International Journal of Applied Environmental Sciences
ISSN 0973-6077 Volume 11, Number 5 (2016), pp. 1273-1284

http://www.ripublication.com

Streambank Erosion Prediction for Natural River Channels

Azlinda Saadon[a,*], Junaidah Ariffin[b] and Jazuri Abdullah[c]

[a] *Faculty of Civil Engineering, Universiti Teknologi Mara (UiTM), Malaysia*

[b] *Faculty of Civil Engineering, Universiti Teknologi Mara (UiTM), Malaysia*

[c] *Faculty of Civil Engineering, Universiti Teknologi Mara (UiTM), Malaysia*

Abstract

Streambank erosion is greatly associated as one of the crucial threat to major scouring of hydraulic structure. It is a complex problem in river engineering studies as it requires integration from various fields of engineering. Studies of streambank erosion provide a key process in fluvial dynamics affecting the physical, ecological, and socio-economic. The interaction between the fluid dynamic factors and the physical characteristics of the streambank soil greatly influence the streambank erosion rates. In consideration to the above, this study intended to establish the streambank erosion prediction for natural river channels.

Keywords: Streambank erosion, rate of streambank erosion, lateral migration.

1. BACKGROUND/ OBJECTIVES AND GOALS

Streambank erosion is one of the complex problems in river engineering studies that require integration of various field of engineering. It is a key process in fluvial dynamics affecting the physical, ecological and socio-economic. The studies of bank erosion serve as a platform to other various studies in fluvial environment. This includes evolution of meandering (e.g. Duan and Julien, 2010), meander or channel migration (e.g. Hasegawa, 1989; Randle, 2006; Constantine et al., 2009; Posner and Duan, 2012), and river bank stability (Osman and Thorne, 1988; Darby et al., 2002; Parker, 1982). One of the earliest approaches addressing bank erosion of alluvial channel on rate of erosion has been conducted by Ikeda et al. (1981). An equation predicting the channel shift has been derived using a simple two-dimensional shallow water flow model. The rate of bank erosion is taken to be proportionally to the excess of near-bank depth-averaged streamwise flow velocity over the cross-sectional mean velocity. Recent studies however shown the contribution of river bank erosion is significant towards the evolution of river and floodplain morphology. It is because the study of river bank erosion will help to quantify the rate of erosion due to fluvial entrainment or the river bank itself. It was indicated that the bank erosion consist of two processes; basal erosion due to fluvial force and bank failure under the influence of gravity. A study conducted by Duan (2006) successfully derived a method for calculating the rate of erosion that integrates both basal erosion and bank failure processes. It includes the effects of hydraulic force, bank geometry, bank materials properties and probability of bank failure, without looking into the physical characteristics of the bank properties. Knowing the effect of such factors to the contribution of the rate of bank erosion, knowledge of the rates, patterns and controls on river bank erosion events is a pre-requisite to a complete understanding of the fluvial system. Field data and experiment data are very useful in presenting the mechanism with regards to bank erosion. They also serve basic fundamental in developing future predictions by means of simulations in any modeling. However, the basic principle in any modeling require the identification of the relevant variables and then relating these variables via known physical laws, and one of the most powerful modeling methods is dimensional analysis. This paper describes the methodology employed for streambank erosion measurement based on fieldwork investigation and streambank erosion rates prediction using empirical method. Fieldwork measurement has been conducted in order to collect and quantify the streambank erosion rates for model development.

2. METHODS

There are two (2) rivers selected for this research and both are located within the river basin in Selangor, namely Sungai (Sg.) Bernam at Tanjung Malim, Hulu Selangor and Sungai (Sg.) Lui at Kampung Sg. Lui, Hulu Langat. Both of the rivers are prone to major flooding within the states of Selangor. Field work of streambank erosion monitoring site has been conducted for duration of 12 months. Data from both sites are used in the streambank erosion empirical model.

2.1 Streambank Erosion Monitoring and Study Area

The assessment of streambank erosion was made at the areas identified as actively undergoing the process of erosion. The reach length for both selected sites are 50m and divided into six transects every 10 meters interval. For Sg. Bernam, the reach is located 200m upstream from the confluence of Sungai Inki. For Sg. Lui, the reach is located five kilometers away from joining the main river, Sungai Langat. The reach evidenced streambank erosion processes due to hydraulic actions from fluvial entrainment and streambank undercutting or scouring. Two different methods were used to determine the rates of streambank erosion and channel lateral changes along the selected reach, namely cross profiling and the used of conventional erosion pins. For this study, it was not considered appropriate to compare the streambank erosion rates measured using repeated cross profiling with those from conventional erosion pin plots, as both methods were measured at different scales of time and space at different location within the reach. Figure 2.1 and Figure 2.2 shows detail aerial image of the study area. Six data categories were measured during the streambank erosion monitoring of the selected reach. Table 2.1 shows the types and description of measured data.

Fig. 2.1: Aerial view of the reach of Sungai Bernam, Hulu Selangor.

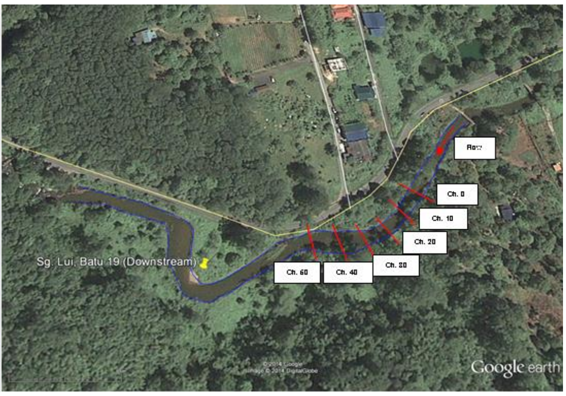

Fig. 2.2: Aerial view of the reach of Sg. Lui, Kampung Sg. Lui, Hulu Langat.

Table 2.1: Data category and detail data description for the assessment and prediction of the streambank erosion rates.

No	Data Category	Detail Data Description	Equipment
1.	Erosion monitoring	▪ Erosion rates	▪ Repeated cross profiling ▪ Vertical streambank profiling ▪ Conventional erosion pins
2.	Hydraulic	▪ Discharge ▪ Flow velocity ▪ Reach slope ▪ Reach length ▪ Reach profile / banklines ▪ Boundary shear stress	▪ OTT Q-liner ▪ Velaport velocity meter ▪ Auto level
3.	Bank Geometry	▪ Height of bank ▪ Bank angle ▪ Bank toe length	▪ Auto level ▪ Direct measurement
4.	Soil Capacity	▪ Critical shear stress ▪ Soil erodibility	▪ Jet test apparatus

5.	Grain Resistance	▪ Bank and bed material ▪ Roughness height	▪ Van Veen grab sampler
6.	Sediment	▪ Suspended load ▪ Bed load	▪ Portable suspended load sampler ▪ Helley Smith sampler

2.2 Streambank Erosion Rates

There are two important parameters related in quantifying the streambank erosion rates (Hooke, 1979): (1) the rate at which the erosive processes are operating, and (2) the proportion of bank under attack (for instance the measurement of erosion pins recording the change). This subject matter has been very popular among researchers, Lawler (1993a) have listed 168 papers based on the measurement techniques for streambank erosion made since 1863. Findings from field measurements that were undertaken in a period of 12 months (October 2014 to October 2015) has been recorded and presented. Three conventional monitoring methods have been employed in a corresponding way to quantify the streambank erosion rates. These methods have been selected to balance up some limitations that the individual methods have. The methods used are (1) repeated cross profiling; (2) vertical streambank profiling; and (3) erosion pins method. This mixture of field monitoring techniques was employed in order to capture the bigger picture on the changes to the selected sites. However, it was not feasible to implement all the methods at each site and observations based on the limitations of these methods. For this study, it was not considered appropriate to compare the streambank erosion rates from one method and another, as it was employed measuring streambank erosion at a different scale of time and space within the watershed.

2.3 Development of Streambank Erosion Rates Model

In order to develop an empirical model for streambank erosion rates, six articles are critically reviewed. The works of previous researchers include Hasegawa (1989), Randle (2006), Duan (2006), Constantine et al. (2009), Duan and Julien (2010) and Posner and Duan (2012). Based on the physical considerations on river bank erosion processes, the primary factors governing the rate of streambank erosion, ξ along a channel can be divided into five (5) major categories; bank geometry, hydraulic, soil capacity (resistance to erosion), grain resistance and others. The parameters for bank geometry consist of channel width, B, water depth, D, streambank height, h_b, streambank angle, β, and channel slope, S_o. The parameters for hydraulic consist of streambank erosion rate, ξ, streambank depth-averaged velocity, u_b, and boundary shear stress, τ_o. The parameters for soil capacity (resistance to erosion) include critical shear stress, τ_c, porosity, p, and plasticity index, PI. The grain resistance includes mean particle diameter, d_{50}, fall velocity, ϖ, shear velocity, u_*, and concentration of suspended load to equilibrium suspended concentration, C. Other variables include gravity acceleration, g, water density, ρ_w, streambank particle density, ρ_s. The rate of bank erosion, ξ serves as the dependent variable. There are eighteen variables selected and three fundamental quantities involved in the relationship.

3. RESULTS

The streambank erosion monitoring data is obtained by employing stream gauging technique, visual inspection of the surveys, cross-sectional profiling survey, and conventional use of erosion pin arrays. The assessment of streambank erosion has been conducted for a period of 12 months beginning October 2014 to October 2015 for Sg. Bernam, Tanjung Malim, Hulu Selangor. A preliminary study has also been conducted at Sg. Lui, Kg. Sg. Lui, Hulu Langat for a period of 3 months between May 2014 until August 2014.

3.1 Summary of streambank erosion monitoring data

Six data categories have been measured at the selected sites; 1) streambank geometry, 2) hydraulic, 3) soil capacity, 4) grain resistance, and 5) others. Table 3.1 shows the summary of streambank erosion monitoring data measured for both sites.

3.2 Empirical equations for streambank erosion rates

Selected streambank erosion parameters from the field measurement are used in the derivation of empirical equations for streambank erosion rates using the Buckingham Pi Theorem. In this analysis, a dimensionless erosion rates function has been derived from dimensional analysis using three repeating variables, as follows:-

a) Dimensional erosion rates function: for repeating variables: u_b, ρ_w, d_{50} :

$$\frac{\xi}{u_b} = f\left[\frac{B}{d_{50}}, \frac{D}{d_{50}}, \frac{h_b}{d_{50}}, \beta, S_o, \frac{\tau_o}{\rho_w\, u_b^{\,2}}, Fr_c, \frac{\omega}{u_b}, \frac{U_*}{u_b}, C, \frac{\rho_s}{\rho_w}, \frac{gd_{50}}{u_b^{\,2}}\right] \qquad (1)$$

These equations were analysed in terms on their correlations between the independent variables to the dependent variable. The dependent variable selected for this study is a dimensionless erosion rates with respect to streambank velocity, $\frac{\xi}{u_b}$. For model development, the data was abstracted from the measurement taken at the selected erosion monitoring sites. The first location is along 50 m reach of Sg. Lui at Kampung Sg. Lui, Hulu Langat and second location is at Sg. Bernam, Tanjung Malim, Hulu Selangor. From the erosion monitoring measurement for a period of 12 months, a total of 494 data has been collected for both rivers accounting for wet and dry seasons. However, only 318 data was used for model development or streambank erosion rates empirical model, and 176 data was used for model performance and verification. Table 3.2 and 3.3 show the distribution of streambank monitoring data and distribution of data for model development and model performance.

Table 3.1: Summary of streambank erosion monitoring data

Location	Date	Width, W	Water Depth, Y	Velocity, V	Discharge, Q	Streambank height, h_b		Streambank slope, S_o		Mean size particle, d_{50}	Suspended load, C	Streambank erosion rates	
		(m)	(m)	(m/s)	(m^3/s)	(m)	(m)	(º)	(º)	(mm)	-	(m/year)	(m/year)
						Left	Right	Left	Right			Left	Right
Sg. Lui (L1)	30/5/14	9.0 – 13.0	0.36 – 0.60	0.39 – 0.79	1.90 – 4.10	2.00 – 2.50	1.54 – 1.98	53.13 – 68.85	51.98 – 65.96	1.350 – 1.812	5.2 x 10^{-7} – 7.4 x 10^{-7}	0.045 – 0.372	0.045 – 0.372
	11/6/14	9.0 – 13.1	0.28 – 0.59	0.37 – 0.69	1.70 – 2.70	1.80 – 2.40	1.54 – 1.98			1.390 – 1.638			
	11/7/14	9.0 – 13.0	0.28 – 0.73	0.39 – 0.73	1.80 – 3.80	1.70 – 2.30	1.54 – 1.98			1.450 – 1.650			
Sg. Bernam (B1)	23/10/14	10.0 – 15.0	0.36 – 0.79	0.24 – 0.33	0.99 – 3.90	3.45 – 3.87	3.30 – 3.83	32.00 – 85.00	35.00 – 91.00	0.904 – 1.870	5.5 x 10^{-7} – 7.4 x 10^{-7}	0.052 – 0.782	0.521 – 1.564
	30/11/14	10.0 – 15.5	0.35 – 0.79	0.36 – 0.42	1.40 – 4.70	3.45 – 3.87	3.30 – 3.83			0.904 – 1.890		0.010 – 0.493	0.020 – 0.168
	24/12/14	10.0 – 15.0	0.35 – 0.79	0.38 – 0.69	1.40 – 4.50	3.45 – 3.85	3.45 – 3.82			0.906 – 1.870		0.006 – 0.677	0.012 – 1.766
	24/1/15	10.0 – 15.5	0.35 – 0.79	0.27 – 0.46	1.20 – 5.20	3.45 – 3.87	2.90 – 3.83			0.904 – 1.870		0.004 – 0.451	0.039 – 1.171
	15/2/15	10.0 – 15.0	0.35 – 0.79	0.27 – 0.40	1.20 – 4.40	3.45 – 3.87	2.90 – 3.83			0.904 – 1.870		0.003 – 0.365	0.032 – 0.952
	5/3/15	12.0 – 15.4	0.34 – 0.47	0.26 – 0.35	1.41 – 1.90	3.34 – 3.88	2.90 – 3.83			0.902 – 1.870		0.003 – 0.316	0.027 – 0.823
	12/4/15	12.0 – 15.4	0.34 – 0.47	0.27 – 0.41	1.30 – 2.40	3.35 – 3.88	2.80 –3.83			0.902 – 1.840		0.002 – 0.245	0.002 – 0.224
	31/5/15	12.0 – 15.4	0.34 – 0.47	0.40 – 1.10	2.20 – 4.50	3.34 – 3.87	2.75 – 3.83			0.902 – 1.880		0.002 – 0.133	0.002 – 0.174
	7/6/15	12.0 – 15.4	0.35 – 0.47	0.40 – 1.10	2.20 – 5.30	3.34 – 3.87	2.90 – 3.83			0.904 – 1.870		0.016 – 0.080	0.002 – 0.273
	31/7/15	12.0 – 15.4	0.35 – 0.47	0.50 – 1.01	2.80 – 4.70	3.35 – 3.87	2.90 – 3.83			0.902 – 1.880		0.006 – 0.260	0.001 – 0.136
	30/8/15	12.0 – 15.4	0.35 – 0.47	0.60 – 0.93	3.20 – 5.50	3.45 – 3.88	2.90 – 3.83			0.904 – 1.870		0.004 – 0.411	0.012 – 0.352
	15/9/15	12.0 – 15.5	0.35 – 0.47	0.60 – 0.99	2.50 – 4.60	3.45 – 3.88	2.90 – 3.83			0.904 – 1.870		0.011 – 0.504	0.011 – 0.336
	24/10/15	10.0 – 15.5	0.35 – 0.75	0.59 – 1.04	1.90 – 4.50	3.34 – 3.88	2.90 – 3.83			0.904 – 1.870		0.010 – 0.450	0.010 – 0.300

Table 3.2: Total number of streambank erosion monitoring data

Location	Monitoring period	Number of data
1. Sg. Lui, Kampung Sg. Lui, Hulu Langat	July 2014 – August 2014	318
2. Sg. Bernam, Tanjung Malim, Hulu Selangor	October 2014 – October 2015	176
Total number of data up to October 2015		**494**

Table 3.3: Distribution of data for empirical model development and model performance

Location	Number of data for empirical model development	Number of data for model performance
1. Sg. Lui, Kampung Sg. Lui, Hulu Langat	58	78
2. Sg. Bernam, Tanjung Malim, Hulu Selangor	260	98
Total number of data	**318**	**176**

3.3 Multiple Linear and Non-Linear Regressions

Multiple regression analysis provides a useful tool for identification of significant relationships. The model predictors described above are used in a multiple linear and non-linear regression analysis. From all 318 data for model the development, total of 21 data were removed due to outliers. The remaining 297 data were used in the model development. Each independent variable individually is regressed against the dependent variable. Multiple linear and non-linear regression analysis is applied to different combinations of the independent variables. A total of 70 regression equations have been derived from the analysis. Out of these 70 equations, 10 equations have been selected to best representation of the analysis employed using this method.

3.4 Model Performance and Verification

Linear and non-linear multiple regression has been conducted in order to obtain a significant relationship between combinations of independent variables to the dependent variable. A total number of 176 data were used in the performance test consist of both sites data. There should be a good agreement between the predicted and measured values. This can be obtained from the distribution pattern of the data in which the scatter plots should be homoscedastic and possess a high positive correlation within the acceptable limit. In addition to the above, the accuracy of the equations are measured based on the discrepancy ratio of 0.5 – 2.0 limit. These ratios of the predicted values to versus measured values are deemed accurate if the data lie within this limit. Based on these criteria, 10 equations fit 78% and above of the discrepancy ratio in the performance test.

The results of the performance test show prediction accuracies for all 10 equations are above 78%. The plots of all equations demonstrate a good correlation between the predicted data and the observed data. Almost all of the data fall within the acceptable range of discrepancy ratio (0.5 – 2.0). Table 3.4 shows the summary of developed equations and performance test results for each equation. Based on the derived model, non-linear regression model (Equation 7) exhibits the highest coefficient of correlation, R^2 value of 0.409 compared to the linear regression model. Although the value of R^2 is lesser than 0.6, indicated that no relation between compared variables, the validations of the discrepancy ratio yield high percentage of accuracy for both methods. Equation 5 exhibits the highest percentage of 84% accuracy with well distributed graphical plots. Therefore, the regressed model was proven to have significant relationship between the dependent parameter against the independent parameters. Figure 3.1 and 3.2 show the scatter plots of the performance test.

Table 3.4: Summary of the derived model using multiple linear regressions

Dependent variable	Derived model (Linear model)	Eqn. No.	R^2	Standard deviation	D.R. (%)
$\frac{\xi}{u_b}$	$6.010\times10^{-9}+3.248\times10^{-7}\left(\frac{\omega}{u_b}\right)+0.003\,(C)$	1	0.277	0.997	77.8
	$7.338\times10^{-11}\left(\frac{\omega}{u_b}\right)^{12.370}\left(\frac{\tau_c}{\rho_w u_b^2}\right)^{-5.609}\left(\frac{B}{d_{50}}\right)^{-1.117}\left(\frac{Y}{d_{50}}\right)^{0.049}(S_o)^{-0.149}$	2	0.398	0.094	80.7
	$1.344\times10^{-10}\left(\frac{\omega}{u_b}\right)^{13.182}\left(\frac{\tau_c}{\rho_w u_b^2}\right)^{-6.040}\left(\frac{B}{d_{50}}\right)^{-1.218}$	3	0.393	0.211	81.3
	$7.644\times10^{-10}\left(\frac{\omega}{u_b}\right)^{4.490}\left(\frac{\tau_c}{\rho_w u_b^2}\right)^{-1.741}\left(\frac{Y}{d_{50}}\right)^{0.123}$	4	0.328	0.128	80.1
	$1.328\times10^{-11}\left(\frac{\omega}{u_b}\right)^{11.319}\left(\frac{\tau_c}{\rho_w u_b^2}\right)^{-5.064}\left(\frac{h_b}{d_{50}}\right)^{-0.717}$	5	0.368	0.159	84.0
	$1.593\times10^{-10}\left(\frac{\omega}{u_b}\right)^{12.914}\left(\frac{\tau_c}{\rho_w u_b^2}\right)^{-5.907}\left(\frac{B}{d_{50}}\right)^{-1.212}\left(\frac{Y}{d_{50}}\right)^{0.034}$	6	0.393	0.121	81.3
	$2.014\times10^{-11}\left(\frac{\omega}{u_b}\right)^{15.660}\left(\frac{\tau_c}{\rho_w u_b^2}\right)^{-7.217}\left(\frac{B}{d_{50}}\right)^{-1.015}\left(\frac{h_b}{d_{50}}\right)^{-0.459}$	7	0.409	0.164	80.1
	$1.333\times10^{-10}\left(\frac{\omega}{u_b}\right)^{13.183}\left(\frac{\tau_c}{\rho_w u_b^2}\right)^{-6.041}\left(\frac{B}{d_{50}}\right)^{-1.216}(\beta)^{0.002}$	8	0.393	0.120	81.3
	$5.832\times10^{-11}\left(\frac{\omega}{u_b}\right)^{12.773}\left(\frac{\tau_c}{\rho_w u_b^2}\right)^{-5.810}\left(\frac{B}{d_{50}}\right)^{-1.128}(S_o)^{-0.145}$	9	0.397	0.093	81.3
	$1.534\times10^{-10}\left(\frac{\omega}{u_b}\right)^{12.840}\left(\frac{\tau_c}{\rho_w u_b^2}\right)^{-5.870}\left(\frac{B}{d_{50}}\right)^{-1.212}\left(\frac{Y}{d_{50}}\right)^{0.044}(\beta)^{0.022}$	10	0.393	0.044	81.2

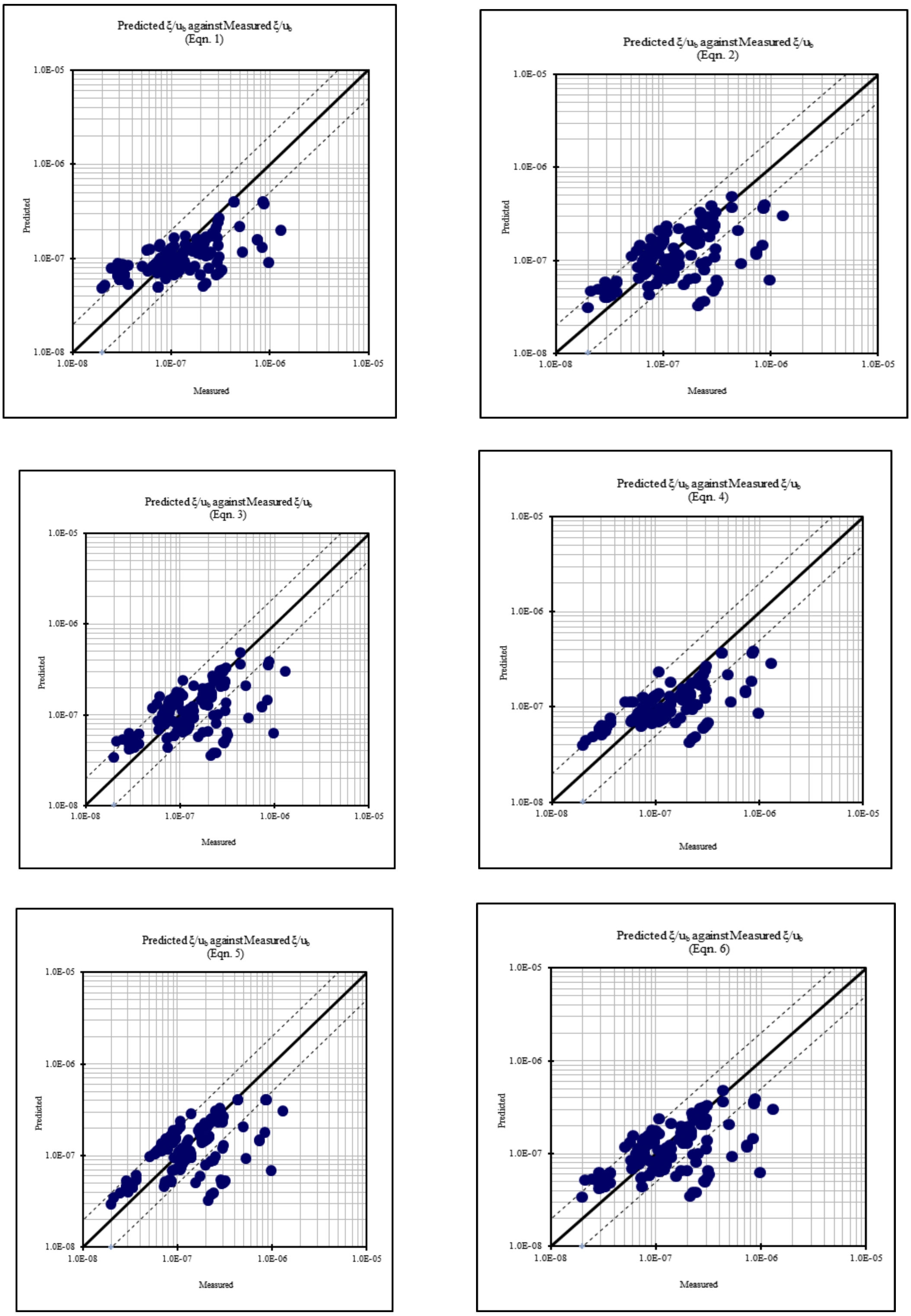

Fig. 3.1: Graphs of predicted against measured dimensionless erosion rates (Equation 1 – 6)

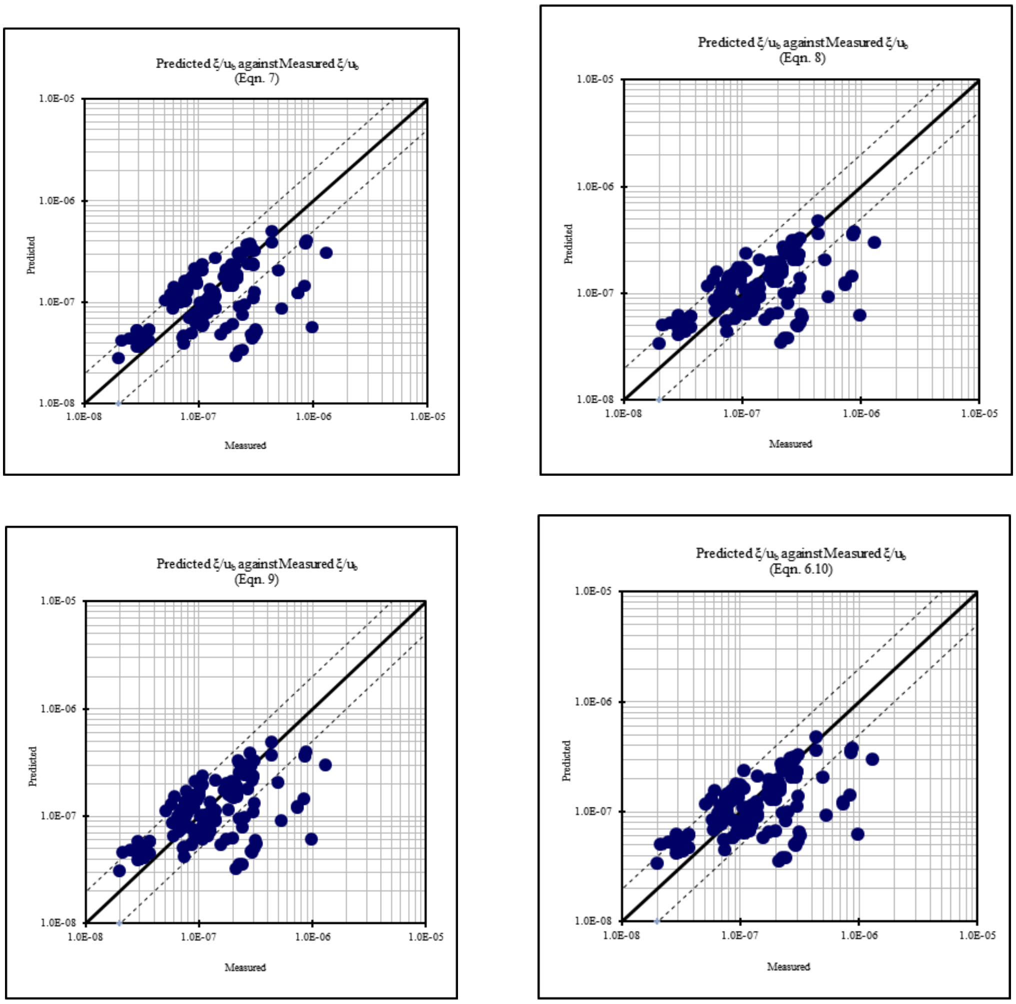

Fig. 3.2: Graphs of predicted against measured dimensionless erosion rates (Equation 7 – 10)

CONCLUSION

This paper describes the methodology employed for streambank erosion measurement based on fieldwork investigation and streambank erosion rates prediction using empirical method. Fieldwork measurement has been conducted in order to collect and quantify the streambank erosion rates for model development. The regression analysis employed in the derivation of factors that governs the rate of streambank erosion. Functional relationship between independent variables to the dependent variables has been established using dimensional analysis. Multiple linear and non-linear regression analysis was performed using 494 data of streambank erosion monitoring for Sg. Lui and Sg. Bernam. From the analysis, a total of 70 regression equations were derived, and 10 equations has been selected to best represent the results. All 10 regression equations were tested for their individual performances based on graphical plot distribution and discrepancy ratio using 176 data. It can be concluded that Equation 5 exhibits the highest prediction accuracies of 84%. Equation 5 demonstrates a relationship of a dimensionless form of fall velocity parameter, critical shear stress parameter and streambank geometry as factors governing the streambank erosion rates.

$$\xi = 1.328 \times 10^{-11} \left(\frac{\omega}{u_b} \right)^{11.319} \left(\frac{\tau_c}{\rho_w u_b{}^2} \right)^{-5.064} \left(\frac{h_b}{d_{50}} \right)^{-0.717} \cdot u_b \qquad (2)$$

This prediction of streambank erosion is important and can greatly be associated with the rate of meander migration and the evolution of the river meandering processes and thus serves as an important aspect in river engineering study.

ACKNOWLEDGEMENT

The first author is a PhD candidate of Faculty of Civil Engineering, Universiti Teknologi MARA, Malaysia and this paper is partially of her research.

REFERENCES

[1] Constantine, C. R., Dunne, T., & Hanson, G. J. (2009). Examining the physical meaning of the bank erosion coefficient used in meander migration modeling, *Geomorphology*, *106* (3-4), 242–252.

[2] Crossato, A. (1987). Simulation model of meandering processes of rivers. Extended Abstract of the International Conference: *Euromech 215-Mechanics of Sediment Transport in Fluvial and Marine Environments,* European Mechanics Society, 158-161.

[3] Crossato, A. (1990). Simulation of meandering river processes, communications on hydraulic and geotechnical engineering, Technical Rep., Civil Engineering Dept., Delft Univ. of Technology, Delft, The Netherlands

[4] Darby, S. E. (2002). A model of equilibrium bed topography for meander, 1085, 1057–1085.

[5] Darby, S.E., Alabyan, A.M., & Van De Wiel, M.J. (2002). Numerical simulation of bank erosion and channel migration in meandering rivers, Water Resources, *38* (9), 2-1-2-12.

[6] Duan, J. G. (2006). Analytical approach to calculate rate of bank erosion, *Journal of Hydraulic Engineering, 131*(11), 980–990.

[7] Duan, J. G., & College, C. (2002). The applications of the enhanced CCHE2D model to study the alluvial channel mi- gration processes les chenaux alluviaux, *39*(5), 469–480.

[8] Duan, J.G., & Julien, P. Y. (2005). Numerical simulation of the inception of channel meandering, *Journal of Hydrology, 30*, 1093 – 1110.

[9] Duan, J.G., & Julien, P. Y. (2010). Numerical simulation of meandering evolution, *Journal of Hydrology, 391*(1-2), 34–46.

[10] Hasegawa, K. (1989). Universal bank erosion coefficient for meandering rivers, *Journal of Hydraulic Engineering,* ASCE, *115* (6), 744-765.

[11] Ikeda, S., Parker, G., & Sawai, K. (1981). Bend theory of river meanders: part I, linear development, *Journal of Fluid Mechanic, 112,* 363-377.

[12] Johannesson, H., & Parker, G. (1989a). Secondary flow in mildly sinuous channel, *Journal of Hydraulic Engineering, 115* (3), 289 – 308.

[13] Johannesson, H., & Parker, G. (1989b). Velocity redistribution in meandering rivers, *Journal of Hydraulic Engineering, 115* (8), 1019 – 1039.

[14] Larsen, E.W., & Greco, S.G. (2002). Modeling channel management impacts on river migration: a case study of Woodson bridge state recreation area, Sacramento River, California, USA, *Environmental Management, 30*(2), 209 – 224.

[15] Larsen, E.W. (1995). Mechanics and modeling of river meandering migration, *PhD. Thesis*, Colorado State Univ., Fort Collins, Colo.

[16] Odgaard, B. A. J., & Asce, M. (1987). Meander flow model. II: applications, *112*(12), 1137–1149.

[17] Odgaard, A.J. (1989a). River meander model. I: development,. *Journal of Hydraulic Engineering, 115* (11). 1433 – 1450.

[18] Odgaard, A.J. (1989b). River meander model. II: applications, *Journal of Hydraulic Engineering, 115* (11). 1451 – 1464.

[19] Osman, M.A. and Thorne, C.R. (1988). Riverbank stability analysis. I: theory, *Journal of Hydraulic Engineering. 114* (2). 134 - 150.

[20] Park, N. (2007). *A Prediction of Meander Migration based on Large-Scale Flume Test in Clay*. A Dissertation. Texas A&M University.

[21] Parker, G. (1982). Stability of the channel of the minnesota river near State Bridge No. 93, Minnesota. Project Report No. 205, St. Anthony Fall Hydr. Lab. Univ. of Minnesota, Minneapolis, Minn, 33.

[22] Posner, & J. Duan. (2012). Simulating river meandering processes using stochastic bank erosion coefficient, *Geomorphology*, (163 - 164), 26 – 36.

[23] Randle, T.J. (2006). Channel migration model for meandering rivers, Proceedings of the Eight Federal Interagency Sedimentation Conference, April 2-6, 2006, Reno, NV, USA.