

Effect of Bulk Viscosity on Interacting Generalized Cosmic Chaplygin Gas Cosmology

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Abstract :

In this paper we study interacting generalized cosmic Chaplygin gas with bulk viscosity for Kaluza-Klein type FRW metric in the frame work of general theory of relativity. We consider sign-changeable interaction between generalized cosmic Chaplygin gas and matter and then study effect of bulk viscosity on the cosmological parameters such as energy density, Hubble parameter and deceleration parameter, for special and general case.

Keywords: Generalized cosmic Chaplygin gas; Kaluza Klein cosmology; bulk viscosity.

1. INTRODUCTION

The cosmological observations namely SNeIa [1, 2], CMB [3] and WMAP [4, 5] indicates that the universe is undergoing accelerated expansion [1]-[9]. This accelerated phase of the universe may be investigated in the context of unknown dark energy (DE) which has positive energy and adequate negative pressure. Several theories have been proposed to describe nature of DE. Among them, one that attracts many workers [10]-[26] is known as Chaplygin gas (CG), which is one of the model for DE [27]-[29]. This model is originally introduced to describe the lifting force on a wing of an air plane in aerodynamics. This CG was not in good agreement with observational data hence later generalized by increasing power of mass density by an arbitrary constant, which is called as generalized Chaplygin gas (GCG) [28, 30]. There is further extension called as generalized cosmic Chaplygin gas (GCCG) [31].

Moreover to study the evolution of the universe, bulk viscosity plays vital role in cosmology, because it provides many interesting facts in the dynamics of homogeneous cosmological models [55, 33, 34, 35]. Also bulk viscosity appears as the only dissipative phenomenon occurring in FRW models and has significant role in getting accelerated expansion of the universe popularly known as inflationary space. Bulk viscosity contributes negative pressure stimulating repulsive gravity. The repulsive gravity overcomes attractive gravity of matter and gives an impetus for rapid expansion of the universe hence cosmological models with bulk viscosity have gained importance in recent years. Zhai et al. [36] first time introduced, the idea that CG may has viscosity and later developed by researches [37] - [40].

Bulk viscous cosmological models are widely discussed by

Barrow [41], Pavon et al. [42], Lima et al. [43] and Mohanty and Pradhan [44] in general theory of relativity. Santhi and Reddy [45] have analyzed field equation for Kaluza-Klein (KK) cosmological model with bulk viscosity in the context of Barber's second self creation cosmology. Samanta and Bishi [46] investigated geometry of the universe described by KK bulk viscous cosmology with modified cosmic Chaplygin gas (MCCG) in general relativity. Big Rip singularity in five dimension viscous cosmology studied by Khadekar and Gharad [47]. Variable viscous generalized cosmic Chaplygin gas was constructed by Naji et al. [48] in the presence of cosmological constant and space curvature.

Kaluza [49] and Klein [50] first time proposed possibility that the world may have one more dimension in addition to existing four dimensions. This extra dimension unify gravity and electromagnetism in general theory of relativity. Modern discoveries shows that higher dimensional gravity theories may provide deep insight to understand the interaction of particle and it plays remarkable role to explain main problem of astrophysics particularly DE. In view of this many authors investigated KK cosmological models with different DE and dark matter (DM). Khadekar and Kamdi [51] analyzed the evolution of the universe it goes from inflationary to radiation dominated phase with variable $\Lambda \propto H^2$ in the presence of perfect fluid in the framework of KK theory of gravitation. Salti [52] describe unified dark matter-energy scenario in the context of KK cosmology by investigating cosmological features of the variable Chaplygin gas (VCG) and shows that, the VCG evolves from the dust-like phase to the phantom or the quintessence phases. Sharif and Saleem [53] investigated an inflationary universe model in the context of the generalized cosmic Chaplygin gas by taking the matter field as standard and tachyon scalar fields. They have evaluated the corresponding scalar fields and scalar potentials by modifying the first Friedmann equation. Naji et al. [54] constructed variable viscous generalized cosmic Chaplygin gas of the form

$$p_d = -\frac{1}{\rho_d^\alpha} \left[\frac{B(H)}{1+\omega} - 1 + \left(\rho_d^{1+\alpha} - \frac{B(H)}{1+\omega} + 1 \right)^{-\omega} \right] - 3\zeta H.$$

in the presence of cosmological constant and space curvature and investigated behavior of some cosmological parameters by using numerical analysis. Stability of model is also discussed. Singh and Baurah [55] considered a model of the universe filled with Generalized Cosmic Chaplygin Gas and another fluid with barotropic equation of state and observed its role in

accelerating phase of the universe. The state finder parameters describe the evolution of the universe in different phases for these two fluid models. Baurah [56] studied the interaction between the generalized cosmic Chaplygin gas energy density and cold dark matter (CDM) and obtained the equation of state for the interacting generalized cosmic Chaplygin energy density with cold dark matter in spatially non-flat universe.

Motivated by the above discussion and investigations in KK cosmology we have extended the work of Naji et al. [54] and studied effect of bulk viscosity on interacting GCCG in the framework of Kaluza Klein theory of gravitation.

This paper is organized as follows: In section 2, we introduced our model and write equation of state for viscous GCCG (VGCCG) with viscosity coefficient $\zeta = \zeta_0 + H\zeta_1$. In section 3, we consider FRW universe and obtain field equations. In section 4 we obtained expression for energy density in terms of scale factor for different CG models. In section 5 we assumed interaction between VGCCG and cold dark matter and discussed effect of bulk viscosity on cosmological parameters. In section 6 we have summarized results obtained.

2. GENERALIZED COSMIC CHAPLYGIN GAS

One of the interesting cosmological model to describe universe is based on CG with the following equation of state [27, 28]

$$p_d = -\frac{B}{\rho_d}, \quad (1)$$

where B is positive constant, p_d is pressure and ρ_d is energy density. The CG overruled the observational data, therefore Eq. (1) has been generalized to,

$$p_d = -\frac{B}{\rho_d^\alpha}, \quad (2)$$

with $0 < \alpha \leq 1$ which is known as GCG. The GCG can unify dark energy and dark matter. From Eq. (2) it is clear that at early time when energy density is high GCG represents pressureless dust ($p_d = 0$). In order to have coincidence with observational data, Gonzalez-Diaz [58] gave another extension of CG, called as generalized cosmic Chaplygin gas (GCCG), of the form

$$p_d = -\frac{1}{\rho_d^\alpha} \left[\frac{B}{1+\omega} - 1 + \left(\rho_d^{1+\alpha} - \frac{B}{1+\omega} + 1 \right)^{-\omega} \right]. \quad (3)$$

This model is stable and free from unphysical behaviors even when the vacuum fluid satisfies the phantom energy condition [59].

We have consider EoS for VGCCG as

$$\begin{aligned} p &= p_d - 3\zeta H \\ &= -\frac{1}{\rho_d^\alpha} \left[\frac{B}{1+\omega} - 1 + \left(\rho_d^{1+\alpha} - \frac{B}{1+\omega} + 1 \right)^{-\omega} \right] - 3\zeta H, \end{aligned} \quad (4)$$

where ζ is bulk viscous coefficient of the form $\zeta = \zeta_0 + \zeta_1$.

3. FRW COSMOLOGY

We have consider Kaluza-Klein type Friedmann Robertson Walker (FRW) universe of the form

$$ds^2 = dt^2 - a^2(t) [dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) + d\Psi^2], \quad (5)$$

where $a(t)$ is the scale factor.

The Einstein field equations are given by

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = T_{\mu\nu}, \quad (6)$$

where $R_{\mu\nu}$, R , $g_{\mu\nu}$ are Ricci tensor, Ricci scalar and metric tensor respectively.

Assuming that matter filling the universe is in the form of a perfect fluid, then

$$T_{00} = \rho, T_{11} = T_{22} = T_{33} = T_{44} = -p, \quad (7)$$

where ρ is the density and p is the pressure.

Using the line element (5), Einstein's field Eq. (6) and Eq. (7), we obtain following set of equations:

$$\frac{\dot{a}^2}{a^2} = \frac{\rho}{6}, \quad (8)$$

$$\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = \frac{-p}{3}, \quad (9)$$

where dot denotes derivative with respect to cosmic time (t). It is assumed that the total matter and energy are conserved with the following conservation equation

$$\dot{\rho} + 4H(\rho + p) = 0, \quad (10)$$

where $H = \frac{\dot{a}}{a}$ is Hubble expansion parameter, $\rho = \rho_m + \rho_d$ and $p = p_d - 3(\zeta_0 + \zeta_1)H$.

4. SCALE FACTOR DEPENDENT DARK ENERGY DENSITY

If matter contribution omitted then we get following energy density for different models.

Inserting EoS (1) in Eq. (10), the energy density takes the form

$$\rho_{CG} = \sqrt{B + \frac{C}{a^8}}, \quad (11)$$

where C is constant of integration.

Also by using Eq. (2) in Eq. (10), one can obtain generalized energy density as

$$\rho_{GCCG} = \left(B + \frac{C}{a^{4(1+\alpha)}} \right)^{\frac{1}{1+\alpha}}. \quad (12)$$

Then, by using the EoS for GCCG in consrvation Eq. (10), we obtain

$$\rho_{GCCG} = \left[\frac{B}{1+\omega} - 1 + \left(1 + \frac{C}{a^{4(1+\omega)(1+\alpha)}} \right)^{\frac{1}{1+\omega}} \right]^{\frac{1}{1+\alpha}}. \quad (13)$$

5. INTERACTING DARK ENERGY

In this section we assume there is an interaction between VGCCG energy density and cold dark matter (CDM) with $\omega_m = 0$. To introduce an interaction between DE and DM, we mathematically split Eq. (10) into two equations where interaction term Q enters explicitly and controls the dynamics of the energy densities of the components as

$$\dot{\rho}_m + 4H\rho_m = Q, \tag{14}$$

$$\dot{\rho}_d + 4H(\rho_d + p) = -Q, \tag{15}$$

where p_d is given by Eq. (4) and Q is the interaction term. Here we consider a sign-changeable interaction of the following form [60]-[63]

$$Q = q(\mu\dot{\rho}_d + 3bH\rho_d), \tag{16}$$

where μ and b are dimensionless parameter and q is deceleration parameter given by

$$q = -1 - \frac{\dot{H}}{H^2}. \tag{17}$$

This type of interaction can change its sign when our universe changes from deceleration $q > 0$ to acceleration $q < 0$.

5.1. Special case

Here we consider simple model for scale factor as

$$a = ct^\sigma. \tag{18}$$

By using Eqs. (16), (17) and (18) in Eq. (15), we obtain

$$\begin{aligned} &\rho_d(\sigma + \mu - \mu\sigma)t^3 + \sigma(4\sigma + 3b - 3b\sigma)t^2\rho_d - 12\sigma^3t\zeta_0 - 12\sigma^4\zeta_1 \\ &- \frac{4\sigma^2t^2}{\rho_d^\alpha} \left[\frac{B}{1+\omega} - 1 + \left(\rho_d^{1+\alpha} - \frac{B}{1+\omega} + 1 \right)^{-\omega} \right] = 0. \end{aligned} \tag{19}$$

For $\alpha = -1$ we get unphysical solution from the above equation as follows

$$\rho_d = \frac{12\sigma^3\zeta_0}{S+2}t^2 + \frac{12\sigma^4\zeta_1}{S+1}t + Ct^{-S}, \tag{20}$$

where $S = \frac{\sigma(4\sigma+3b-3b\sigma)}{(\sigma+\mu-\mu\sigma)} - 4\sigma^2 \left[\left(\frac{B}{1+\omega} - 1 \right) + \left(2 - \frac{B}{1+\omega} \right)^{-\omega} \right]$ and C is constant of integration .

In the physical case, we can use numerical method to solve Eq. (19) and illustrate behavior of dark energy density. In figs. (1) and (2) we have plotted energy density versus time for varying bulk viscosity coefficient ζ_1 and ζ_0 respectively. Fig. (1) shows energy density is decreasing function and fig. (2) shows that energy density is increasing with increasing ζ_0 . In fig. (3) we have shown variation of energy density versus time for different values of interaction parameter and found that as the value of b increases energy density also increases.

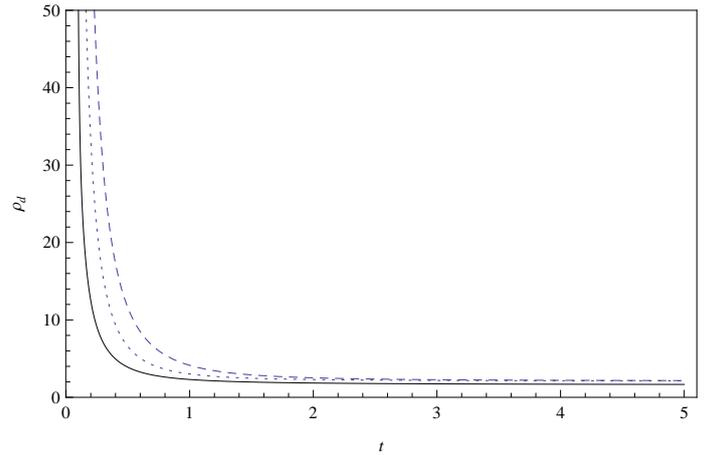


Figure 1: Dark energy density ρ_d versus time for $B = 3.4$, $b = 0.5$, $\alpha = 0.5$, $\mu = 1$, $\sigma = 2$, $\omega = 1$, $\zeta_0 = 0.1$, $\zeta_1 = 0.01$ (solid line), $\zeta_1 = 0.05$ (dotted line), $\zeta_1 = 0.1$ (dashed line).

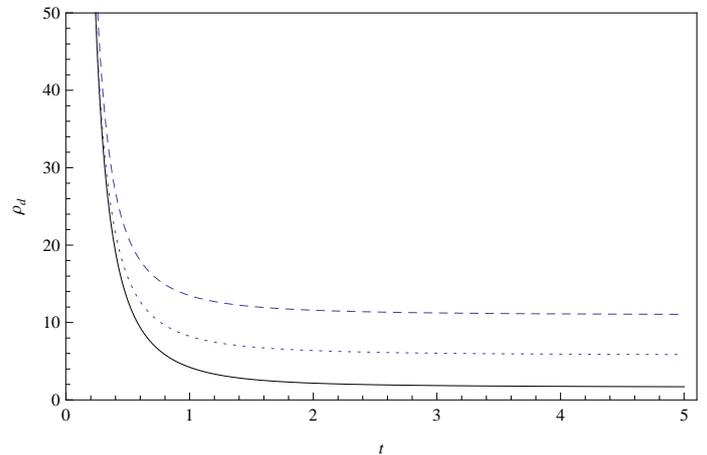


Figure 2: Dark energy density ρ_d versus time for $B = 3.4$, $b = 0.5$, $\alpha = 0.5$, $\mu = 1$, $\sigma = 2$, $\omega = 1$, $\zeta_1 = 0.1$, $\zeta_0 = 0.1$ (solid line), $\zeta_0 = 0.5$ (dotted line), $\zeta_0 = 1$ (dashed line).

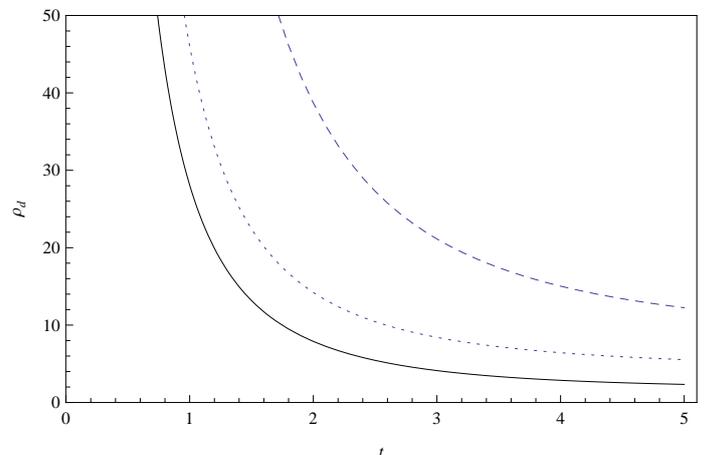


Figure 3: Dark Energy density ρ_d versus time for $B = 3.4$, $\alpha = 0.5$, $\mu = 1$, $\sigma = 2$, $\omega = 1$, $\zeta_0 = 2$, $\zeta_1 = 0.01$, $b = 0.5$ (solid line), $b = 1$ (dotted line), $b = 1.5$ (dashed line).

5.2. General Case

In order to get general solution we obtained following nonlinear differential equation by using Eqs. (4), (8), (16) and (17) in Eq. (15)

$$\frac{\mu}{2\sqrt{6}}\rho_d^{-3/2}\dot{\rho}_d^2 + \left(\frac{3b}{2} + \mu - 1\right)\dot{\rho}_d + (4 + 2\zeta_1 + 3b)\frac{\rho_d^{3/2}}{\sqrt{6}} + 2\zeta_0\rho_d$$

$$\frac{4}{\sqrt{6}} \left[\frac{B}{1+\omega} - 1 + \left(\rho_d^{1+\alpha} - \frac{B}{1+\omega} + 1 \right)^{-\omega} \right] \rho_d^{\frac{1}{2}-\alpha} = 0. \quad (21)$$

Since the above equation is highly non linear and can not be solved easily, hence for simplicity of the calculation we choose only two stages namely early time universe ($t \ll 1$) and late time universe ($t \gg 1$) as follows:

5.2.1 Early time universe ($t \ll 1$)

At the early time universe which has high energy density, we may neglect first term of the Eq. (21). To investigate the effect of viscosity, first we choose $B = 0$ (this is appropriate choice in the early time when energy density is high) and $\alpha = \frac{1}{2}$. Thus Eq. (21) reduces to

$$\dot{\rho}_d + \frac{2}{\sqrt{6}} \left(\frac{4 + 2\zeta_1 + 3b - 4\omega}{2\mu + 3b - 2} \right) \rho_d^{3/2} + \frac{4\zeta_0\rho_d}{2\mu + 3b - 2} = 0, \quad (22)$$

on solving above equation we get

$$\rho_d = \left[-\frac{4 + 2\zeta_1 + 3b - 4\omega}{2\sqrt{6}\zeta_0} + C \exp\left(\frac{2\zeta_0 t}{2\mu + 3b - 2}\right) \right]^{-2}, \quad (23)$$

where C is constant of integration.

Inserting Eq. (23) in Eq. (8) we get Hubble parameter as

$$H = \frac{1}{\sqrt{6}} \left[-\frac{4 + 2\zeta_1 + 3b - 4\omega}{2\sqrt{6}\zeta_0} + C \exp\left(\frac{2\zeta_0 t}{2\mu + 3b - 2}\right) \right]^{-1}. \quad (24)$$

To get real Hubble parameter which leads to constant after long time we have following conditions:

$$2\mu + 3b - 2 > 0,$$

$$4 + 2\zeta_1 + 3b - 4\omega > 0, \quad (25)$$

$$\zeta_0 > 0.$$

With these conditions we have investigated behavior of Hubble parameter and deceleration parameter at the early universe.

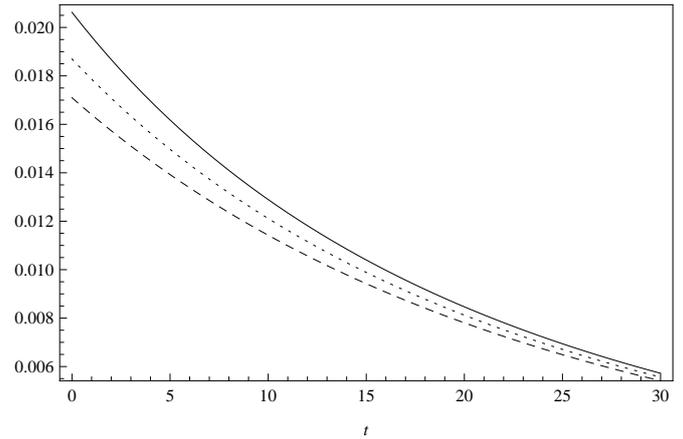


Figure 4: Hubble parameter versus time for $b = 2$, $\mu = 1$, $\zeta_0 = 0.1$, $\omega = 1$, $C = 30$, $\zeta_1 = 0.5$ (solid line), $\zeta_1 = 1$ (dotted line), $\zeta_1 = 1.5$ (dashed line).

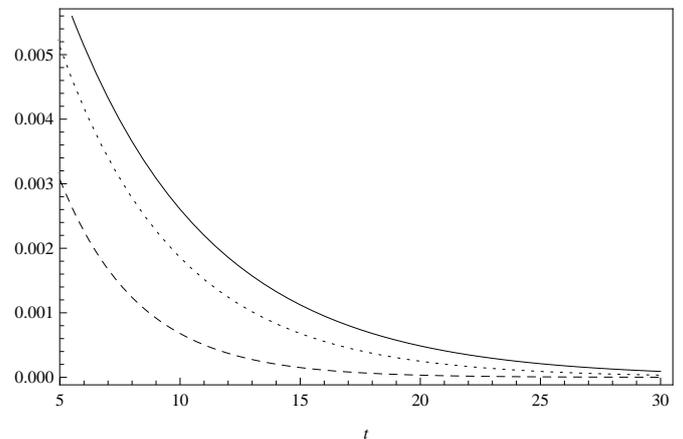


Figure 5: Hubble parameter versus time for $b = 2$, $\mu = 1$, $\zeta_1 = 0.5$, $\omega = 1$, $C = 30$, $\zeta_0 = 0.3$ (solid line), $\zeta_0 = 0.6$ (dotted line), $\zeta_0 = 0.9$ (dashed line).

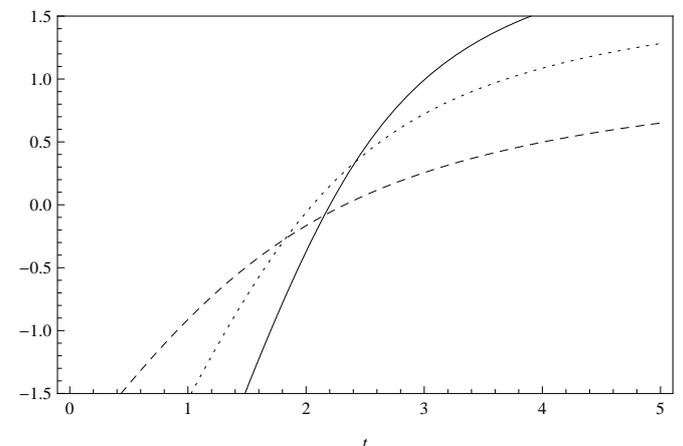


Figure 6: Deceleration parameter versus time for $b = 1$, $\mu = 1$, $\zeta_0 = 0.5$, $\omega = 1$, $C = 0.9$, $\zeta_1 = 0.6$ (solid line), $\zeta_1 = 1$ (dotted line), $\zeta_1 = 1.5$ (dashed line).

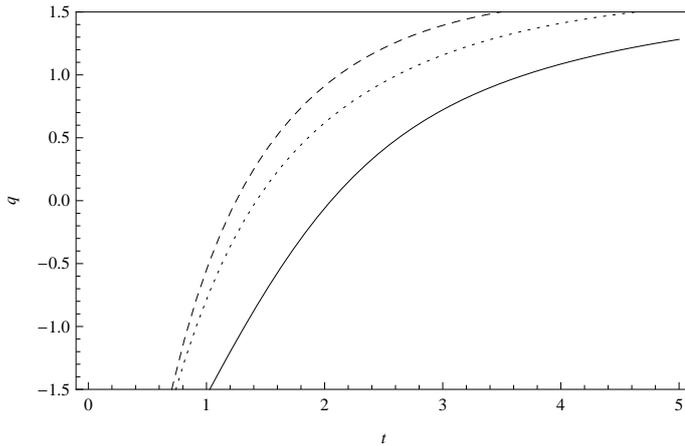


Figure 7: Deceleration parameter versus time for $b = 1, \mu = 1, \zeta_1 = 1, \omega = 1, C = 0.9, \zeta_0 = 0.5$ (solid line), $\zeta_0 = 1$ (dotted line), $\zeta_0 = 1.5$ (dashed line).

In figs. (4) and (5) we plotted Hubble expansion parameter versus time for varying viscosity coefficients ζ_1 and ζ_0 respectively, where H decreases with increasing value of viscosity coefficients. In figs. (6) and (7), we have shown effect of bulk viscosity coefficients on deceleration parameter and observed that q decreases with increasing ζ_1 and increases with increasing ζ_0 . Further it is observed that the model which we obtain undergoes decelerated expansion to an accelerated expansion at $t = 2 \sim 2.5$ for increasing ζ_1 and at $t = 1 \sim 2$ for increasing ζ_0 .

5.2.2 Late time universe ($t \gg 1$)

At the late time universe which has very small energy density, from Eq. (21) [for $\alpha = \frac{1}{2}$] we obtain

$$\rho_d = \frac{-2B}{\zeta_0 \sqrt{6}}(2\mu + 3b - 2) + C \exp\left(\frac{-4\zeta_0}{2\mu + 3b - 2}t\right). \quad (26)$$

Therefore Hubble parameter is defined as

$$H = \sqrt{\frac{-2B}{6\zeta_0 \sqrt{6}}(2\mu + 3b - 2) + \frac{C}{6} \exp\left(\frac{-4\zeta_0}{2\mu + 3b - 2}t\right)} \quad (27)$$

6. CONCLUSION

In this paper we considered generalized cosmic Chaplygin gas with bulk viscosity interacting with ordinary matter with sign-changeable form. We have studied behavior of time dependent density in special case and general case. In the special case we assumed $\Lambda = 0$ and $k = 0$ (universe is flat), $8\pi G = 1$ and some simple model for scale factor in order to discuss numerical and analytical analysis of energy density from conservation Eq. (15). In Figs. (1) - (3) we have shown the variation of energy density with time for varying viscosity coefficients ζ_1, ζ_0 and interacting parameter b and found that ρ_d is decreasing function of time. From figs. (1)-(3) it is clear that energy density increases with increasing viscosity coefficients and interaction parameter.

In general case, we obtain highly nonlinear differential equation in terms of ρ_d which can not be solved easily, therefore to get expression for energy density we considered only two cases: namely early time and late time. In early time energy density is high and in late time energy density is very low, with this assumption we simplified Eq. (21) and obtained expression for energy density and Hubble expansion parameter. The behavior of Hubble parameter and deceleration parameter versus time is shown in figs. (4) - (7) for varying viscosity coefficients and interaction parameter, where H decreases with increasing value of viscosity coefficients ζ_1 and ζ_0 and q decreases with increasing ζ_1 and increases with increasing ζ_0 . It is also observed that model which we obtain undergoes decelerated expansion to an accelerated expansion.

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