International Journal of Applied Engineering Research ISSN 0973-4562 Volume 18, Number 4 (2023) pp. 262-265
© Research India Publications. https://dx.doi.org/10.37622/IJAER/18.4.2023.262-265

On Some New Forms of Fsgb-Continuous Mappings in Fuzzy Topological Spaces.

Megha Kulkarni* and Jenifer J Karnel**
** Department of Mathematics, East West Institute of Technology, Bengaluru.,*
E-mail:-meghavkulkarni92@gmail.com
*** Department of Mathematics, SDM College of Engineering & Technology,*
Dharwad-580003 Karnataka, India.E-mail:-jeniferjk17@gmail.com

Abstract

This article introduces the new class of functions called stronger forms of fuzzy strongly generalized b-continuous mappings namely strongly fsgb-continuous , perfectl

y fsgb-continuous and completely fsgb-continuous functions on fuzzy topological spaces. We investigate some of their characterization and also the connection between the mappings.

Keywords: fb-OS, fsgb-CS , fg-OS ,FS ,strongly fsgb-ℂℕ , perfectly fsgb- ℂℕ and completely fsgb- ℂℕ.

2010 AMS subject classification: 54A40

1.Introduction

Zadeh[10] developed the fundamental idea of fuzzy sets. Fuzzy topology was first introduced by C.L.Chang[6]. The theory of FTS was developed by several scholars. The concept of b-open sets in general topology was first developed by Andrejevic[1].

Jenifer and Megha introduced the fsgb-closed sets concepts in [7] and also the concept of fsgb-continuous , fsgb-irresolute, fsgb-open and fsgb-closed mappings in FTS[8]. The development of FTS has been aided by numerous researchers, including P. Sundaram, K. K. Azad, M. N. Mukharjee, and others.The objective of this article is introduce and investigate some stronger forms of fuzzy strongly generalized b-continuous functions namely strongly fsgb-continuous , perfectly fsgb-continuous and completely fsgb-continuous mappings in FTS

2. Preliminaries

Throughout this study(L,τ),(M,σ) and (N,γ)(or simply *L*,*M* and *N*) are fuzzy topological spaces(in-short as fts). The closure , interior and compliment of a fuzzy subset P of (L,τ) are denoted by *Cl*(P),*Int*(P) and P^c respectively.

***2*.1 Definition** [3] A fuzzy set P in a fts L is called fuzzy b-open set(fb-OS) iff $P \leq (IntCl(P)\ v\ ClInt(P))$.

2.2 Definition [3] Fuzzy b-interior and fuzzy b-closure of a Fuzzy set P is given by

(i) bInt(P) $= \vee\{Q: Q$ is a fb-open set of L and $P \geq Q\}$.

(ii) bCl(P) = $\wedge\{R: R$ is a fb-closed set of L and $R \geq P\}$.

2.3Definition [7]A fuzzy set P in a fts L is called a fsgb-closed set(fsgb-CS) if $bCl(P) \leq Q$, whenever $P \leq Q$ and Q is fuzzy generalized open set(fg-OS) in *L*.

2.4Definition [7] A fuzzy set P in a fts L is called a fsgb-open set(fsgb-OS) if $bInt(P) \geq Q$, whenever $P \geq Q$ and Q is fg -OS in *L*.

2.5Definition Let *L*, *M* be 2 FTS . A mapping $g: L \to M$ is known as

i)f-continuous map(in short f-ℂℕ map)[3] if $g^{-1}(P)$ is fuzzy-OS in L , for every f-OS P of *M*.

ii)f-completely continuous map(in short fc-ℂℕ map)[9] if $g^{-1}(P)$ is fuzzy regular open set in L , for every f -OS P in *M*.

iii) f-perfectly continuous map(in short fp-ℂℕ map)[3] if $g^{-1}(P)$ is f-OS and f-CS in L , for every f-OS P in *M*.

iv) fuzzy strongly generalized b-continuous(in short fsgb- ℂℕ map)[7] if $g^{-1}(P)$ is fsgb-CS in *L*, for every f-CS P in *M*.

) Fuzzy strongly generalized b-irresolute(in short fsgb-irr)[7] if $g^{-1}(P)$ is fsgb-CS in L for every fsgb-CS P in *M*.

3. Strongly Fsgb-continuous mappings in FTS.

Defintion3.1: A mapping $g: L \to M$ is strongly fsgb-continuous (in short strongly fsgb- ℂℕ) if and only if the inverse of every fsgb-OS in *M* is f-OS in *L*.

Theorem3.2: A mapping $g: L \to M$ is strongly fsgb- ℂℕ map if and only if the inverse of every fsgb-CS in *M* is f-CS in *L*.

Proof: Consider that g is strongly fsgb- ℂℕ map. Let P be fsgb-CS in *M*. Then 1-*P* is fsgb-OS in *L*.As g is strongly fsgb-ℂℕ , $g^{-1}(1-P)$ is f-OS in L. And $g^{-1}(1-P) = 1 - g^{-1}(P)$, so $g^{-1}(P)$ is f-CS in L.

Conversely, consider that the inverse of every fsgb-CS in *M* is f-CS in *L*. Let *Q* be fsgb-OS in *M*, then 1-*Q* is fsgb-CS in *M*. By proposition, $g^{-1}(1-P)$ is f-CS in L. And $g^{-1}(1-Q) =$

$1-g^{-1}(Q)$, so $g^{-1}(Q)$ is fuzzy-OS in L. Thus g is strongly fsgb-ℂℕ map.

Theorem3.3: Every strongly fsgb-ℂℕ map is a f-ℂℕ map.

Proof: Suppose $g: L \to M$ be strongly fsgb- ℂℕ map. Let Q be f-OS in M, so Q is fsgb-OS in M. Then $g^{-1}(Q)$ is f-OS in L. Therefore g is f-ℂℕ map.

The illustration below shows that the inverse implication of the above theorem is incorrect.

Example3.4 Let L=M=$\{x, y, z\}$ and f-sets P, Q and R be defined as follows.

$P=\{(x, 0.5),(y, 0.7),(z, 0.4)\}$, $Q=\{(x, 1),(y, 0.6),(z, 0.9)\}$, $R=\{(x, 0.7),(y, 0.7),(z, 1)\}$, let $\tau=\{0,1, P, Q\}$ and $\sigma=\{0,1, Q\}$ .Then $(L, \tau),(M, \sigma)$ are fts. Thus mapping $g: L \to M$ defined by $g(x)=x$, $g(y)=y$ and $g(z)=z$.Then g is f-ℂℕ but not strongly fsgb-ℂℕ as $g(z)=z$.

Theorem3.5: Every fuzzy strongly-ℂℕ map is a strongly fsgb- ℂℕ map.

Proof: Consider $g: L \to M$ be fuzzy strongly-ℂℕ map. Let P be fsgb-OS in M. Then $g^{-1}(P)$ is f-OS and also f-CS in L as g is fuzzy strongly-ℂℕ.Thus g is strongly fsgb-ℂℕ map.

The example below shows that the inverse implication of the above theorem is incorrect.

Example3.6: Let L=M=$\{x, y, z\}$ and f-sets P, Q and R be defined as follows.

$P=\{(x, 0.5),(y, 0.7),(z, 0.4)\}$, $Q=\{(x, 1),(y, 0.6),(z, 0.9)\}$ .Let $\tau=\{0,1, P\}$ and $\sigma=\{0,1, Q\}$ .Then$(L, \tau),(M, \sigma)$ are fts. Thus mapping $g: L \to M$ defined by $g(x)=x$, $g(y)=y$ and $g(z)=z$.Then g is strongly fsgb-ℂℕ but not strongly f-ℂℕ as$g^{-1}(Q)=Q$ is f-OS but not f-CS in L.

Theorem3.7: Let $g: L \to M, \hbar: M \to N$ be 2 strongly fsgb-ℂℕ then the composition $\hbar \circ g: L \to N$ is strongly fsgb- ℂℕ map.

Proof: Let R be fsgb-OS in N. Then $\hbar^{-1}(R)$ is fuzzy-OS in M , as $\hbar$ is strongly fsgb-ℂℕ. Thus $\hbar^{-1}(R)$ is fsgb-OS in M. Also g is strongly fsgb-ℂℕ, $g^{-1}(\hbar^{-1}(R))=(\hbar \circ g)^{-1}(R)$ is f-OS in L .Therefore $(\hbar \circ g)$ is strongly fsgb- ℂℕ map.

Theorem3.8: Consider $g: L \to M$, $\hbar: M \to N$ be maps such that g is strongly fsgb-ℂℕ and $\hbar$ is fsgb-ℂℕ then $\hbar \circ g: L \to N$ is fuzzy-ℂℕ map.

Proof: Assume that Q be a f-CS in N. Then $\hbar^{-1}(Q)$ is fsgb-CS in M.As $\hbar$ is fsgb- ℂℕ. Also g is strongly fsgb- ℂℕ , $g^{-1}(\hbar^{-1}(Q))=(\hbar \circ g)^{-1}(Q)$ is f-CS in L. Thus $(\hbar \circ g)$ is f-ℂℕ map.

Theorem3.9: If $g: L \to M$ be strongly fsgb-ℂℕ and $\hbar: M \to N$ be fsgb-irr map, then $\hbar \circ g: L \to N$ is strongly fsgb-ℂℕ map.

Proof: Suppose that Q be fsgb-OS in N. Then $\hbar^{-1}(Q)$ is fsgb-OS in M .As $\hbar$ is fsgb-irr map, then $g^{-1}(\hbar^{-1}(Q))=(\hbar \circ g)^{-1}(Q)$ is f-OS in L. Also g is strongly fsgb-ℂℕ. Therefore $\hbar \circ g$ is strongly fsgb-ℂℕ map.

4. Perfectly Fsgb-continuous mappings in FTS

Defintion4.1: A mapping $g: L \to M$ is known as perfectly fsgb- ℂℕ if the inverse of every fsgb-OS in M is f-OS and also f-CS in L.

Theorem4.2: A mapping $g: L \to M$ is perfectly fsgb-ℂℕ iff the inverse of every fsgb-CS in M is f-OS and also f-CS in L.

Proof: Suppose that g is perfectly fsgb-ℂℕ. Let P be fsgb-CS in M. Then 1-P is fsgb-OS in M. Then $1-P$ is fsgb-OS.And thus $g^{-1}(1-P)$ is f-OS and also f-CS in L. And $g^{-1}(1-P)=1-g^{-1}(P)$, so $g^{-1}(P)$ is f-OS and also f-CS in L.

Conversely, consider that the inverse of every fsgb-CS in M is f-OS and also f-CS in L. Let Q be fsgb-OS in M, then 1-Q is fsgb-CS in M . By proposition, $g^{-1}(1-P)$ is f-OS and also f-CS in L. Now $g^{-1}(1-Q)=1-g^{-1}(Q)$ and so $g^{-1}(Q)$ is f-OS and f-CS in L. Therefore g is perfectly fsgb-ℂℕ map.

Theorem4.3: Every perfectly fsgb-ℂℕ is a fuzzy-ℂℕ map.

Proof: Consider $g: L \to M$ be perfectly fsgb-ℂℕ map. Let Q be f-OS in M, so Q is fsgb-OS in M. As g is perfectly fsgb-ℂℕ , then $g^{-1}(Q)$ is f-OS and also f-CS in L. Therefore $g^{-1}(Q)$ is f-OS in L. Thus g is f-ℂℕ map.

The example below shows that the inverse implication of the above theorem is incorrect.

Example4.4: From the example3.4, the map g is f-ℂℕ but not perfectly fsgb-ℂℕ as the f- set $1-R=\{(x, 0.3),(y, 0.3),(z, 0)\}$ is fsgb-OS in M and $g^{-1}(1-R)=1-R$ which is not f-OS and also f-CS in L.

Theorem4.5: Every perfectly fsgb-ℂℕ map is a fuzzy perfectly-ℂℕ map.

Proof: Consider $g: L \to M$ be perfectly fsgb-ℂℕ map. Let P be f-OS in M .Then P be fsgb-OS in M , as g is perfectly fsgb- ℂℕ.Then $g^{-1}(P)$ is f-OS and also f-CS in L . Thus g is fuzzy perfectly ℂℕ map.

The opposite of the above theorem need not be true as seen from the following illustration.

Example4.6: Let L=M=$\{x, y, z\}$ and f-sets P, Q , R and S be defined as follows. $P=\{(x, 0.3),(y, 0.5),(z, 0.7)\}$, $Q=\{(x, 0.2),(y, 1),(z, 0.9)\}$, $R=\{(x, 0.5),(y, 0.7),(z, 0)\}$ and $S=\{(x, 0.1),(y, 0.4),(z, 0.4)\}$ let $\tau=\{0,1, P, Q, R\}$ and $\sigma=\{0,1, S\}$ .Then $(L, \tau),(M, \sigma)$ are fts. Thus mapping $g: L \to M$ defined by $g(x)=x$, $g(y)=y$ and $g(z)=z$.Then g is perfectly f-ℂℕ. Since the f-set S is f-OS in M. But g is not perfectly fsgb-ℂℕ. Also since S is fsgb-OS in M and $g^{-1}(S)=S$ is not f-OS and f-CS in L.

Theorem4.7: Every perfectly fsgb-ℂℕ map is strongly fsgb-ℂℕ map.

Proof: Consider $g: L \to M$ be prefectly fsgb-ℂℕ map. Let Q be fsgb-open set in M. Then $g^{-1}(Q)$ is f-OS and f-CS in L. Thus $g^{-1}(Q)$ is f-OS in L. Therefore g is strongly fsgb-ℂℕ map.

The opposite of the above theorem need not be true as seen from the following illustration.

Example4.8: From the example 3.6, the mapping g is strongly fsgb-ℂℕ map but not perfectly fsgb-ℂℕ , since the f-set Q is fsgb-OS in M and $g^{-1}(Q) = Q$ is f-OS but not f-CS in L.

Theorem4.9: Let $g: L \to M, \hbar: M \to N$ be 2 perfectly fsgb- ℂℕ then the composition $\hbar \circ g: L \to N$ is perfectly fsgb-ℂℕ map.

Proof: Let R be fsgb-OS in N. Then $\hbar^{-1}(R)$ is f-OS and also f-CS in M , as $\hbar$ is perfectly fsgb-ℂℕ. And hence $\hbar^{-1}(R)$ is fsgb-OS in M. Also g is perfectly fsgb-ℂℕ, $g^{-1}(\hbar^{-1}(R)) = (\hbar \circ g)^{-1}(R)$ is f-OS and f-CS in L .Thus $\hbar \circ g$ is perfectly fsgb-ℂℕ map.

Theorem4.10: If $g: L \to M$ be perfectly fsgb-ℂℕ and $\hbar: M \to N$ be fsgb-irr maps, then the composition map $\hbar \circ g: L \to N$ is perfectly fsgb-ℂℕ map.

Proof : Let Q be fsgb-OS in N. Then $\hbar^{-1}(Q)$ is fsgb-OS in M , as $\hbar$ is fsgb-irr map . Also g is perfectly fsgb-ℂℕ , $g^{-1}(\hbar^{-1}(Q)) = (\hbar \circ g)^{-1}(Q)$ is f-OS and f-CS in L. Thus $\hbar \circ g$ is perfectly fsgb-ℂℕ map.

5. Completely Fsgb-continuous mappings in FTS.

Definition 5.1: A mapping $g: L \to M$ is known as completely fsgb- ℂℕ , if the inverse of every fsgb-OS in M is fg-OS in L.

Theorem5.2: A mapping $g: L \to M$ is completely fsgb-ℂℕ iff the inverse of every fsgb-CS in M is f-OS and also f-CS in L.

Proof: Consider g is completely fsgb-ℂℕ map. Let P be fsgb-CS in M. Then 1-P is fsgb-OS in L. Hence $g^{-1}(1 - P)$ is f-OS and also f-CS in L. And $g^{-1}(1 - P) = 1 - g^{-1}(P)$, so $g^{-1}(P)$ is f-OS and also f-CS in L.

Conversely, consider that the inverse of every fsgb-CS in M is f-OS and also f-CS in L. Let Q be fsgb-OS in M , then 1-Q is fsgb-CS in M . By proposition, $g^{-1}(1 - P)$ is f-OS and f-CS in L. Now $g^{-1}(1 - Q) = 1 - g^{-1}(Q)$, so $g^{-1}(Q)$ is f-OS and f-CS in L. Therefore g is completely fsgb-ℂℕ map.

Theorem5.3: Every completely fsgb-ℂℕ is a fuzzy-ℂℕ map.

Proof: Let $g: L \to M$ be completely fsgb-ℂℕ map. Let Q be f-OS in M, and so Q is fsgb-OS in M. Then $g^{-1}(Q)$ is f-OS and f-CS in L. Therefore $g^{-1}(Q)$ is fuzzy-OS in L. Hence g is f-ℂℕ map.

The illustration below shows that the inverse implication of the above theorem is incorrect.

Example5.4: From the example3.4, the map g is f-ℂℕ but not completely fsgb-ℂℕ .Since the f- set $1 - R = \{(x, 0.3), (y, 0.3), (z, 0)\}$ is fsgb-OS in M and $g^{-1}(1 - R) = 1 - R$ which is not f-OS and also f-CS in L.

Theorem5.5: Every completely fsgb-ℂℕ map is a fuzzy completely-ℂℕ map.

Proof: Consider $g: L \to M$ be completely fsgb-ℂℕ map. Let P be f-OS in M .And then g be fsgb-OS in M .Then $g^{-1}(P)$ is fuzzy generalized-OS in L. Thus g is f- completely ℂℕ map.

The illustration below shows that the inverse implication of the above theorem is incorrect.

Example5.6: Let L=M=$\{x, y, z\}$ and f-sets P, Q , R and S be defined as follows. $P = \{(x, 0.3), (y, 0.5), (z, 0.7)\}$, $Q = \{(x, 0.2), (y, 1), (z, 0.9)\}$, $R = \{(x, 0.5), (y, 0.7), (z, 0)\}$ and $S = \{(x, 0.1), (y, 0.4), (z, 0.4)\}$ let $\tau = \{0,1, P, Q, R\}$ and $\sigma = \{0,1, S\}$ .Then (L, τ), (M, σ) are fts. Thus mapping $g: L \to M$ defined by $g(x) = x$, $g(y) = y$ and $g(z) = z$.Then g is completely f-ℂℕ. Since the f-set Q is f-OS in M and $g^{-1}(Q) = Q$ is fg-OS in L . But g is not completely fsgb-ℂℕ , since S is fsgb-OS in M and $g^{-1}(1 - R) = 1 - R$ is not fg-OS in L.

Theorem5.7: Every completely fsgb-ℂℕ map is strongly fsgb- ℂℕ map.

Proof: Suppose $g: L \to M$ be completely fsgb-ℂℕ map. Let Q be fsgb-open set in M. Then $g^{-1}(Q)$ is fuzzy generalized-OS in L. Thus $g^{-1}(Q)$ is fuzzy-OS in L. Therefore g is strongly fsgb-ℂℕ map.

The illustration below shows that the inverse implication of the above theorem is incorrect.

Example5.8: From the example 3.6, the map g is strongly fsgb-ℂℕ map but not completely fsgb-ℂℕ , since the f-set Q is fsgb-OS in M and $g^{-1}(Q) = Q$ is not fuzzy generalized-CS in L.

Theorem5.9: Let $g: L \to M, \hbar: M \to N$ be 2 completely fsgb-ℂℕ maps, then $\hbar \circ g: L \to N$ is completely fsgb-ℂℕ map.

Proof: Let R be fsgb-OS in N. Then $\hbar^{-1}(R)$ is fuzzy generalized-OS in M. As $\hbar$ is completely fsgb-ℂℕ , $\hbar^{-1}(R)$ is f-OS and then fsgb-OS in M. Also g is completely fsgb-ℂℕ, $g^{-1}(\hbar^{-1}(R)) = (\hbar \circ g)^{-1}(R)$ is fuzzy generalized-OS in L .Thus $\hbar \circ g$ is completely fsgb-ℂℕ map.

Theorem5.10: If $g: L \to M$ be completely fsgb-ℂℕ and $\hbar: M \to N$ be fsgb-irresolute maps, then the composition map $\hbar \circ g: L \to N$ is completely fsgb-ℂℕ map.

Proof: Let Q be fsgb-OS in N. Then $\hbar^{-1}(Q)$ is fsgb-OS in M , as $\hbar$ is fsgb-irr map. Also g is completely fsgb- ℂℕ , $g^{-1}(\hbar^{-1}(Q)) = (\hbar \circ g)^{-1}(Q)$ is fuzzy generalized-OS in L. Therefore $\hbar \circ g$ is completely fsgb-ℂℕ map.

Acknowledgements

The authors are grateful to principal of SDMCET, Dharwad and management SDM society for their support.

References

[1] D.Andrijevic,On b-open sets, Mat.vesnik ,48 (1996), 59-64.

[2] K. K. Azad, Fuzzy Hausdroff spaces and fuzzy perfect mappings, JI. Math. Anal. Appl. 82 No.1 (1981), 297–305.

[3] G. Balasubramanian and P.Sundaram, On some generalizations of fuzzy continuous, Fuzzy Sets and

International Journal of Applied Engineering Research ISSN 0973-4562 Volume 18, Number 4 (2023) pp. 262-265
© Research India Publications. https://dx.doi.org/10.37622/IJAER/18.4.2023.262-265

Systems, 86 (1997), 93-100.

[4] S.S.Benchalli and Jenifer Karnel,On Fuzzy b-open sets in Fuzzy Topological Spaces,J.Computer and Mathematical Sciences,1,2010,127-134.

[5] S.S.Benchalli and Jenifer Karnel,On Fuzzy b-Neighbourhoods and Fuzzy b-Mappings in Fuzzy Topological Spaces,J.Computer and Mathematical Sciences1(6),2010,696-701.

[6] A. S. Bin Shahna, On fuzzy strong continuity and fuzzy pre-continuity, Fuzzy sets and systems, 44 (1991), 303–308.

[7] C.L.Chang, Fuzzy topological spaces, J. Math. Anal. Appl. (1968), 182-190.

[8] Jenifer J Karnel and Megha Kulkarni, On Fuzzy Strongly Generalized b-closed sets in fuzzy topological spaces , International Journal of Advanced and Innovative Research Volume 6,Issue 2, April-June 2019.

[9] Jenifer J Karnel and Megha Kulkarni , On Fsgb-Continuous , Irresolute , Open and Closed Mappings in Fuzzy Topological Spaces.(communicated).

[10] M. N. Mukherjee and B. Ghosh , Some stronger forms of fuzzy continuous mappings in fuzzy topological spaces, Fuzzy sets and systems, 38(1990) 375–387.

[11] L.A.Zadeh, Fuzzy sets,Inform.Contr.,8,1965, 338-353.