/1 \leq i \leq \frac{n-5}{2}\}$ be the geodetic set and $<$ V ($\widetilde{C_n^a}$) $- S >$ is connected. Clearly S forms the g_{ns} -set. We have case (ii) of Theorem 4.3, |S| $=\frac{n-1}{2}$. Therefore $g_r(\widetilde{C_n^a}) = g_{ns}(\widetilde{C_n^a}) = \frac{n-1}{2} = \alpha_0(\widetilde{C_n^a}) - 1$.

Corollary 4.6. For any cycle C_n of order n ≥ 6, $g_r(\widetilde{C_n^a}) = \alpha_0(\widetilde{C_n^a}) - 1$.

Theorem 4.7. If $\widetilde{C_n^a}$ be the graph obtained by switching of an arbitrary vertex of cycle C_n, n ≥ 6,

$$g_s(\widetilde{C_n^a}) = \begin{cases} \beta_0(\widetilde{C_n^a}) & for\ n\ is\ even, \\ \beta_0(\widetilde{C_n^a}) + 1 & for\ n\ is\ odd. \end{cases}$$

Proof. Let $\widetilde{C_n^a}$ be the graph obtained by switching of an arbitrary vertex (say v_1) and β_0 be the vertex independent number and V ($\widetilde{C_n^a}$) = {v_1 , v_2 , ..., v_n }, $\Delta(G) = n - 2$. We consider the following cases.

Case(i) Suppose n be an even and v_1 be a switching vertex. Let $S = \{v_2, v_n, v_{2i+3}/1 \leq i \leq \frac{n-6}{2}\}$ be the geodetic set of $\widetilde{C_n^a}$, but $<$ V ($\widetilde{C_n^a}$) $- S >$ is connected. Also for even cycle, vertex independent number $\beta_0(\widetilde{C_n^a}) = \frac{n}{2}$. Consider $S_1 = S \cup \{v_1\}$ which makes $<$ V ($\widetilde{C_n^a}$) $- S >$ disconnected. Clearly S_1 is a $g_s - set$. Therefore, $g_s(\widetilde{C_n^a}) = |S_1| = |S \cup \{v_1\}| = \beta_0(\widetilde{C_n^a})$.

Case(ii) Suppose n be an odd. Let $S = \{v_2, v_n, v_{2i+3}/1 \leq i \leq \frac{n-5}{2}\}$ be the geodetic set of $\widetilde{C_n^a}$, but $<$ V ($\widetilde{C_n^a}$) $- S >$ is connected. Also for odd cycle, vertex independent number $\beta_0(\widetilde{C_n^a}) = \frac{n-1}{2}$. Consider $S_1 = S \cup \{v_1\}$ which makes $<$ V ($\widetilde{C_n^a}$) $- S_1 >$ disconnected. Clearly S_1 is a $g_s - set$. Therefore $g_s(\widetilde{C_n^a}) = |S_1| = |S \cup \{v_1\}| = \frac{n-1}{2} + 1 = \beta_0(\widetilde{C_n^a}) + 1$.

Theorem 4.8. For any cycle C_n, $n \geq 8$,

$$g_{ss}(\widetilde{C_n^a}) = \begin{cases} \beta_0(\widetilde{C_n^a}) + 2 & for\ n\ is\ even, \\ \beta_0(\widetilde{C_n^a}) + 3 & for\ n\ is\ odd. \end{cases}$$

Proof. Let V ($\widetilde{C_n^a}$) = {v_1, v_2, ..., v_n}. We discuss the following cases.

Case(i) When n is even. Let $S = \{v_1, v_2, v_n, v_{2i+3}/1 \leq i \leq \frac{n-6}{2}\}$ be the split geodetic set of $\widetilde{C_n^a}$ such that $<$ V ($\widetilde{C_n^a}$) $- S >$ is disconnected. Consider $S' = \{v_3, v_{n-1}\}$ be the set, where v_3, v_{n-1} are adjacent to v_2, v_n, then $S \cup S'$ forms $<$ V ($\widetilde{C_n^a}$) $- (S \cup S') >$ is totally disconnected. Which is a $g_{ss} - set$. By Theorem 4.7, case(i), $|S| = \beta_0(\widetilde{C_n^a})$. Therefore $g_{ss}(\widetilde{C_n^a}) = |S \cup S'| = |S| + |S'| = \beta_0(\widetilde{C_n^a}) + 2$.

Case(ii) When n is odd. Let $S = \left\{v_1, v_2, v_n, v_{2i+3} \leq i \leq \frac{n-5}{2}\right\}$ be the split geodetic set of $\widetilde{C_n^a}$ such that $<$ V $(\widetilde{C_n^a}) - S >$ is disconnected. Consider $S' = \{v_3, v_{n-1}\}$ be the set, where v_3, v_{n-1} are adjacent to v_2, v_n, then $S \cup S'$ forms $<$ V $(\widetilde{C_n^a}) - (S \cup S') >$ is totally disconnected. Which is a $g_{ss} - set$. By Theorem 4.7, case(ii), $|S| = \beta_0(\widetilde{C_n^a}) + 1$. Therefore $g_{ss}(\widetilde{C_n^a}) = |S \cup S'| = |S| + |S'| = \beta_0(\widetilde{C_n^a}) + 3$.

Theorem 4.9. For any cycle $C_n = \{v_1, v_2, v_3, \ldots, v_n\}$ and v_1 be the switching vertex, $n \geq 7$,

$$g_c(\widetilde{C_n^a}) = \begin{cases} \beta_0(\widetilde{C_n^a}) + d - 2 & for\ n\ is\ even, \\ \beta_0(\widetilde{C_n^a}) + d - 1 & for\ n\ is\ odd. \end{cases}$$

Proof. Let V$(\widetilde{C_n^a}) = \{v_1, v_2, v_3, \ldots, v_n\}$. To prove this result we discuss the following cases.

Case(i) When n is even. Let $\beta_0(\widetilde{C_n^a}) = \frac{n-2}{2} + 1$, diameter $d = 4$ and by Theorem 4.3, case i) we have $S = \left\{v_2, v_n, v_{2i+3} / 1 \le i \le \frac{n-6}{2}\right\} = \frac{n-2}{2}$ be the geodetic set of $\widetilde{C_n^a}$ but $< S >$ is totally disconnected.Consider set $S' = \{v_1, v_3, v_{n-1}\} = 3 = d - 1$ be the vertex set, where v_3, v_{n-1} are adjacent to the pendant vertex and $deg_{\widetilde{C_n^a}}(v_1) = n - 3 = \Delta(\widetilde{C_n^a})$, then $S_1 = S \cup S'$ is a connected geodetic set of $\widetilde{C_n^a}$. Therefore, $g_c(\widetilde{C_n^a}) = |S_1| = \beta_0(\widetilde{C_n^a}) + d - 2$.

Case(ii) When n is odd. Let $\beta_0(\widetilde{C_n^a}) = \frac{n-1}{2}$, diameter $d = 4$ and by Theorem 4.3, case ii) we have $S = \left\{v_2, v_n, v_{2i+3} / 1 \le i \le \frac{n-5}{2}\right\} = \frac{n-1}{2}$ be the geodetic set of $\widetilde{C_n^a}$ but $< S >$ is totally disconnected. Consider a set $S' = \{v_1, v_3, v_{n-1}\} = 3 = d - 1$ be the vertex set, where v_3, v_{n-1} are adjacent to pendant vertex and $deg_{\widetilde{C_n^a}}(v_1) = n - 3 = \Delta(\widetilde{C_n^a})$, then $S_1 = S \cup S'$ is a connected geodetic set of $\widetilde{C_n^a}$. Therefore, $g_c(\widetilde{C_n^a}) = |S_1| = \beta_0(\widetilde{C_n^a}) + d - 1$.

Corollary 4.10. *For any cycle* C_n, $n \ge 8$,

$$g_t(\widetilde{C_n^a}) = g_c(\widetilde{C_n^a}).$$

Corollary 4.11. *For any cycle* C_n, $n \ge 8$,

$$g_{toi}(\widetilde{C_n^a}) = g_t(\widetilde{C_n^a}).$$

Theorem 4.12 *If the graph* $\widetilde{C_n^a}$ *is obtained by switching of an arbitrary* vertex *of order* $n \ge 6$, *then* $g_{dc}(\widetilde{C_n^a}) = n - 1$.

Proof. Let $V(\widetilde{C_n^a}) = \{v_1, v_2, v_3, \dots, v_n\}$. Consider $S = V(\widetilde{C_n^a}) - \{v_l\}$, where $v_l / 3 \le l \le n - 2$ is any arbitrary internal vertex which is not adjacent to pendant vertices of $\widetilde{C_n^a}$, be the set which makes $< S >$ and $< V(\widetilde{C_n^a}) - S >$ connected. Clearly S is a $g_{dc} - set$. Therefore, $g_{dc}(\widetilde{C_n^a}) = |S| = n - 1$.

5. SWITCHING OF AN ARBITRARY VERTEX OF W_n

Observation 5.1. Let $\widetilde{W_n^a}$ is the graph obtained by switching of an arbitrary vertex of wheel W_n. Without loss of generality, we switch the arbitrary vertex v_1. Let $V(\widetilde{W_n^a}) = \{x, v_1, v_2, \dots, v_{n-1}\}$, $deg_{\widetilde{W_n^a}}(x) = n - 2 = \Delta(\widetilde{W_n^a})$, $deg_{\widetilde{W_n^a}}(v_1) = n - 4$, $deg_{\widetilde{W_n^a}}(v_2) = 2 = deg_{\widetilde{W_n^a}}(v_{n-2})$ and $deg_{\widetilde{W_n^a}}(v_i) = 4, for\ all\ i \in \{3,4, \dots, n-2\}$.

Example 5.2. Let $\widetilde{W_9^a}$ be the graph obtained by switching of arbitrary vertex of wheel W_9 is shown in figure 4. The darkened vertices is its geodetic set.

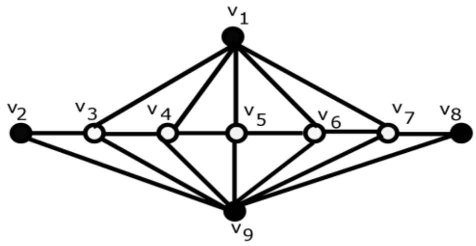

Figure 4: $\widetilde{W_9^a}$

The geodetic set $S = \{v_1, v_2, v_8, v_9\}$ and the geodetic number is $g(\widetilde{W_n^a}) = 4$

Observation 5.3. If $\widetilde{W_n^a}$ be the graph obtained by switching of an arbitrary vertex of wheel W_n, $n \ge 6$, then $g(\widetilde{W_n^a}) = 4$.

Theorem 5.4. For any wheel W_n of order $n \ge 6$, then $g_t(\widetilde{W_n^a}) = g(\widetilde{W_n^a}) + 1$.

Proof. Let $V(\widetilde{W_n^a}) = \{x, v_1, v_2, v_3, \dots, v_{n-1}\}$ and $S = \{x, v_1, v_2, v_{n-1}\}$ be the geodetic set such that $< S >$ has isolated vertex. Let $N_{\widetilde{W_n^a}}(v_1) = \{v_2, v_3, \dots, v_{n-2}\} \subseteq V(\widetilde{W_n^a}) - S$. Consider $v_k \in N(v_1)$, where v_k be the any one vertex with $deg_{\widetilde{W_n^a}}(v_k) = 4$. Thus $S_1 = S \cup \{v_k\}$ makes $< S_1 >$ has no isolated vertex. Hence S_1 forms $g_t - set$. By observation 5.3, $|S| = 4 = g(\widetilde{W_n^a})$. Therefore $g_t(\widetilde{W_n^a}) = |S_1| = g(\widetilde{W_n^a}) + 1$.

Corollary 5 .5. For any wheel W_n of order $n \ge 6$, then $g_c(\widetilde{W_n^a}) = g(\widetilde{W_n^a}) + 1$.

Corollary5.6. For any wheel W_n of order $n \ge 6$, then $g_{tr}(\widetilde{W_n^a}) = g(\widetilde{W_n^a}) + 1$.

Theorem 5.7. The split geodetic number of $\widetilde{W_n^a}$ is 5.

(i.e) $g_s(\widetilde{W_n^a}) = 5, n \ge 6$.

Proof. Let $S = \{x, v_1, v_2, v_{n-1}\}$ be the geodetic set such that $< V(\widetilde{W_n^a}) - S >$ is connected. Consider $S' = \{v_4, v_5, \dots, v_{n-4}\} \subset V(\widetilde{W_n^a}) - S$. Let $v_k \in S'$. Thus $S_1 = S \cup \{v_k\}$ which makes $< V(\widetilde{W_n^a}) - S_1 >$ disconnected. Hence S_1 is a split geodetic set. By observation 5.3, $|S| = 4$. Therefore, $g_s(\widetilde{W_n^a}) = |S_1| = 5$.

Theorem 5.8 For any wheel W_n of order $n \ge 6$, $g_{dc}(\widetilde{W_n^a}) = 5$.

Proof. Let $V(\widetilde{W_n^a}) = \{x, v_1, v_2, v_3, \dots, v_{n-1}\}$ with $deg_{\widetilde{W_n^a}}(v_2) = 2 = deg_{\widetilde{W_n^a}}(v_{n-1})$ which is joining to $deg_{\widetilde{W_n^a}}(v_3) = 4 = deg_{\widetilde{W_n^a}}(v_{n-2})$ *Let* $S = \{x, v_1, v_2, v_{n-1}\}$ be the geodetic set, but $< S >$ is not connected. Consider $S_1 = S \cup \{v\}$, where $v = v_2$ *or* $v = v_{n-2}$ such that $< S_1 >$ and $< V(\widetilde{W_n^a}) - S_1 >$ are connected. Therefore, $g_{dc}(\widetilde{W_n^a}) = |S_1| = 5$.

Theorem 5.9. If $\widetilde{W_n^a}$ is the graph obtained by switching of an arbitrary vertex (say v_1) of $W_n$of order $n \geq 6$, then

$$g_{toi}(\widetilde{W_n^a}) = \begin{cases} \dfrac{n+4}{2} & for\ n\ is\ even, \\ \dfrac{n+3}{2} & for\ n\ is\ odd. \end{cases}$$

Where d is the diameter.

Proof. Let $V(\widetilde{W_n^a}) = \{x, v_1, v_2, v_3, \dots, v_{n-1}\}$. We consider the following cases.

Case(i) When n is even. Let $S = \{x, v_1, v_2, v_{n-1}\}$ be the geodetic set of $\widetilde{W_n^a}$, but $< S >$ has isolated vertex and $< V(\widetilde{W_n^a}) - S >$ is connected. Consider $S' = \{v_{2i+2}/2 \leq i \leq \frac{n-4}{2}\}$ be the internal non-adjacent vertices of $\widetilde{W_n^a}$. Then $S \cup S'$ is a total outer-independent geodetic set. We have by observation 5.3, $|S| = 4$. Hence $g_{toi}(\widetilde{W_n^a}) = |S \cup S'| = 4 + \frac{n-4}{2} = \frac{n+4}{2}$.

Case(ii) When n is odd. Let $S = \{x, v_1, v_2, v_{n-1}\}$ be the geodetic set of $\widetilde{W_n^a}$, but $< S >$ has isolated vertex and $< V(\widetilde{W_n^a}) - S >$ is connected. Consider $S' = \{v_{2i+2}/2 \leq i \leq \frac{n-5}{2}\}$ be the internal non-adjacent vertices of $\widetilde{W_n^a}$. Then $S \cup S'$ is a total outer-independent geodetic set. We have by observation 5.3, $|S| = 4$. Hence $g_{toi}(\widetilde{W_n^a}) = |S \cup S'| = 4 + \frac{n-5}{2} = \frac{n+3}{2}$.

6 SWITCHING OF APEX VERTEX OF H_n

Observation 6.1. Let $\widetilde{H_n^l}$ be the graph obtained by switching of an apex vertex of helm graph H_n, where l be the apex vertex, $V(\widetilde{H_n^l}) = \{x, v_1, v_2, v_3, \dots, v_n, v'_1, v'_2, v'_3, \dots, v'_n\}$, $deg_{\widetilde{H_n^l}}(x) = n$, $deg_{\widetilde{H_n^l}}(v_i) = 3, deg_{\widetilde{H_n^l}}(v'_j) = 2\ and\ d = 4$.

Example 6.2. Let $\widetilde{H_6^l}$ be the graph obtained by switching of the apex vertex x is shown in figure 5. The darkened vertices is its geodetic set.

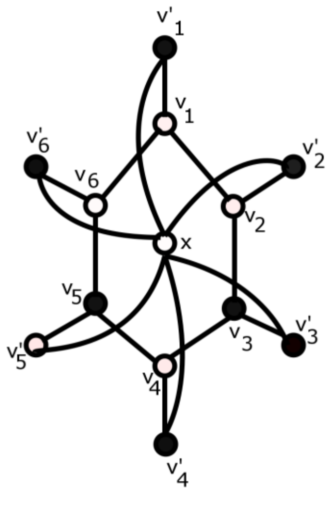

Figure 5: $\widetilde{H_6^l}$

The geodetic set $S = \{v_3, v_5, v'_1, v'_2, v'_4, v'_6\}$ and the geodetic number is $g(\widetilde{H_6^l}) = 6$.

Theorem 6.3. If $\widetilde{H_n^l}$ be the graph obtained by switching of an apex vertex of helm graph H_n , $n \geq 6$, then $g(\widetilde{H_n^l}) = ig(\widetilde{H_n^l}) = n$.

Proof. Let $V(\widetilde{H_n^l}) = \{x, v_1, v_2, v_3, \dots, v_n, v'_1, v'_2, v'_3, \dots, v'_n\}$ and $|V(\widetilde{H_n^l})| = 2n + 1$. Consider the vertex sets $A = \{v_1, v_{4i+1}/1 \leq i \leq \lceil \frac{n}{4} \rceil - 1\}$ and $B = \{v'_j/ 1 \leq j \leq n\}$ with $|B| = n - \lceil \frac{n}{4} \rceil$, where $j \neq 1, 4i + 1, 1 \leq i \leq \lceil \frac{n}{4} \rceil - 1$, $deg_{\widetilde{H_n^l}}(v_{4i+1}) = 3, deg_{\widetilde{H_n^l}}(v'_j) = 2$, then $S = A \cup B$ forms $I[S] = V(\widetilde{H_n^l})$ such that S is the geodetic set and $< S >$ has isolated vertex, then S itself forms a independent geodetic set. Therefore, $g(\widetilde{H_n^l}) = ig(\widetilde{H_n^l}) = |S| = n$.

Corollary 6.4. If $\widetilde{H_n^l}$ be the graph obtained by switching of an apex vertex of helm graph H_n , $n \geq 6$, then $g_s(\widetilde{H_n^l}) = n$.

Theorem 6.5. *For any helm graph* H_n , $n \geq 6$, $g_{ns}(\widetilde{H_n^l}) = 2\beta_0(\widetilde{H_n^l}) - \left\lceil \frac{\beta_0(\widetilde{H_n^l})}{d} \right\rceil$, where d is the diameter and $\widetilde{H_n^l}$ be the graph obtained from switching of apex vertex of H_n.

Proof. Let x be an apex vertex, $v_1, v_2, v_3, \dots, v_n$ be the rim vertices and $v'_1, v'_2, \dots, v'_n$ be the pendant vertices then $|V(\widetilde{H_n^l})| = 2n + 1$, $\beta_0(\widetilde{H_n^l}) = n$ and d=4. We have by Theorem 6.3, $S = \{v_1, v_{4i+1}/1 \leq i \leq \lceil \frac{n}{4} \rceil - 1\} \cup \{v'_j/ 1 \leq j \leq n\}$, where $j \neq 1, 4i + 1, 1 \leq i \leq \lceil \frac{n}{4} \rceil - 1$, be the geodetic set of $\widetilde{H_n^l}$, but $< V(\widetilde{H_n^l}) - S >$ is not connected. Consider $S_1 = \{v_k/ 1 \leq k \leq n\}$, where $k \neq v_1$ and $k \neq \bigcup_{i=1}^{\lceil \frac{n}{4} \rceil - 1} v_{4i+1}$. Then $S \cup S_1$ be the vertex set which makes $< V(\widetilde{H_n^l}) - S_1 >$ is connected. Clearly $g_{ns}(\widetilde{H_n^l}) = |S \cup S_1| = 2\beta_0(\widetilde{H_n^l}) - \left\lceil \frac{\beta_0(\widetilde{H_n^l})}{d} \right\rceil$.

Theorem 6.6. *For any helm graph* H_n , $n \geq 8$, , $g_c(\widetilde{H_n^l}) = \alpha_0(\widetilde{H_n^l}) + \left\lceil \frac{\beta_0(\widetilde{H_n^l})}{4} \right\rceil$.

Proof. Let x be an apex vertex, $v_1, v_2, v_3, \dots, v_n$ be the rim vertices and $v'_2, v'_3, \dots, v'_n$ be the pendant vertices. Let $S = \{v_1, v_{4i+1}/1 \leq i \leq \lceil \frac{n}{4} \rceil - 1\} \cup \{v'_j/ 1 \leq j \leq n\} = \alpha_0(\widetilde{H_n^l}) - 1$, where $j \neq 1, 4i + 1, 1 \leq i \leq \lceil \frac{n}{4} \rceil - 1$, be the geodetic set of $\widetilde{H_n^l}$, but $< S >$ is not connected.Consider $S' = \{x, v'_k/ 1 \leq k \leq \lceil \frac{n}{4} \rceil\}$. Then $S \cup S'$ makes $< S' >$ is connected. Clearly $g_{ns}(\widetilde{H_n^l}) = |S \cup S'| = g_c(\widetilde{H_n^l}) = \alpha_0(\widetilde{H_n^l}) + \left\lceil \frac{\beta_0(\widetilde{H_n^l})}{4} \right\rceil$.

Theorem 6.7. *For any helm graph* H_n *,* $n \geq 4$, $g_{ss}(\widetilde{H_n^l}) = \alpha_0(\widetilde{H_n^l}) + \left\lceil\frac{n}{2}\right\rceil - \left\lceil\frac{n}{4}\right\rceil$.

Proof. Let $V(\widetilde{H_n^l}) = \{x, v_1, v_2, v_3, \ldots, v_n, v'_1, v'_2, v'_3, \ldots, v'_n\}$, $deg_{\widetilde{H_n^l}}(x) = n$, $deg_{\widetilde{H_n^l}}(v_i) = 3, deg_{\widetilde{H_n^l}}(v'_j) = 2$. We construct vertex set S as follows; $S = \{v_1, v_{4i+1}/1 \leq i \leq \left\lceil\frac{n}{4}\right\rceil - 1\} \cup \{v'_j/\ 1 \leq j \leq n\} \cup \{x\} = \alpha_0(\widetilde{H_n^l}) - 1$, where $j \neq 1, 4i+1, 1 \leq i \leq \left\lceil\frac{n}{4}\right\rceil - 1$. Then $|S| = n + 1 + \left\lceil\frac{n}{2}\right\rceil - \left\lceil\frac{n}{4}\right\rceil$, where $i \neq j$. Also every vertex $v \in V(\widetilde{H_n^l}) - S$ is adjacent to two vertices of S. Moreover $< V(\widetilde{H_n^l}) - S >$ is totally disconnected. Thus S is a $g_{ss} - set$ of $\widetilde{H_n^l}$. Therefore $g_{ss}(\widetilde{H_n^l}) = |S| = \alpha_0(\widetilde{H_n^l}) + \left\lceil\frac{n}{2}\right\rceil - \left\lceil\frac{n}{4}\right\rceil$.

7 CONCLUSION

In this paper, we established geodetic number, split, non-split, restrained, connected, doubly connected, total and total outer independent geodetic number on switching of path P_n, cycle C_n,wheel W_n and helm graph H_n.

8 ACKNOWLEDGMENT

We are very thankful to the referees for reading the paper and giving valuable suggestions.

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