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Effects of Heat Source / Sink and Non-uniform Temperature Gradients on Non-Darcian-Benard-Magneto-Marangoni Convection in an Infinite Horizontal Composite Layer

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Abstract

The physical configuration of the problem is a composite layer which is horizontally unbounded, in the presence of uniform heat source/sink in both the layers enclosed by adiabatic boundaries. The problem of Benard-Magneto-Marangoni convection is investigated on this composite layer for non-Darcian case and is subjected to uniform and nonuniform temperature gradients. The eigenvalue, thermal Marangoni number is obtained in the closed form for lower rigid and upper free with surface tension velocity boundary conditions. The influence of various parameters on the Marangoni number against thermal ratio is discussed. It is observed that the heat absorption in the fluid layer and the applied magnetic field play an important role in controlling Benard-Magneto-Marangoni convection. The parameters which direct this convection are determined and the effect of porous parameter is relatively interesting.

Keywords: Heat source (sink), Thermal ratio, Magnetic field, Exact method, Temperature gradients, Adiabatic boundaries.

AMS Subject Classification: 80-XX, 80Axx, 80A20.

1. INTRODUCTION

Marangoni convection is a kind of heat transfer by the surface tension. Surface tension is a property of a liquid and depends on temperature and concentrations of mineralisers added to the liquid. Italian physician Carlo Marangoni was the first one to have investigated Marangoni convection in 19th century. Marangoni convection is a critical matter in the industry of Manufacture of crystals which are used in the integrated chips in all electronic goods and plays an important role in the purity of the crystals. It is an important incidence which occurs mainly in geophysics, bio-convection, nuclear reactors, solid matrix heat exchanger, solidification of alloys, melting of ice and electronic cooling. Marangoni convection with additional possessions like heat sources/ sinks, magnetic field, rotation, concentration and temperature/salinity gradients also has many engineering applications exemplified by moisture migration in thermal insulation and stored grain, underground spreading chemical pollutant and waste and fertilizer migration in saturated soil, energy storage applications, oil recovery process in petroleum industry and so on.

Some of the works done on Marangoni convection in single / fluid /porous layers, is by Abdullah *et al.* [1], Niccolo Giannetti *et al.* [11], Sankaran and Yarin [18], Mahanthesh and Gireesha [10] and Pejman Abolhosseini *et al.* [14] and the references there in. A very few papers on Marangoni convection in composite layers, Shivakumara *et al.*[19] have investigated the onset of surface-tension-driven space convection in a composite layer with the lower rigid surface of the porous layer is either perfectly heat conducting or insulating, while the upper heat insulation fluid boundary is a free and at which the surface tension effects are allowed using both Beavers-Joseph and the Jones conditions at the interface. The resulting Eigen value problem is solved exactly. Besides analytical expression for the critical Marangoni number is obtained for insulating boundaries by using regular perturbation technique. Sumithra [21] obtained exact solution of triple diffusive Marangoni convection in a composite layer. Sumithra and Manjunatha [22] have investigated effects of non uniform temperature gradients on two component magneto convection in a composite layer system.

The works on the convective instability in the presence of heat source/sink have been considered by Rionero and Straughan [17], Rao and Wang [15], Rees and Pop [16], Parthiban and Patil [13], Khalili and Shivakumara [8], Herron [5], Khalili *et al.* [9], Joshi *et al.* [7], Nouri-Borujerdi *et al.* [12], Grosnan *et al.* [4], Cookey *et al.* [3] and Jawdat and Hashim [6]. Bhadauria [2] investigated double diffusive convection in a saturated anisotropic porous layer with internal heat source. Siddheshwar and Vanishree [20] have obtained Lorenz and Ginzburg Landau equations for thermal convection in a high porosity medium with heat source. Sumithra *et al.* [23] have demonstrated the study of Marangoni convection along with the presence of heat sources in both fluid and porous layers of the composite layer. Sumithra and Manjunatha [24] discussed Benard-Magneto-Marangoni convection in flow past a densely packed porous layer along with heat sources in both the layers.

It is evident that not much work is done on convective instabilities on composite layers and the same in the presence of additional heat source/sink and presence of temperature gradients is rarely found. An attempt is made to study the Benard-Marangoni convection in a composite layer enclosed by adiabatic boundaries with heat source/sink along with a vertical magnetic field and non uniform temperature gradients using non-Darcy model for the porous layer. The eigenvalue, thermal Marangoni number is obtained in the closed form for lower rigid and upper free with surface tension velocity boundary conditions. The influence of various parameters on the Marangoni number against thermal ratio is discussed. The parameters which regulate Benard-Magneto-Marangoni convection in composite layer are discussed.

2. FORMULATION OF THE PROBLEM

Consider a horizontal single component, electrically conducting fluid saturated isotropic, incompressible sparsely packed porous layer of thickness d_m underlying a single component fluid layer of thickness d with an imposed magnetic field intensity H_0 in the vertical z-direction and with heat sources Φ_m and Φ respectively. The lower surface of the porous layer rigid and the upper surface of the fluid layer is free with surface tension effects depending on temperature. A Cartesian coordinate system is chosen with the origin at the interface between porous and fluid layers and the z-axis, vertically upwards. The basic equations for fluid and porous layer respectively governing such a system are,

$$\nabla.\overrightarrow{q} = 0 \quad (1)$$

$$\nabla.\overrightarrow{H} = 0 \quad (2)$$

$$\rho_0[\frac{\partial \overrightarrow{q}}{\partial t} + (\overrightarrow{q}.\nabla)\overrightarrow{q}] = -\nabla P + \mu\nabla^2\overrightarrow{q} + \mu_p(\overrightarrow{H}.\nabla)\overrightarrow{H} \quad (3)$$

$$\frac{\partial T}{\partial t} + (\overrightarrow{q}.\nabla)T = \kappa\nabla^2 T + \Phi \quad (4)$$

$$\frac{\partial \overrightarrow{H}}{\partial t} = \nabla \times \overrightarrow{q} \times \overrightarrow{H} + \nu_m\nabla^2\overrightarrow{H} \quad (5)$$

$$\nabla_m.\overrightarrow{q_m} = 0 \quad (6)$$

$$\nabla_m.\overrightarrow{H} = 0 \quad (7)$$

$$\rho_0[\frac{1}{\varepsilon}\frac{\partial \overrightarrow{q_m}}{\partial t} + \frac{1}{\varepsilon^2}(\overrightarrow{q_m}.\nabla_m)\overrightarrow{q_m}] = -\nabla_m P_m - \frac{\mu}{K}\overrightarrow{q_m} + \mu_m\nabla_m^2\overrightarrow{q_m} + \mu_p(\overrightarrow{H}.\nabla_m)\overrightarrow{H} \quad (8)$$

$$A\frac{\partial T_m}{\partial t} + (\overrightarrow{q_m}.\nabla_m)T_m = \kappa_m\nabla_m^2 T_m + \Phi_m \quad (9)$$

$$\varepsilon\frac{\partial \overrightarrow{H}}{\partial t} = \nabla_m \times \overrightarrow{q_m} \times \overrightarrow{H} + \nu_{em}\nabla_m^2\overrightarrow{H} \quad (10)$$

where for fluid layer, $\overrightarrow{q}$ is the velocity vector, ρ_0 is the fluid density, t is time, μ is fluid viscosity, $P = p + \frac{\mu_p H^2}{2}$ is the total pressure, $\overrightarrow{H}$ is the magnetic field, T is temperature, κ thermal diffusivity of the fluid, $\nu_m = \frac{1}{\mu_p\sigma}$ is the magnetic viscosity and μ_p is the magnetic permeability. For porous layer, ε is the porosity, μ_m is the effective viscosity of the fluid in the porous layer, K permeability of the porous medium, $A = \frac{(\rho_0 C_p)_m}{(\rho_0 C_p)_f}$ ratio of heat capacities, C_p specific heat, κ_m thermal diffusivity, $\nu_{em} = \frac{\nu_m}{\varepsilon}$ is the effective magnetic viscosity and the subscripts $'m'$ and $'f'$ (in these equations) denotes the quantities in porous layer and fluid layer respectively.

The aim of this paper is to investigate the stability of a quiescent state to infinitesimal perturbations superposed on the basic state. The basic state of the liquid being quiescent is described by

$$\overrightarrow{q} = \overrightarrow{q_b} = 0, P = P_b(z), T = T_b(z), \overrightarrow{H} = H_0(z) \quad (11)$$

$$\overrightarrow{q_m} = \overrightarrow{q_{mb}}, P_m = P_{mb}(z_m), T_m = T_{mb}(z_m), \overrightarrow{H} = H_0(z_m) \quad (12)$$

The basic state temperatures of $T_b(z)$ and $T_{mb}(z_m)$ are obtained as

$$T_b(z) = \frac{-\Phi z(z-d)}{2\kappa} + \frac{(T_u - T_0)h(z)}{d} + T_0 \quad 0 \le z \le d \quad (13)$$

$$T_{mb}(z_m) = \frac{-\Phi_m z_m(z_m + d_m)}{2\kappa_m} + \frac{(T_0 - T_l)h_m(z_m)}{d_m} + T_0 \quad -d_m \le z_m \le 0 \quad (14)$$

where $T_0 = \frac{\kappa d_m T_u + \kappa_m d T_l}{\kappa d_m + \kappa_m d} + \frac{d d_m(\Phi_m d_m + \Phi d)}{2(\kappa d_m + \kappa_m d)}$ is the interface temperature and $h(z)$ and $h_m(z_m)$ are the non-dimensional temperature gradients with $\int_0^1 h(z)dz = 1$ and $\int_0^1 h_m(z_m)dz_m = 1$ and subscript $'b'$ denote the basic state.

We superimpose infinitesimal disturbances on the basic state for fluid and porous layer respectively

$$\overrightarrow{q} = \overrightarrow{q_b} + \overrightarrow{q}', P = P_b + P', \quad T = T_b(z) + \theta, \overrightarrow{H} = H_0(z) + \overrightarrow{H'} \quad (15)$$

$$\overrightarrow{q_m} = \overrightarrow{q_{mb}} + \overrightarrow{q_m}', P_m = P_{mb} + P'_m, \quad T_m = T_{mb}(z_m) + \theta_m, \overrightarrow{H} = H_0(z_m) + \overrightarrow{H'} \quad (16)$$

where the prime indicates the perturbations. Introducing (15) and (16) in (1) - (10), operating curl twice and eliminate the pressure term from equations (3) and (8).

The following dimensionless equations (after neglecting the primes) are in $0 \le z \le 1$ and $-1 \le z_m \le 0$

$$\frac{1}{Pr}\frac{\partial(\nabla^2 W)}{\partial t} = \nabla^4 W + Q\tau_{fm}\frac{\partial(\nabla^2 H_z)}{\partial z} \quad (17)$$

$$\frac{\partial\theta}{\partial t} = \nabla^2\theta + [h(z) + R_I\frac{(2z-1)}{2(T_0 - T_u)}]W \quad (18)$$

$$\frac{\partial H_z}{\partial t} = \frac{\partial W}{\partial t} + \tau_{fm}\nabla^2 H_z \quad (19)$$

$$\frac{\beta^2}{Pr_m}\frac{\partial(\nabla_m^2 W_m)}{\partial t} = \widehat{\mu}\beta^2\nabla_m^4 W_m - \nabla_m^2 W_m + Q_m\tau_{mm}\beta^2\frac{\partial(\nabla_m^2 H_{zm})}{\partial z_m} \quad (20)$$

$$A\frac{\partial\theta_m}{\partial t} = \nabla_m^2\theta_m + [h_m(z_m) + R_{Im}\frac{(2z_m+1)}{2(T_l - T_0)}]W_m \quad (21)$$

$$\varepsilon\frac{\partial H_{zm}}{\partial t} = \frac{\partial W_m}{\partial t} + \tau_{mm}\nabla^2 H_{zm} \quad (22)$$

where, for fluid layer $Pr = \frac{\nu}{\kappa}$ = Prandtl number, $Q = \frac{\mu_p H_0^2 d^2}{\mu\kappa\tau_{fm}}$ = Chandrasekhar number, $\tau_{fm} = \frac{\nu_{mv}}{\kappa}$ and $R_I = \dfrac{R_i}{d(T_0 - T_u)}$ = internal Rayleigh number. For porous layer, $Pr_m = \frac{\varepsilon\nu_m}{\kappa_m}$ = Prandtl number, $\beta^2 = \frac{K}{d_m^2} = Da$ = Darcy number, β = porous parameter, $\widehat{\mu} = \frac{\mu_m}{\mu}$ = viscosity ratio, $Q_m = \frac{\mu_p H_0^2 d_m^2}{\mu\kappa_m\tau_{mm}} = Q\varepsilon\widehat{d}^2$ = Chandrasekhar number, $\tau_{mm} = \frac{\nu_{em}}{\kappa_m}$ and $R_{Im} = \dfrac{R_{im}}{d_m(T_l - T_0)}$ = internal Rayleigh number with $R_i = \dfrac{\Phi d^2}{\kappa}$ and $R_{im} = \dfrac{\Phi_m d_m^2}{\kappa_m}$.
The dimensionless equations are then subjected to normal mode analysis as follows

$$\begin{bmatrix} W \\ \theta \\ H \end{bmatrix} = \begin{bmatrix} W(z) \\ \theta(z) \\ H(z) \end{bmatrix} f(x,y)e^{nt} \tag{23}$$

$$\begin{bmatrix} W_m \\ \theta_m \\ H \end{bmatrix} = \begin{bmatrix} W_m(z_m) \\ \theta_m(z_m) \\ H(z_m) \end{bmatrix} f_m(x_m,y_m)e^{n_m t} \tag{24}$$

with $\nabla_2^2 f + a^2 f = 0$ and $\nabla_{2m}^2 f_m + a_m^2 f_m = 0$, where a and a_m are the horizontal wave numbers, n and n_m are the frequencies, $W(z)$ and $W_m(z_m)$ are the dimensionless vertical velocities and $\theta(z)$ and $\theta_m(z_m)$ are the temperature in fluid and porous layers respectively and obtain the following equations
in $0 \le z \le 1$

$$(D^2 - a^2 + \frac{n}{Pr})(D^2 - a^2)W(z) = -Q\tau_{fm}D(D^2 - a^2)H(z) \tag{25}$$

$$(D^2 - a^2 + n)\theta(z) + [h(z) + R_I^*(2z-1)]W(z) = 0 \tag{26}$$

$$(\tau_{fm}(D^2 - a^2) + n)H(z) + DW(z) = 0 \tag{27}$$

in $-1 \le z_m \le 0$

$$[(D_m^2 - a_m^2)\widehat{\mu}\beta^2 + \frac{n_m\beta^2}{Pr_m} - 1](D_m^2 - a_m^2)W_m(z_m) = -Q_m\tau_{mm}\beta^2 D_m(D_m^2 - a_m^2)H(z_m) \tag{28}$$

$$(D_m^2 - a_m^2 + An_m)\theta_m(z_m) + [h_m(z_m) + R_{Im}^*(2z_m+1)]W_m(z_m) = 0 \tag{29}$$

$$\tau_{mm}(D_m^2 - a_m^2 + n_m\varepsilon)H(z_m) + D_mW_m(z_m) = 0 \tag{30}$$

where $R_I^* = \dfrac{R_I}{2(T_0 - T_u)}$ and $R_{Im}^* = \dfrac{R_{Im}}{2(T_l - T_0)}$.
Substituting the equation (27) in (25) and (30) in (28) to eliminate the magnetic field in these equations and assume that the present problem is satisfies the principle of exchange of stability, so putting $n = n_m = 0$. We get,
in $0 \le z \le 1$

$$(D^2 - a^2)^2 W(z) = QD^2W(z) \tag{31}$$

$$(D^2 - a^2)\theta(z) + [h(z) + R_I^*(2z-1)]W(z) = 0 \tag{32}$$

in $-1 \le z_m \le 0$

$$[(D_m^2 - a_m^2)\widehat{\mu}\beta^2 - 1](D_m^2 - a_m^2)W_m(z_m) = Q_m\beta^2 D_m^2 W_m(z_m) \tag{33}$$

$$(D_m^2 - a_m^2)\theta_m(z_m) + [h_m(z_m) + R_{Im}^*(2z_m+1)]W_m(z_m) = 0 \tag{34}$$

3. BOUNDARY CONDITIONS

The boundary conditions are nondimensionalized and then subjected to normal mode expansion and are

$$\begin{aligned} &D^2W(1) + Ma^2\theta(1) = 0, \\ &W(1) = 0, W_m(-1) = 0, D_mW_m(-1) = 0, \\ &\widehat{T}\widehat{d}^2(D^2 + a^2)W(0) = \widehat{\mu}(D_m^2 + a_m^2)W_m(0), \\ &\widehat{T}W(0) = W_m(0), \widehat{T}\widehat{d}DW(0) = D_mW_m(0), \\ &\widehat{T}\widehat{d}^3\beta^2(D^3W(0) - 3a^2DW(0)) = -D_mW_m(0) \\ &+\widehat{\mu}\beta^2(D_m^3W_m(0) - 3a_m^2D_mW_m(0)), \\ &D\theta(1) = 0, \theta(0) = \widehat{T}\theta_m(0), \\ &D\theta(0) = D_m\theta_m(0), D_m\theta_m(-1) = 0 \end{aligned} \tag{35}$$

where
$\widehat{T} = \dfrac{T_l - T_0}{T_0 - T_u}$ is the thermal ratio, $M = -\dfrac{\partial\sigma_t}{\partial T}\dfrac{(T_0 - T_u)d}{\mu\kappa}$ is the thermal Marangoni number and $\widehat{d} = \dfrac{d_m}{d}$ is the depth ratio.

4. METHOD OF SOLUTION

The solutions of $W(z)$ and $W_m(z_m)$ are obtained by solving (31) and (33) using the velocity boundary conditions (35)

$$W(z) = A_1[\cosh\delta z + a_1\sinh\delta z + a_2\cosh\zeta z + a_3\sinh\zeta z] \tag{36}$$

$$W_m(z_m) = A_1[a_4\cosh\eta_m z_m + a_5\sinh\eta_m z_m + a_6\cosh\psi_m z_m + a_7\sinh\psi_m z_m] \tag{37}$$

where
$\delta = \dfrac{\sqrt{Q} + \sqrt{Q + 4a^2}}{2}, \zeta = \dfrac{\sqrt{Q} - \sqrt{Q + 4a^2}}{2}$,
$\eta_m = \sqrt{\dfrac{E + \sqrt{E^2 - 4F}}{2}}, \psi_m = \sqrt{\dfrac{E - \sqrt{E^2 - 4F}}{2}}$,
$E = \dfrac{(2\widehat{\mu}\beta^2 a_m^2 + 1 + Q_m\beta^2)}{\widehat{\mu}\beta^2}, F = \dfrac{(a_m^2 + a_m^4\widehat{\mu}\beta^2)}{\widehat{\mu}\beta^2}$,
$a_1 = \frac{1}{\delta_{14}}(a_6\delta_{15} + a_7\delta_{16} + \delta_{17}), a_2 = a_6\delta_5 + \delta_6$,
$a_3 = \frac{1}{\delta_9}(a_1\delta_{10} + a_7\delta_{11}), a_4 = \delta_7 + a_6\delta_8, a_5 = a_1\delta_{12} + a_7\delta_{13}$,
$a_6 = \dfrac{\delta_{23}\delta_{25} - \delta_{26}\delta_{22}}{\delta_{25}\delta_{21} - \delta_{24}\delta_{22}}, a_7 = \dfrac{\delta_{23}\delta_{24} - \delta_{26}\delta_{21}}{\delta_{24}\delta_{22} - \delta_{25}\delta_{21}}$,
$\delta_1 = \widehat{T}\beta^2\widehat{d}^3(\delta^3 - 3a^2\delta), \delta_2 = \widehat{T}\beta^2\widehat{d}^3(\zeta^3 - 3a^2\zeta)$,
$\delta_3 = \widehat{\mu}\beta^2(\eta_m^3 - 3a_m^2\eta_m) - \eta_m, \delta_4 = \widehat{\mu}\beta^2(\psi_m^3 - 3a_m^2\psi_m) - \psi_m$,
$\delta_5 = \dfrac{\widehat{\mu}[(\psi_m^2 + a_m^2) - \widehat{T}(\eta_m^2 + a_m^2)]}{[\widehat{T}\widehat{d}^2(\zeta^2 + a^2) - \widehat{\mu}\widehat{T}(\eta_m^2 + a_m^2)]}$,
$\delta_6 = \dfrac{[\widehat{\mu}(\eta_m^2 + a_m^2) - \widehat{d}^2(\delta^2 + a^2)]}{[\widehat{d}^2(\zeta^2 + a^2) - \widehat{\mu}(\eta_m^2 + a_m^2)]}$,

$\delta_7 = \widehat{T}(1+\delta_6), \delta_8 = \widehat{T}\delta_5 - 1, \delta_9 = \delta_2 - \frac{\widehat{T}\widehat{d}\zeta\delta_3}{\eta_m}, \delta_{10} = -\delta_1 + \frac{\widehat{T}\widehat{d}\delta\delta_3}{\eta_m},$
$\delta_{11} = \delta_4 - \frac{\psi\delta_3}{\eta_m}, \delta_{12} = \frac{1}{\eta_m}(\widehat{T}\widehat{d}\delta + \frac{\zeta\delta_{10}}{\delta_9}), \delta_{13} = \frac{1}{\eta_m}[(\frac{\widehat{T}\widehat{d}\zeta\delta_{11}}{\delta_9} - \psi_m)],$
$\delta_{14} = \sinh\delta + \frac{\delta_{10}\sinh\zeta}{\delta_9}, \delta_{15} = \delta_5\cosh\zeta, \delta_{16} = \frac{\delta_{11}\sinh\zeta}{\delta_9},$
$\delta_{17} = \delta_6\cosh\zeta + \cosh\delta, \delta_{18} = \frac{\delta_{12}\delta_{15}}{\delta_{14}},$
$\delta_{19} = \frac{\delta_{12}\delta_{16}}{\delta_{14}} + \delta_{13}, \delta_{20} = \frac{\delta_{12}\delta_{17}}{\delta_{14}},$
$\delta_{21} = \delta_8\cosh\eta_m - \delta_{18}\sinh\eta_m + \cosh\psi_m,$
$\delta_{22} = -\delta_{19}\sinh\eta_m - \sinh\psi_m, \delta_{23} = \delta_{20}\sinh\eta_m - \delta_7\cosh\eta_m,$
$\delta_{24} = -\eta_m\delta_8\sinh\eta_m - \delta_{18}\eta_m\cosh\eta_m - \psi_m\sinh\psi_m,$
$\delta_{25} = -\eta_m\delta_{19}\cosh\eta_m + \psi_m\cosh\psi_m,$
$\delta_{26} = \delta_{20}\eta_m\cosh\eta_m + \delta_7\eta_m\sinh\eta_m$

4.1. Linear temperature profile

Consider the linear profile,

$$h(z) = 1 \quad and \quad h_m(z_m) = 1 \tag{38}$$

substituting equation (38) into (32) and (34), the temperature distributions $\theta(z)$ and $\theta_m(z_m)$ are obtained using the temperature boundary conditions, as follows

$$\theta(z) = A_1[c_1\cosh az + c_2\sinh az + g_1(z)] \tag{39}$$
$$\theta_m(z_m) = A_1[c_3\cosh a_m z_m + c_4\sinh a_m z_m + g_{m1}(z_m)] \tag{40}$$

where
$g_1(z) = A_1[\delta_{27} - \delta_{28} + \delta_{29} - \delta_{30}],$
$g_{m1}(z_m) = A_1[\delta_{31} - \delta_{32} + \delta_{33} - \delta_{34}]$
$\delta_{27} = \frac{(E_2 z + E_1)}{(\delta^2 - a^2)}(\cosh\delta z + a_1\sinh\delta z)$
$\delta_{28} = \frac{2\delta E_2}{(\delta^2 - a^2)^2}(a_1\cosh\delta z + \sinh\delta z)$
$\delta_{29} = \frac{(E_2 z + E_1)}{(\zeta^2 - a^2)}(a_2\cosh\zeta z + a_3\sinh\zeta z)$
$\delta_{30} = \frac{2\zeta E_2}{(\zeta^2 - a^2)^2}(a_3\cosh\zeta z + a_2\sinh\zeta z)$
$\delta_{31} = \frac{(E_{2m} z_m + E_{1m})}{(\eta_m^2 - a_m^2)}(a_4\cosh\eta_m z_m + a_5\sinh\eta_m z_m)$
$\delta_{32} = \frac{2\eta_m E_{2m}}{(\eta_m^2 - a_m^2)^2}(a_5\cosh\eta_m z_m + a_4\sinh\eta_m z_m)$
$\delta_{33} = \frac{(E_{2m} z_m + E_{1m})}{(\psi_m^2 - a_m^2)}(a_6\cosh\psi_m z_m + a_7\sinh\psi_m z_m)$
$\delta_{34} = \frac{2\psi_m E_{2m}}{(\psi_m^2 - a_m^2)^2}(a_7\cosh\psi_m z_m + a_6\sinh\psi_m z_m)$
$E_1 = R_I^* - 1, E_2 = -2R_I^*,$
$E_{1m} = -(R_{Im}^* + 1), E_{2m} = -2R_{Im}^*$
$c_1 = c_3\widehat{T} + \Delta_2 - \Delta_3, c_2 = \frac{1}{a}(c_4 a_m + \Delta_4 - \Delta_5),$
$c_3 = \frac{\Delta_8\Delta_{10} - \Delta_{11}\Delta_6}{-\Delta_7\Delta_{10} - \Delta_9\Delta_6}, c_4 = \frac{\Delta_8\Delta_9 + \Delta_{11}\Delta_7}{\Delta_6\Delta_9 + \Delta_{10}\Delta_7},$
$\Delta_1 = -[\delta_{35} + \delta_{36} + \delta_{37} + \delta_{38}],$
$\delta_{35} = \frac{\delta(E_2 + E_1)}{(\delta^2 - a^2)}(a_1\cosh\delta + \sinh\delta),$
$\delta_{36} = [\frac{E_2}{(\delta^2 - a^2)} - \frac{2\delta^2 E_2}{(\delta^2 - a^2)^2}](\cosh\delta + a_1\sinh\delta),$
$\delta_{37} = \frac{\zeta(E_2 + E_1)}{(\zeta^2 - a^2)}(a_3\cosh\zeta + a_2\sinh\zeta),$
$\delta_{38} = [\frac{E_2}{(\zeta^2 - a^2)} - \frac{2\zeta^2 E_2}{(\zeta^2 - a^2)^2}](a_2\cosh\zeta + a_3\sinh\zeta),$
$\Delta_2 = \widehat{T}[\frac{E_{1m}a_4}{(\eta_m^2 - a_m^2)} - \frac{2E_{2m}\eta_m a_5}{(\eta_m^2 - a_m^2)^2} + \frac{E_{1m}a_6}{(\psi_m^2 - a_m^2)} - \frac{2E_{2m}\psi_m a_7}{(\psi_m^2 - a_m^2)^2}],$
$\Delta_3 = \frac{E_1}{(\delta^2 - a^2)} - \frac{2\delta a_1 E_2}{(\delta^2 - a^2)^2} + \frac{a_2 E_1}{(\zeta^2 - a^2)} - \frac{2\zeta a_3 E_2}{(\zeta^2 - a^2)^2},$
$\Delta_4 = [\frac{E_{2m}}{(\eta_m^2 - a_m^2)} - \frac{2\eta_m^2 E_{2m}}{(\eta_m^2 - a_m^2)^2}]a_4 + \frac{\eta_m a_5 E_{1m}}{(\eta_m^2 - a_m^2)} + \Delta_{400}$
$\Delta_{400} = [\frac{E_{2m}}{(\psi_m^2 - a_m^2)} - \frac{2\psi_m^2 E_{2m}}{(\psi_m^2 - a_m^2)^2}]a_6 + \frac{\psi_m a_7 E_{1m}}{(\psi_m^2 - a_m^2)}$
$\Delta_5 = \frac{E_1\delta a_1 + E_2}{(\delta^2 - a^2)} - \frac{2E_2\delta^2}{(\delta^2 - a^2)^2} + \frac{E_1\zeta a_3 + E_2 a_2}{(\zeta^2 - a^2)} - \frac{2a_2 E_2\zeta^2}{(\zeta^2 - a^2)^2},$
$\Delta_6 = a_m\cosh a_m, \Delta_7 = a_m\sinh a_m,$
$\Delta_8 = -[\delta_{39} + \delta_{40} + \delta_{41} + \delta_{42}]$
$\delta_{39} = [\frac{E_{2m}}{(\eta_m^2 - a_m^2)} - \frac{2\eta_m^2 E_{2m}}{(\eta_m^2 - a_m^2)^2}](a_4\cosh\eta_m - a_5\sinh\eta_m)$
$\delta_{40} = \eta_m\frac{(E_{1m} - E_{2m})}{(\eta_m^2 - a_m^2)}(a_5\cosh\eta_m - a_4\sinh\eta_m)$
$\delta_{40} = [\frac{E_{2m}}{(\psi_m^2 - a_m^2)} - \frac{2\psi_m^2 E_{2m}}{(\psi_m^2 - a_m^2)^2}](a_6\cosh\psi_m - a_7\sinh\psi_m)$
$\delta_{41} = \psi_m\frac{(E_{1m} - E_{2m})}{(\psi_m^2 - a_m^2)}(a_7\cosh\psi_m - a_6\sinh\psi_m)$
$\Delta_9 = \widehat{T}a\sinh a, \Delta_{10} = a_m\cosh a,$
$\Delta_{11} = \Delta_1 - a(\Delta_2 - \Delta_3)\sinh a - (\Delta_4 - \Delta_5)\cosh a$
From the boundary condition (35), we have

$$M = \frac{-D^2W(1)}{a^2\theta(1)}$$

The thermal Marangoni number for the linear temperature profile is as follows

$$M_1 = -\frac{[\delta^2(\cosh\delta + a_1\sinh\delta) + \zeta^2(a_2\cosh\zeta + a_3\sinh\zeta)]}{a^2(c_1\cosh a + c_2\sinh a + \Lambda_1 + \Lambda_2)} \tag{41}$$

where
$\Lambda_1 = \frac{(E_2 + E_1)}{(\delta^2 - a^2)}(\cosh\delta + a_1\sinh\delta) - \frac{2\delta E_2}{(\delta^2 - a^2)^2}(a_1\cosh\delta + \sinh\delta)$
$\Lambda_2 = \frac{(E_2 + E_1)}{(\zeta^2 - a^2)}(a_2\cosh\zeta + a_3\sinh\zeta) - \frac{2\zeta E_2}{(\zeta^2 - a^2)^2}(a_3\cosh\zeta + a_2\sinh\zeta)$

4.2. Parabolic temperature profile

For the parabolic temperature profile,

$$h(z) = 2z \quad and \quad h_m(z_m) = 2z_m \tag{42}$$

Substituting (42) into (32) and (34), the temperature distributions $\theta(z)$ and $\theta_m(z_m)$ are obtained using the temperature boundary conditions is as follows

$$\theta(z) = A_1[c_5\cosh az + c_6\sinh az + g_2(z)] \tag{43}$$
$$\theta_m(z_m) = A_1[c_7\cosh a_m z_m + c_8\sinh a_m z_m + g_{m2}(z_m)] \tag{44}$$

where
$g_2(z) = A_1[\delta_{43} - \delta_{44} + \delta_{45} - \delta_{46}],$
$g_{m2}(z_m) = A_1[\delta_{47} - \delta_{48} + \delta_{49} - \delta_{50}]$
$\delta_{43} = \frac{(E_4 z + E_3)}{(\delta^2 - a^2)}(\cosh\delta z + a_1\sinh\delta z)$
$\delta_{44} = \frac{2\delta E_4}{(\delta^2 - a^2)^2}(a_1\cosh\delta z + \sinh\delta z)$
$\delta_{45} = \frac{(E_4 z + E_3)}{(\zeta^2 - a^2)}(a_2\cosh\zeta z + a_3\sinh\zeta z)$
$\delta_{46} = \frac{2\zeta E_4}{(\zeta^2 - a^2)^2}(a_3\cosh\zeta z + a_2\sinh\zeta z)$
$\delta_{47} = \frac{(E_{4m} z_m + E_{3m})}{(\eta_m^2 - a_m^2)}(a_4\cosh\eta_m z_m + a_5\sinh\eta_m z_m)$

$\delta_{48} = \frac{2\eta_m E_{4m}}{(\eta_m^2 - a_m^2)^2}(a_5 \cosh \eta_m z_m + a_4 \sinh \eta_m z_m)$
$\delta_{49} = \frac{(E_{4m} z_m + E_{3m})}{(\psi_m^2 - a_m^2)}(a_6 \cosh \psi_m z_m + a_7 \sinh \psi_m z_m)$
$\delta_{50} = \frac{2\psi_m E_{4m}}{(\psi_m^2 - a_m^2)^2}(a_7 \cosh \psi_m z_m + a_6 \sinh \psi_m z_m)$
$E_3 = R_I^*, E_4 = -2(R_I^* + 1),$
$E_{3m} = -R_{Im}^*, E_{4m} = -2(R_{Im}^* + 1)$
$c_5 = c_7 \widehat{T} + \Delta_{13} - \Delta_{14}, c_6 = \frac{1}{a}(c_8 a_m + \Delta_{15} - \Delta_{16}),$
$c_7 = \frac{\Delta_{19}\Delta_{20} - \Delta_{22}\Delta_{17}}{-\Delta_{18}\Delta_{20} - \Delta_{21}\Delta_{17}}, c_8 = \frac{\Delta_{19}\Delta_{21} + \Delta_{22}\Delta_{18}}{\Delta_{17}\Delta_{21} + \Delta_{20}\Delta_{18}},$
$\Delta_{12} = -[\delta_{51} + \delta_{52} + \delta_{53} + \delta_{54}],$
$\delta_{51} = \frac{\delta(E_4 + E_3)}{(\delta^2 - a^2)}(a_1 \cosh \delta + \sinh \delta),$
$\delta_{52} = [\frac{E_4}{(\delta^2 - a^2)} - \frac{2\delta^2 E_4}{(\delta^2 - a^2)^2}](\cosh \delta + a_1 \sinh \delta),$
$\delta_{53} = \frac{\zeta(E_4 + E_3)}{(\zeta^2 - a^2)}(a_3 \cosh \zeta + a_2 \sinh \zeta),$
$\delta_{54} = [\frac{E_4}{(\zeta^2 - a^2)} - \frac{2\zeta^2 E_4}{(\zeta^2 - a^2)^2}](a_2 \cosh \zeta + a_3 \sinh \zeta),$
$\Delta_{13} = \widehat{T}[\frac{E_{3m} a_4}{(\eta_m^2 - a_m^2)} - \frac{2E_{4m}\eta_m a_5}{(\eta_m^2 - a_m^2)^2} + \frac{E_{3m} a_6}{(\psi_m^2 - a_m^2)} - \frac{2E_{4m}\psi_m a_7}{(\psi_m^2 - a_m^2)^2}],$
$\Delta_{14} = \frac{E_3}{(\delta^2 - a^2)} - \frac{2\delta a_1 E_4}{(\delta^2 - a^2)^2} + \frac{a_2 E_3}{(\zeta^2 - a^2)} - \frac{2\zeta a_3 E_4}{(\zeta^2 - a^2)^2},$
$\Delta_{15} = [\frac{E_{4m}}{(\eta_m^2 - a_m^2)} - \frac{2\eta_m^2 E_{4m}}{(\eta_m^2 - a_m^2)^2}]a_4 + \frac{\eta_m a_5 E_{3m}}{(\eta_m^2 - a_m^2)} + \Delta_{150}$
$\Delta_{150} = [\frac{E_{4m}}{(\psi_m^2 - a_m^2)} - \frac{2\psi_m^2 E_{4m}}{(\psi_m^2 - a_m^2)^2}]a_6 + \frac{\psi_m a_7 E_{3m}}{(\psi_m^2 - a_m^2)}$
$\Delta_{16} = \frac{E_3 \delta a_1 + E_4}{(\delta^2 - a^2)} - \frac{2E_4 \delta^2}{(\delta^2 - a^2)^2} + \frac{E_3 \zeta a_3 + E_4 a_2}{(\zeta^2 - a^2)} - \frac{2a_2 E_4 \zeta^2}{(\zeta^2 - a^2)^2},$
$\Delta_{17} = a_m \cosh a_m, \Delta_{18} = a_m \sinh a_m,$
$\Delta_{19} = -[\delta_{55} + \delta_{56} + \delta_{57} + \delta_{58}]$
$\delta_{55} = [\frac{E_{4m}}{(\eta_m^2 - a_m^2)} - \frac{2\eta_m^2 E_{4m}}{(\eta_m^2 - a_m^2)^2}](a_4 \cosh \eta_m - a_5 \sinh \eta_m)$
$\delta_{56} = \eta_m \frac{(E_{3m} - E_{4m})}{(\eta_m^2 - a_m^2)}(a_5 \cosh \eta_m - a_4 \sinh \eta_m)$
$\delta_{57} = [\frac{E_{4m}}{(\psi_m^2 - a_m^2)} - \frac{2\psi_m^2 E_{4m}}{(\psi_m^2 - a_m^2)^2}](a_6 \cosh \psi_m - a_7 \sinh \psi_m)$
$\delta_{58} = \psi_m \frac{(E_{3m} - E_{4m})}{(\psi_m^2 - a_m^2)}(a_7 \cosh \psi_m - a_6 \sinh \psi_m)$
$\Delta_{20} = \widehat{T} a \sinh a, \Delta_{21} = a_m \cosh a,$
$\Delta_{22} = \Delta_{12} - a(\Delta_{13} - \Delta_{14}) \sinh a - (\Delta_{15} - \Delta_{16}) \cosh a$
From the boundary condition (35), the thermal Marangoni number for parabolic temperature profile is as follows

$$M_2 = -\frac{[\delta^2(\cosh \delta + a_1 \sinh \delta) + \zeta^2(a_2 \cosh \zeta + a_3 \sinh \zeta)]}{a^2(c_5 \cosh a + c_6 \sinh a + \Lambda_3 + \Lambda_4)} \quad (45)$$

where
$\Lambda_3 = \frac{(E_4 + E_3)}{(\delta^2 - a^2)}(\cosh \delta + a_1 \sinh \delta) - \frac{2\delta E_4}{(\delta^2 - a^2)^2}(a_1 \cosh \delta + \sinh \delta)$
$\Lambda_4 = \frac{(E_4 + E_3)}{(\zeta^2 - a^2)}(a_2 \cosh \zeta + a_3 \sinh \zeta) - \frac{2\zeta E_4}{(\zeta^2 - a^2)^2}(a_3 \cosh \zeta + a_2 \sinh \zeta)$

4.3. Inverted Parabolic temperature profile

Consider inverted parabolic profile as

$$h(z) = 2(1 - z) \quad and \quad h_m(z_m) = 2(1 - z_m) \quad (46)$$

Substituting (46) into (32) and (34), the temperature distributions $\theta(z)$ and $\theta_m(z_m)$ are obtained using the temperature boundary conditions, as follows

$$\theta(z) = A_1[c_9 \cosh az + c_{10} \sinh az + g_3(z)] \quad (47)$$
$$\theta_m(z_m) = A_1[c_{11} \cosh a_m z_m + c_{12} \sinh a_m z_m + g_{m3}(z_m)] \quad (48)$$

where
$g_3(z) = A_1[\delta_{59} - \delta_{60} + \delta_{61} - \delta_{62}],$
$g_{m3}(z_m) = A_1[\delta_{63} - \delta_{64} + \delta_{65} - \delta_{66}]$
$\delta_{59} = \frac{(E_6 z + E_5)}{(\delta^2 - a^2)}(\cosh \delta z + a_1 \sinh \delta z)$
$\delta_{60} = \frac{2\delta E_6}{(\delta^2 - a^2)^2}(a_1 \cosh \delta z + \sinh \delta z)$
$\delta_{61} = \frac{(E_6 z + E_5)}{(\zeta^2 - a^2)}(a_2 \cosh \zeta z + a_3 \sinh \zeta z)$
$\delta_{62} = \frac{2\zeta E_6}{(\zeta^2 - a^2)^2}(a_3 \cosh \zeta z + a_2 \sinh \zeta z)$
$\delta_{63} = \frac{(E_{6m} z_m + E_{5m})}{(\eta_m^2 - a_m^2)}(a_4 \cosh \eta_m z_m + a_5 \sinh \eta_m z_m)$
$\delta_{64} = \frac{2\eta_m E_{6m}}{(\eta_m^2 - a_m^2)^2}(a_5 \cosh \eta_m z_m + a_4 \sinh \eta_m z_m)$
$\delta_{65} = \frac{(E_{6m} z_m + E_{5m})}{(\psi_m^2 - a_m^2)}(a_6 \cosh \psi_m z_m + a_7 \sinh \psi_m z_m)$
$\delta_{66} = \frac{2\psi_m E_{6m}}{(\psi_m^2 - a_m^2)^2}(a_7 \cosh \psi_m z_m + a_6 \sinh \psi_m z_m)$
$E_5 = R_I^* - 2, E_6 = 2(1 - R_I^*),$
$E_{5m} = -2 - R_{Im}^*, E_{6m} = 2(1 - R_{Im}^*)$
$c_9 = c_{11} \widehat{T} + \Delta_{24} - \Delta_{25}, c_{10} = \frac{1}{a}(c_{12} a_m + \Delta_{26} - \Delta_{27}),$
$c_{11} = \frac{\Delta_{30}\Delta_{31} - \Delta_{33}\Delta_{28}}{-\Delta_{29}\Delta_{31} - \Delta_{32}\Delta_{28}}, c_{12} = \frac{\Delta_{30}\Delta_{32} + \Delta_{33}\Delta_{29}}{\Delta_{28}\Delta_{32} + \Delta_{31}\Delta_{29}},$
$\Delta_{23} = -[\delta_{67} + \delta_{68} + \delta_{69} + \delta_{70}],$
$\delta_{67} = \frac{\delta(E_6 + E_5)}{(\delta^2 - a^2)}(a_1 \cosh \delta + \sinh \delta),$
$\delta_{68} = [\frac{E_6}{(\delta^2 - a^2)} - \frac{2\delta^2 E_6}{(\delta^2 - a^2)^2}](\cosh \delta + a_1 \sinh \delta),$
$\delta_{69} = \frac{\zeta(E_6 + E_5)}{(\zeta^2 - a^2)}(a_3 \cosh \zeta + a_2 \sinh \zeta),$
$\delta_{70} = [\frac{E_6}{(\zeta^2 - a^2)} - \frac{2\zeta^2 E_6}{(\zeta^2 - a^2)^2}](a_2 \cosh \zeta + a_3 \sinh \zeta),$
$\Delta_{24} = \widehat{T}[\frac{E_{5m} a_4}{(\eta_m^2 - a_m^2)} - \frac{2E_{6m}\eta_m a_5}{(\eta_m^2 - a_m^2)^2} + \frac{E_{5m} a_6}{(\psi_m^2 - a_m^2)} - \frac{2E_{6m}\psi_m a_7}{(\psi_m^2 - a_m^2)^2}],$
$\Delta_{25} = \frac{E_5}{(\delta^2 - a^2)} - \frac{2\delta a_1 E_6}{(\delta^2 - a^2)^2} + \frac{a_2 E_5}{(\zeta^2 - a^2)} - \frac{2\zeta a_3 E_6}{(\zeta^2 - a^2)^2},$
$\Delta_{26} = [\frac{E_{6m}}{(\eta_m^2 - a_m^2)} - \frac{2\eta_m^2 E_{6m}}{(\eta_m^2 - a_m^2)^2}]a_4 + \frac{\eta_m a_5 E_{5m}}{(\eta_m^2 - a_m^2)} + \Delta_{260}$
$\Delta_{260} = [\frac{E_{6m}}{(\psi_m^2 - a_m^2)} - \frac{2\psi_m^2 E_{6m}}{(\psi_m^2 - a_m^2)^2}]a_6 + \frac{\psi_m a_7 E_{5m}}{(\psi_m^2 - a_m^2)}$
$\Delta_{27} = \frac{E_5 \delta a_1 + E_6}{(\delta^2 - a^2)} - \frac{2E_6 \delta^2}{(\delta^2 - a^2)^2} + \frac{E_5 \zeta a_3 + E_6 a_2}{(\zeta^2 - a^2)} - \frac{2a_2 E_6 \zeta^2}{(\zeta^2 - a^2)^2},$
$\Delta_{28} = a_m \cosh a_m, \Delta_{29} = a_m \sinh a_m,$
$\Delta_{30} = -[\delta_{71} + \delta_{72} + \delta_{73} + \delta_{74}]$
$\delta_{71} = [\frac{E_{6m}}{(\eta_m^2 - a_m^2)} - \frac{2\eta_m^2 E_{6m}}{(\eta_m^2 - a_m^2)^2}](a_4 \cosh \eta_m - a_5 \sinh \eta_m)$
$\delta_{72} = \eta_m \frac{(E_{5m} - E_{6m})}{(\eta_m^2 - a_m^2)}(a_5 \cosh \eta_m - a_4 \sinh \eta_m)$

$$\delta_{73} = [\frac{E_{6m}}{(\psi_m^2 - a_m^2)} - \frac{2\psi_m^2 E_{6m}}{(\psi_m^2 - a_m^2)^2}](a_6 \cosh\psi_m - a_7 \sinh\psi_m)$$

$$\delta_{74} = \psi_m \frac{(E_{5m} - E_{6m})}{(\psi_m^2 - a_m^2)}(a_7 \cosh\psi_m - a_6 \sinh\psi_m)$$

$$\Delta_{31} = \widehat{T} a \sinh a, \Delta_{32} = a_m \cosh a,$$

$$\Delta_{33} = \Delta_{23} - a(\Delta_{24} - \Delta_{25}) \sinh a - (\Delta_{26} - \Delta_{27}) \cosh a$$

From the boundary condition (35), the thermal Marangoni number for inverted parabolic temperature profile is as follows

$$M_3 = -\frac{[\delta^2(\cosh\delta + a_1 \sinh\delta) + \zeta^2(a_2 \cosh\zeta + a_3 \sinh\zeta)]}{a^2(c_9 \cosh a + c_{10} \sinh a + \Lambda_5 + \Lambda_6)} \quad (49)$$

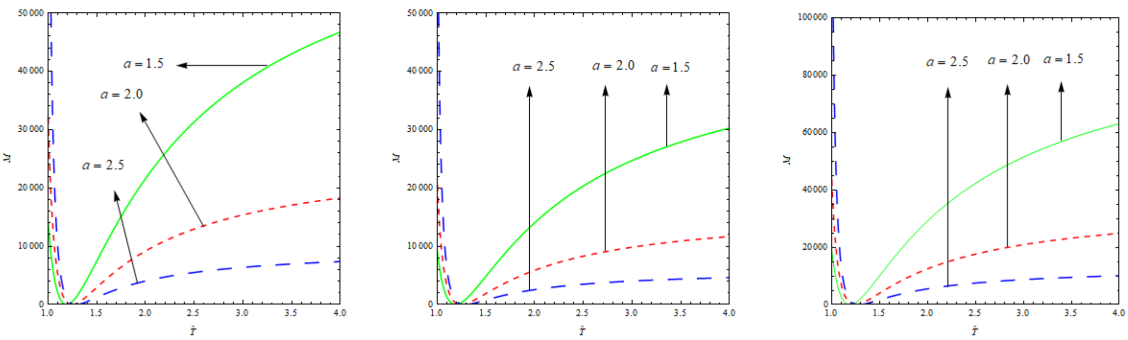
where

$$\Lambda_5 = \frac{(E_6 + E_5)}{(\delta^2 - a^2)}(\cosh\delta + a_1 \sinh\delta) - \frac{2\delta E_6}{(\delta^2 - a^2)^2}(a_1 \cosh\delta + \sinh\delta)$$

$$\Lambda_6 = \frac{(E_6 + E_5)}{(\zeta^2 - a^2)}(a_2 \cosh\zeta + a_3 \sinh\zeta) - \frac{2\zeta E_6}{(\zeta^2 - a^2)^2}(a_3 \cosh\zeta + a_2 \sinh\zeta)$$

5. RESULTS AND DISCUSSION

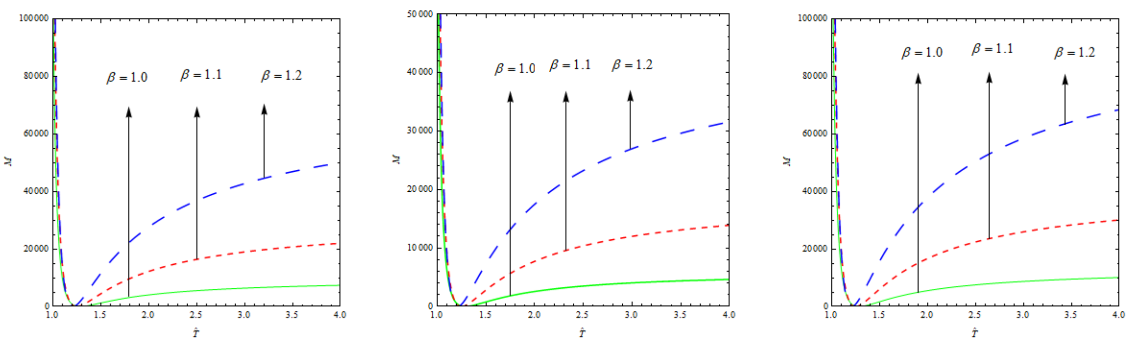
The thermal Marangoni number M is obtained as an expression of the depth ratio $\widehat{d}$, the horizontal wave numbers a and a_m both for the fluid and porous layers, the porous parameter β, the thermal ratio $\widehat{T}$, the viscosity ratio $\widehat{\mu}$, R_I and R_{Im}, the internal Rayleigh numbers for the fluid and porous layers and the Chandrasekhar number Q. This thermal Marangoni number M is drawn versus the thermal ratio $\widehat{T}$. From the figures 1-5 it is evident that, for smaller values of $\widehat{T}$, the thermal Marangoni number M decreases and then after some value of $\widehat{T}$ again thermal Marangoni slightly increases as the value of thermal ratio is further increased. The effects of the horizontal wave number a, the porous parameter β, the Chandrasekhar number Q, the viscosity ratio $\widehat{\mu}$, the internal Rayleigh number R_I on the thermal Marangoni number are displayed in the following figures where the supplementary parameters are fixed. They are $Q = 10, a = 1.5, \varepsilon = 1, \beta = 0.1, \widehat{d} = 2.5, \widehat{\mu} = 2.5, R_I = -3$ and $R_{Im} = 1$.



(a) (b) (c)

Figure 1: Variations of Horizontal wave number a

Fig.1 exhibits the effects of the horizontal wave number a on the values of thermal Marangoni number M for the values of a are $1.5, 2.0$ and 2.5. From the figure it is clear that for smaller values of thermal ratio there is no much effect of this parameter on thermal Marangoni number. For larger values of thermal ratio, there is drastic effect of this parameter on thermal Marangoni number. For a fixed value of thermal ratio, increase in the value of a, decreases the thermal Marangoni number. Hence the system becomes stable by decreasing the horizontal wave number. Similar effects are observed for both uniform and non uniform (parabolic and inverted parabolic) temperature profiles.



(a) (b) (c)

Figure 2: The effects of Porous parameter β

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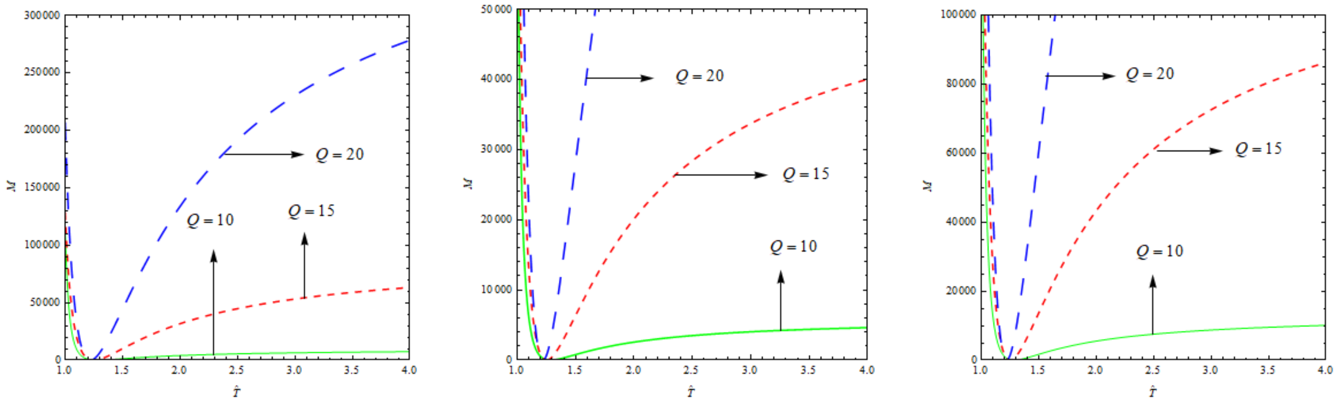
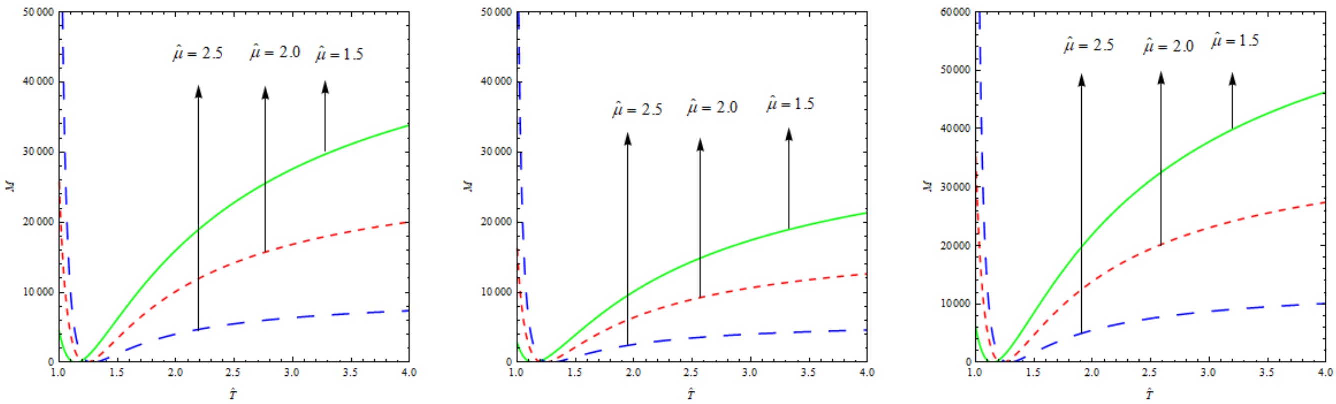
Fig. 2 depicts the effects of β the porous parameter on the thermal Marangoni number and it is for $\beta = 1.0, 1.1$ and 1.2. The curves are diverging for all the temperature profiles, which indicates that the effect of the porous parameter is dominant for larger values of thermal ratio. For fixed values of thermal ratio, the increase in the value of β , shows the increase in the Marangoni number. Hence the system can be stabilized by increasing the value of β. Though there is more permeability for the fluid, the system still remains stabilized, this may be due to presence of vertical magnetic field and presence of heat source in the porous layer.



(a) (b) (c)

Figure 3: The effects of Chandrasekhar number Q

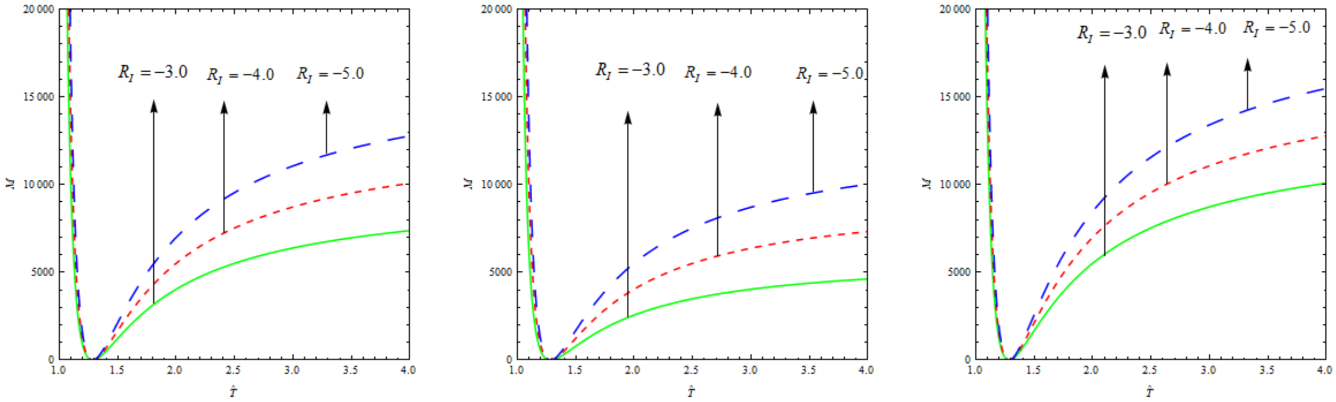
The effects of Chandrasekhar number Q is displayed in Fig. 3 for the linear, parabolic and inverted parabolic temperature profiles. The values of Q taken are $10, 15$ and 20. The curves are for all the three profiles are diverging drastically, indicating the prominence of Q for larger thermal ratio values. For a fixed thermal ratio, the increase in the value of Q increases the thermal Marangoni number, hence the non-Darcy-Benard-Magneto-Marangoni convection can be advanced by decreasing the values of Q and hence the system can be destabilized.



(a) (b) (c)

Figure 4: The effects of Viscosity ratio $\widehat{\mu}$

The effects of the viscosity ratio $\widehat{\mu}$ on the thermal Marangoni number is shown in the Fig. 4 , for all the three temperature profiles the curves are diverging for larger values of thermal ratio, which indicates that the effect of $\widehat{\mu}$ is effective only for the larger values of thermal ratio and the increase in the value of viscosity ratio decreases the thermal Marangoni number M hence, the system can be stabilized by decreasing the value of viscosity ratio $\widehat{\mu}$.



(a) (b) (c)

Figure 5: Effects of internal Rayleigh number R_I

The effect of internal Rayleigh number R_I on the Marangoni number is similar for all the three temperature profiles depicted the Fig. 5 for $R_I = -3, -4$ and -5. Decreasing the values of R_I, the Marangoni number increases, hence the non-Darcy-Benard-Magneto-Marangoni convection can be delayed by decreasing the values of R_I. That is heat absorption supports stability of the system.

6. CONCLUSIONS

Following conclusions are drawn from this study

(i) Larger values of porous parameter and Chandrasekhar number postpone non-Darcy-Benard-Magneto-Marangoni convection.

(ii) Larger values of the horizontal wavenumber a and viscosity ratio $\widehat{\mu}$ prepone non-Darcy-Benard-Magneto-Marangoni convection.

(iii) The presence of heat sink in the fluid layer postpones non-Darcy-Benard-Magneto-Marangoni convection.

(iv) There is no effect of internal Rayleigh number R_{Im} on the non-Darcy-Benard-Magneto-Marangoni convection.

(v) The inverted parabolic temperature gradient is the highly stable of all the three gradients.

(vi) The effects of the physical parameters is analogous to all the three temperature gradients considered in the study.

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