International Journal of Agriculture and Food Science Technology.
ISSN 2249-3050, Volume 4, Number 9 (2013), pp. 851-858

http://www.ripublication.com/ ijafst.htm

Performance Modeling and Availability Analysis of Skim Milk Powder Production System of a Dairy Plant

Anil Kr. Aggarwal[1], Sanjeev Kumar[2] and Vikram Singh[3]

[1]*Mech. Engg. Deptt., YMCAUST, Faridabad, India.*
[2]*Mech. Engg. Deptt., YMCAUST, Faridabad, India.*
[3]*Mech. Engg. Deptt., YMCAUST, Faridabad, India.*

Abstract

The present paper deals with the performance modeling and availability analysis of the serial processes in the skim milk powder production system of a dairy plant. The skim milk powder system consist of six sub-systems namely chiller, cream separator, pasteurizer, evaporator, drying chamber and packaging sub-system. These sub-systems are connected either in series or parallel with each other. Two sub-systems namely evaporator and drying chamber are supported by a standby sub-system with perfect switch over devices and by assuming the non-failure of packaging sub-system, the remaining five sub-systems are subjected to failure. By considering the exponential distribution of failure and repair rate of sub-systems, the mathematical formulation of the model is developed with the use of mnemonic rule for these five sub-systems and Chapman-Kolmogorov differential equations are derived from the transition diagram. These differential equations are solved by using normalizing conditions to compute the availability under steady state condition. Finally, the performance of each sub-system of the system has been analyzed for selecting the best possible maintenance strategies in the plant. The findings of the paper will be highly beneficial to the plant personnel to enhance the system performance by adopting the best possible maintenance strategies.

Keywords: Performance modeling, Chapman-Kolmogorov differential equations, steady state availability..

1. Introduction

Now days, due to automation in the process industries, maintenance is considered to be an integral part of the production. It is done by ensuring high availability level. The long-run availability and reliability of the system helps the management to understand the effects of change in the failure or repair rates of the components in a system. With the increase in the complexity of the system, high reliability can be obtained either by providing sufficient redundant parts or by increasing the capacity of the system.

2. Literature Review

The available literature reveals that several approaches have been used to analyze the system performance in terms of availability and reliability. Singer (1990), Arora and Kumar (1997) discussed the availability analysis of steam and powder generation systems of thermal power plant. Dhillon and Singh (1981) , Kumar et al. (1988, 1989, 1993 and 2007) used the Markov modeling in the analysis and evaluation of the performances of paper and urea fertilizer plants. Gupta et al. (2008) developed the performance models and decision support system for a feed water unit of thermal power plant. Gupta and Tewari (2011) presented the availability model for a thermal power plant. Khanduja et al. (2012) presented the steady state behavior and maintenance planning of the bleaching system of paper plant. Kumar and Tewari (2011) discussed the mathematical modeling and performance optimization of CO_2 cooling system of a fertilizer plant using genetic algorithm.

3. System Configuration and Assumptions

The skim milk powder system comprises of the following five sub-systems in series;

(1) *Sub-system A (Chiller)*: The milk get filtered after testing and chilled to about 5^oC and stored in silos. It is single sub-system connected in series. Failure of this sub-system causes the complete failure of the system. It is provided with pump, compressor etc connected in series.

(2) *Sub-system B (Cream separator)*: The fat from the milk get separated in the form of cream due to difference in density when acted upon by centrifugal force. It is single unit connected in series. Failure of this sub-system causes the complete failure of the system. It is provided with bearings, motor, gearbox connected in series.

(3) *Subsystem C (Pasteurizer)*: Pasteurization is the process of heating milk to 72^oC for 16 seconds then quickly cooling it to 4^oC. Thus, by destroying the bacteria, milk becomes safe to drink. It is single sub-system connected in series. Failure of this sub-system causes the complete failure of the system. It is provided with bearings, motor etc.

(4) *Subsystem D (Evaporator)*: The skim milk is heated with saturated steam under low pressure to reduce the water content. It consists of two sub-systems connected in parallel; one operative and other in cold standby condition. Complete failure of the system will occur when both sub-systems get failed at a time. It is provided with pump, temp. and pressure measuring devices.

(5) *Subsystem E (Drying chamber)*: The concentrated skim milk is passed under pressure through a distributed nozzle; it converts in the form of fine spray or mist. When the particles of mist/spray are subjected to high pressure steam, it gets converted in to powder form. It consists of two sub-systems connected in

parallel; one operative and other in cold standby condition. Complete failure of the system will occur when both sub-systems get failed at a time. It is provided with mechanical vibrator, motor, atomizing devices connected in series.

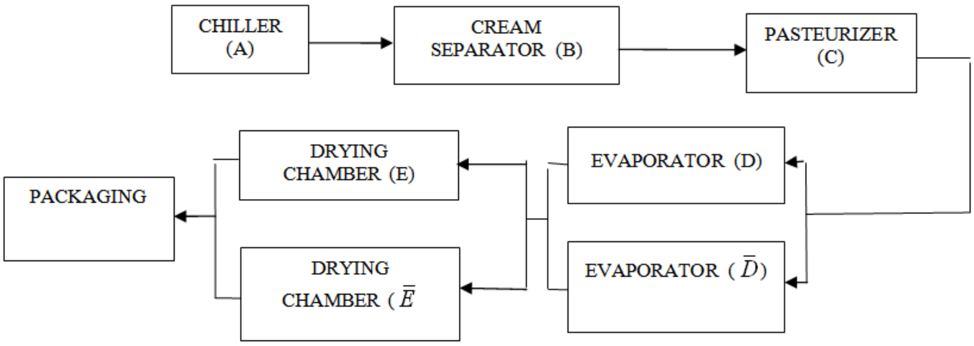


Fig. 1: Schematic flow diagram of skim milk powder system with standby units.

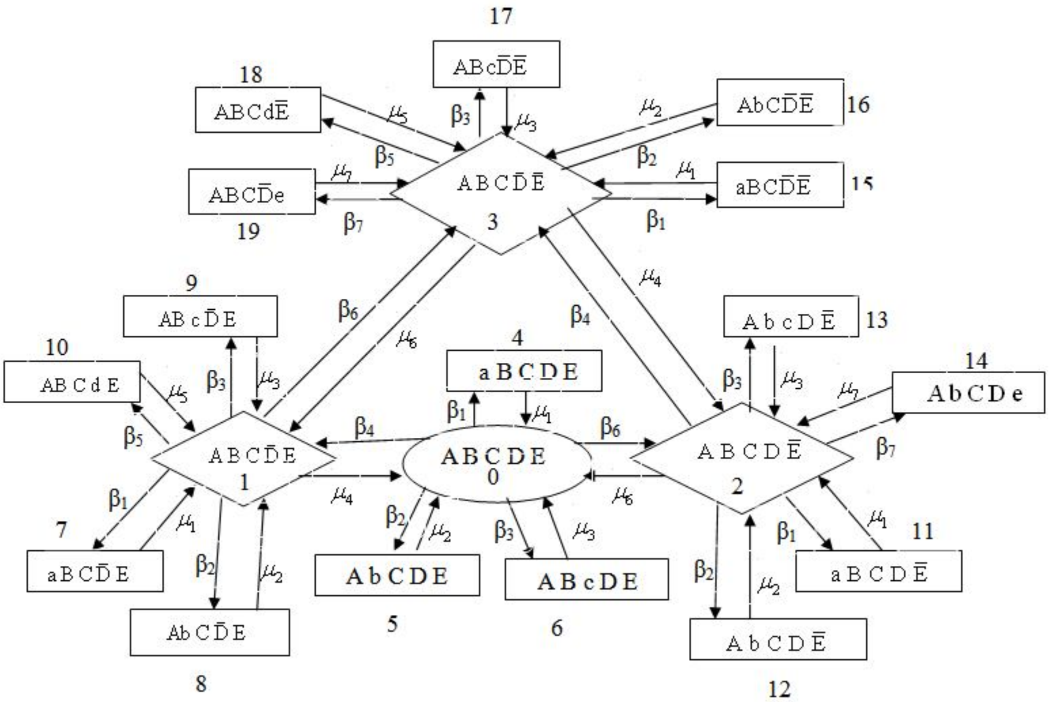
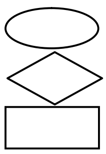


Fig. 2: Transition diagram of skim milk powder system

Notations



: Indicates the system is in full working state.

: Indicates the system is in standby state.

: Indicates the system is in failed state.

: A, B,C,D,E :Indicates full working states of sub-systems.

$\bar{D}$ and $\bar{E}$: Indicates that the sub-system D and E are working under cold standby state.

a,b,c,d,e : Indicates the failed states of sub-systems.
$P_o(t)$:Probability of the system working with full capacity.
P_1 (t), P_2 (t), P_3(t) : Probability of the system working under standby state.

- β_i , i=1,2,3……7, represents, the constant failures rates of sub-systems A, B, C, D, $\bar{D}$, E, $\bar{E}$ resp.
- $\mu_i, i = 1, 2, 3.....7$ represents, the constant repair rates of sub-systems A,B,C,D, $\bar{D}$,E, $\bar{E}$ resp.
- $P_j(t)$, j=1,2,3,…..19 as the probability that the system is in jth state at time t. p' represents its derivative with respect to time (t).
- The symbols a,b,c,d and e represent the failed state of the sub-system A,B,C,D and E resp.

Assumptions;

- The states of all subsystems are mutually independent and the failure and repair rates are constant over time and follows exponential distribution.
- There are sufficient repair or replacement facilities available and there are no simultaneous failures among the sub-systems.
- When one subsystem fails, it is instantaneously replaced by one of the standby subsystems (if any) and the switchover devices are perfect and the repaired sub-system behaves as new sub-system.

4. Mathematical Formulation of the System

The mathematical modeling of the system is carried out to determine the long run availability of skim milk powder system and Chapman-Kolmogorov differential equations are developed on the basis of Markov birth-death process as stated by Kumar et al. (2007 and 2011). The transition diagram (fig.: 2) depicts a simulation model showing all the possible states of skim milk powder system. Mathematical equations, (1) to (8) are developed by applying markov-birth death process to each state one by one out of 19 states of transition diagram (figure 2) as explained by Garg et al. (2008 and 2010).

$$P'_o(t) = - X_oP_o(t) + \mu_1P_4(t) + \mu_2P_5(t) + \mu_3P_6(t) + \mu_4P_1(t) + \mu_6P_2(t) \qquad (1)$$

Similarly;

$$P'_1(t) = - X_1P_1(t) + \mu_1P_7(t) + \mu_2P_8(t) + \mu_3P_9(t) + \beta_4P_o(t) + \mu_5P_{10}(t) + \mu_6P_3(t) \qquad (2)$$

$$P'_2(t) = - X_2P_2(t) + \mu_1P_{11}(t) + \mu_2P_{12}(t) + \mu_3P_{13}(t) + \mu_4P_3(t) + \beta_6P_0(t) + \mu_7P_{14}(t) \qquad (3)$$

$$P'_3(t) = - X_3P_3(t) + \mu_1P_{15}(t) + \mu_2P_{16}(t) + \mu_3P_{17}(t) + \beta_4P_2(t) + \mu_5P_{18}(t) + \beta_6P_1(t) \qquad (4)$$

Where: $X_o= (\beta_1+\beta_2+\beta_3+\beta_4+\beta_6)$; $X_1= (\beta_1+\beta_2+\beta_3+ \mu_4+\beta_5+\beta_6)$; $X_2= (\beta_1+\beta_2+\beta_3+\beta_4+ \mu_6+\beta_7)$

$X_3= (\beta_1+\beta_2+\beta_3+ \mu_4+ \beta_5 + \mu_6+\beta_7)$. Similarly;

$$P'_i(t) + \mu_j P_i(t) = \beta_j P_o(t) \qquad (5);$$

Where, i=4, 5, 6; j=1,2,3

$$P'_i(t) + \mu_j P_i(t) = \beta_j P_1(t) \qquad (6);$$

Where, i=7, 8, 9, 10; j=1, 2, 3, 5

$$P'_i(t) + \mu_j P_i(t) = \beta_j P_2(t) \qquad (7);$$

Where, i=11, 12, 13, 14; j=1, 2, 3, 7

$P'_i(t) + \mu_j P_i(t) = \beta_j P_3(t)$. …(8);

Where, i=15, 16, 17, 18, 19; j=1, 2,3,5,7

With initial conditions; $P_j(0)=1$ if j=1 and 0 otherwise… …(9)

In process industries, the management is interested to get long run availability of the system. The steady state condition is applied to get long run availability of the system. Under steady state condition; d/dt→0, as t→0. Therefore, the equations (1) to (8) get reduced to the following system of equations;

$X_0P_0= \mu_1P_4 + \mu_2P_5 + \mu_2P_5 + \mu_3P_{6+} \mu_4P_1 + \mu_6P_2$……… …..(11)

$X_1P_1= \mu_1P_7 + \mu_2P_8 + \mu_2P_5 + \mu_3P_9+ \beta_4P_0 + \mu_5P_{10} + \mu_6P_3$……….(12)

$X_2P_2= \mu_1P_{11} + \mu_2P_{12} + \mu_3P_{13} + \mu_4P_3+ \beta_6P_0 + \mu_7P_{14}$……… ……(13)

$X_3P_3= \mu_1P_{15} + \mu_2P_{16} + \mu_3P_{17} + \beta_4P_2+ \mu_5P_{18} + \beta_6P_1$……… … (14)

$\mu_j P_i = \beta_j P_0$………… .(15);

where, i=4,5,6 ;j=1,2,3

$\mu_j P_i = \beta_j P_1$……… ….(16);

where, i=7,8,9,10 ;j=1,2,3,5

$\mu_j P_i = \beta_j P_2$… ……….(17);

where, i=11,12,13,14 ;j=1,2,3,7

$\mu_j P_i = \beta_j P_3$…………(18) ; where, i=15,16,17,18,19 ;j=1,2,3,5,7

The values of P_1, P_2 and P_3 are obtained by solving the equations from (11) to (18) by recursive method.

$P_1= P_0 A$ …….(19) ; $P_2= P_0 B$ ……… (20)

and

$P_3= P_0 C$ ………… ….(21)

Where; $A= (K_1 K_4- \lambda_6 \mu_6+ \lambda_4\mu_4)/(\mu_4 K_4-\mu_4K_3)$; $B= (K_1 K_3+ \lambda_6 \mu_6- \lambda_4\mu_4)/(\mu_6 K_4+ \mu_6K_3)$;

$C= (K_4 \lambda_6 \lambda_4+ \lambda_6 \lambda_4 K_3)/(K_2K_3 K_4- K_4\lambda_6 \mu_6 - \mu_4 \lambda_4K_3)$; $K_1= \lambda_4+\lambda_6$; $K_2= \mu_4+ \mu_6$; $K_3= \lambda_6+ \mu_4$; $K_4= \lambda_4+ \mu_6$

Under normalized conditions i.e. sum of all the probabilities is equal to one.

$$\therefore \sum_{i=0}^{19} P_i = 1 \text{ i.e. } P_o+P_1+P_2+\dots\dots\dots+P_{19}=1 \Rightarrow P_o\left[1+\frac{P_1}{P_o}+\frac{P_2}{P_o}+\frac{P_3}{P_o}+\frac{P_4}{P_o}+\dots\dots\dots+\frac{P_{19}}{P_o}\right]=1$$

$$P_o=\frac{1}{\left[1+A+B+C+4\left(\lambda_1/\mu_1+\lambda_2/\mu_2+\lambda_3/\mu_3\right)+2\left(\lambda_5/\mu_5+\lambda_7/\mu_7\right)\right]} \quad \dots\dots \quad \dots\dots.(22)$$

The long run availability i.e. A(∞) of the system is summation of working and standby states i.e.

$A(\infty)= P_o + P_1 + P_2 + P_3$ …… ……….(23)

The equation (23) gives the long run availability of the system. The numerical computations have been carried out with MATLAB 7.1 software by taking;

a) The failure and repair rates of evaporator (λ_4, μ_4) and its standby unit (λ_6, μ_6) are same.
b) The failure and repair rates of drying chamber (λ_5, μ_5) and its standby unit (λ_7, μ_7) are same.

5. Performance Analysis

The long run availability of the system is calculated by using equation (23) for various values – combinations of failure and repair rates and presented in table 1-5;

- ***Effect of failure and repair rates of Chiller on long run availability of the system:*** The long run availability of the system is studied by varying their values as; β_1=0.0033, 0.0038, 0.0043, 0.0048 and μ_1=0.316, 0.321, 0.326, 0.331; β_2=0.0057; β_3=0.0073; β_4=0.0048; β_5=0.0451, μ_2=0.073; μ_3=0.281; μ_4=0.092; μ_5=0.089.

Table 1: Effect of Failure and repair rate of Chiller on long run availability of the system.

β_1 μ_1	**0.0033**	**0.0038**	**0.0043**	**0.0048**
0.316	0.88517355	0.88388693	0.88260416	0.88132523
0.321	0.88530103	0.88403330	0.88276931	0.88150904
0.326	0.88542462	0.88417521	0.88292944	0.88168729
0.331	0.88554452	0.88431289	0.88308480	0.88186022

- ***Effect of failure and repair rates of cream-separator on long run availability of the system: The*** long run availability of the system is studied by varying their values as; β_2=0.0052, 0.0057, 0.0062, 0.0067 and μ_2=0.068, 0.073, 0.078, 0.083; β_1=0.0038, β_3=0.0073, β_4=0.0048, β_5=0.00451, μ_1=0.321, μ_3=0.281, μ_4=0.092, μ_5=0.089.

Table 2: Effect of Failure and repair rate of Cream separator on long run availability of the system.

β_2 μ_2	**0.0052**	**0.0057**	**0.0062**	**0.0067**
0.068	0.88534333	0.87956901	0.87386965	0.86824377
0.073	0.88946794	0.88403330	0.87866477	0.87336116
0.078	0.89309525	0.88796237	0.88288828	0.87787195
0.083	0.89631007	0.89144705	0.88663663	0.88187796

- ***Effect of failure and repair rates of pasteurizer on long run availability of the system:*** The long run availability of the system is studied by varying their values as; β_3=0.0068, 0.0073, 0.0078, 0.0083 and μ_3=0.276, 0.281, 0.286, 0.291; β_1=0.0038, β_2=0.0057, β_4=0.0048, β_5=0.00451, μ_1=0.321, μ_2=0.073, μ_4=0.092, μ_5=0.089.

Table 3: Effect of Failure and repair rate of Pasteurizer on long run availability of the system

β 3 μ 3	**0.0068**	**0.0073**	**0.0078**	**0.0083**
0.276	0.88513124	0.88366565	0.88220501	0.88074930
0.281	0.88547484	0.88403330	0.88259655	0.88116459
0.286	0.88580667	0.88438838	0.88297474	0.88156572
0.291	0.88612734	0.88473153	0.88334023	0.88195341

- ***Effect of failure and repair rates of evaporator on long run availability of the system:*** The long run availability of the system is studied by varying their values as; β_4=0.0043, 0.0048, 0.0053, 0.0058 and μ_4=0.087, 0.092, 0.097, 0.102; β_1=0.0038, β_2=0.0057, β_3=0.0073, β_5=0.00451; μ_1=0.321, μ_2=0.073, μ_3=0.281, μ_5=0.089.

Table 4: Effect of Failure and repair rate of evaporator on long run availability of the system.

β_4 μ_4	0.0043	0.0048	0.0053	0.0058
0.087	0.88238917	0.88232903	0.88226908	0.88220934
0.092	0.88201243	0.88195341	0.88189458	0.88183595
0.097	0.88164193	0.88158400	0.88152626	0.88146871
0.102	0.88127750	0.88122064	0.88116396	0.88110746

- ***Effect of failure and repair rates of drying chamber on long run availability of the system:*** The long run availability of the system is studied by varying their values as; β_5=0.00446, 0.00451, 0.00456, 0.00461 and μ_5=0.084, 0.089, 0.094, 0.099. The failure and repair rates of other sub-systems have been taken as : β_1=0.0038, β_2=0.0057, β_3=0.0073, β_4=0.0048, ;μ_1=0.321, μ_2=0.073, μ_3=0.281, μ_4=0.092.

Table 5: Effect of Failure and repair rate of drying chamber on long run availability of the system.

β_5 μ_5	0.00446	0.00451	0.00456	0.00461
0.084	0.85428054	0.85385353	0.85342693	0.85300074
0.089	0.85642335	0.85601829	0.85561360	0.85520928
0.094	0.85834731	0.85796206	0.85757715	0.85719257
0.099	0.86008433	0.85971705	0.85935007	0.85898339

6. Discussion and Conclusion

The table 1 shows that increase in failure rate (β_1) of chiller from 0.0033 to 0.0048 decreases the system availability from 0.4348% to 0.4160% while increase in repair rate (μ_1) of chiller from 0.316 to 0.331 increases the system availability from 0.0419% to 0.0607%. The table 2 shows that increase in failure rate (β_2) of cream separator from 0.0052 to 0.0067 decreases the system availability from 1.9314% to 1.6102% while increase in repair rate (μ_2) of cream separator from 0.068 to 0.083 increases the system availability from 1.2387% to 1.5703%. The table 3 shows that increase in failure rate (β_3) of pasteurizer from 0.0068 to 0.0083 decreases the system availability from 0.4951% to 0.4710% while increase in repair rate (μ_3) of cream separator from 0.276 to 0.291 increases the system availability from 0.1125% to 0.1367%. The table 4 shows that increase in failure rate (β_4) of evaporator from 0.0043 to 0.0058 decreases the system availability from 0.0204% to 0.0193% while increase in repair rate (μ_4) of evaporator from 0.087 to 0.102 increases the system availability from 0.1249% to 0.1260%. The table 5 shows that increase in failure rate (β_5) of drying chamber from 0.00446 to 0.00461 decreases the system availability from 0.1498% to 0.1280% while increase in repair rate (μ_5) of drying chamber from 0.084 to 0.099 increases the system availability from 0.6794% to 0.7014%.

Thus, the sub-system namely, cream separator has the maximum effect on long run availability of the complete system. Hence, it is suggested that the management should take utmost care of this sub-system to enhance the performance of skim milk powder system of dairy plant.

References

[1] Adhikary, D. D., Bose, G. K., Chattopadhyay, S. D., Bose, D. and Mitrad ,S. (2012), RAM investigation of coal-fired thermal power plants: A case study", *International Journal of Industrial Engineering Computations*, **3**, pp.423-434.

[2] Arora, N. and Kumar, D. (1997), Availability analysis of steam and power generation systems in thermal power plant", *Microelectronic Reliability*, **37**, pp.795-799.

[3] B.S. Dhillon and C. Singh (1981), *Engineering reliability-new techniques and applications*. New York: John Wiley and Sons.

[4] E. Balaguruswamy (1984), *Reliability Engineering*, Tata McGraw hill, New Delhi.

[5] Gupta, P., Lal, A.K., Sharma, R.K. and Singh, J. (2005), Behavioral Study of the Cement manufacturing Plant – A Numerical Approach", *Journal of Mathematics and Systems Sciences*, **1**, pp.50-70.

[6] Gupta, S. and Tewari ,P.C. (2011), Simulation Modeling In a Availability Thermal Power Plant, *Journal of Engineering Science and Technology Review*, **4**, *2* ,pp. 110-117 .

[7] Gupta, S., Kumar, A., Sharma, R. and Tewari, P.C. (2008), A performance modeling and decision support system for a feed water unit of a thermal power plant, *S A J Ind. Engg.*, **19**, *2* pp.125–34.

[8] Khanduja, R.,Tewari, P.C. and Kumar, D. (2012), Steady state behavior and maintenance planning of bleaching system in a paper plant, *International Journal of Industrial Engg.* , **7**, *12*, pp., 39-44.

[9] Kumar, S., Tewari, P.C. and Sharma, R. (2007), Simulated availability of CO2 cooling system in a fertilizer plant, *Ind. Eng. J. (Indian Inst Ind Eng, Mumbai)*, **36**, *10*, pp.19–23.

[10] Kumar,S. and Tewari, P.C. (2011) , Mathematical modeling and performance optimization of CO_2 cooling system of a fertilizer plant, *International Journal of Industrial Engineering Computations*, **2**, pp. 689–98.

[11] L.S. Srinath (1994), *Reliability Engineering*; 2nd edition, East-West Press Pvt. Ltd; New Delhi, India.