

Simulation Studies on Conjugate Mixed Convection with Radiation from a Vertical Channel with Multiple Non-Identical Discrete Heat Sources

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Abstract

The present paper reports the parametric studies on the problem of combined conduction-mixed convection-radiation from a non-identically and discretely heated vertical channel. Five discrete heat sources of non-identical heights with identical rates of volumetric heat generation are assumed to be flush-mounted in the descending order of their heights in the left wall of the channel starting from the entry of the channel to its exit. The channel wall spacing could be varied by varying its aspect ratio. The cooling medium is air that is considered to be radiatively transparent. The governing equations for fluid flow and heat transfer are considered in their full strength without the boundary layer approximations and are first transformed into vorticity-stream function form and then are appropriately normalized. The resulting algebraic equations are solved by finite volume method. A computer code is written for the purpose in C++. The influence of parameters like modified Richardson number, thermal conductivity, surface emissivity and aspect ratio on local wall temperature distribution, peak channel temperature and relative contributions of mixed convection and radiation to heat dissipation have been probed into. The studies reveal the prominence radiation assumes in the present kind of problems in different regimes of mixed convection.

Keywords - Mixed convection, Conduction, Radiation, Finite volume method, Vertical channel, Non-identical discrete heat sources.

Introduction

A vertical channel is a commonly encountered geometry in the analyses pertaining to thermal control of electronic equipment. It very closely simulates the cooling passages

of a series of printed circuit boards (PCBs) with heat generating components. The geometry of a vertical channel has been explored by many researchers. Elenbaas [1] is one of those initial researchers, who presented experimental results of free convection in a vertical, symmetrically heated, isothermal channel with air as the cooling medium. Quintiere and Mueller [2] presented approximate analytical solutions for constant-property laminar mixed convection between finite vertical parallel plates. Bar-Cohen and Rohsenow [3] reported some useful composite relations for the average Nusselt number for natural convection in a vertical channel for both symmetric and asymmetric isothermal or isoflux boundary conditions. Kim and Anand [4] numerically studied laminar developing fluid flow and heat transfer between a series of conducting parallel plates with surface-mounted heat generating blocks making use of air as the cooling agent. Gururaja Rao et al. [5] solved, numerically, the problem of conjugate mixed convection with surface radiation from a vertical channel with a solitary discrete heat source in each wall. Bousetta et al. [6] have probed into numerical solutions on the influence of surface radiation on the laminar air flow induced by natural convection in vertical, asymmetrically-heated channels.

The review of the literature, a brief summary of which is provided above, indicates that enough information is not available about the interplay between the three modes of heat transfer for the geometry of a vertical channel with multiple, non-identical, discrete heat sources in its left wall. On account of this, a numerical simulation of interaction of radiation with conjugate mixed convection from the above geometry is attempted in the present paper.

Statement of the Problem and its Formulation

The problem geometry considered in the present study consists of a vertical parallel-plate channel of height L and width W , with each wall of thickness t ($t \ll L$ or W), is shown in Fig. 1, along with the system of coordinates and extended computational domain. For a given height L , an aspect ratio (AR) is defined as L/W such that a smaller AR represents a wider channel, while a larger AR indicates a narrower channel. There are five flush-mounted discrete heat sources along the left wall of the channel arranged in the descending order of their heights from the entry of the channel to its exit with q_v being the rate of volumetric heat generation in each of the heat sources. All the surfaces of the left and right walls are adiabatic except the wall surfaces facing each other. The cooling medium (air) is assumed to be radiatively transparent and of constant thermo physical properties with the Boussinesq approximation considered to be valid. It enters from the bottom of the channel with a uniform velocity u_∞ and a uniform temperature T_∞ . The heat generated in the five heat sources gets conducted axially along the left wall and is later dissipated by mixed convection and radiation. The thermal conductivity and the surface emissivity of the channel walls are, respectively, k_s and ϵ .

The governing equations for fluid flow and heat transfer, viz., the continuity equation, the Navier-Stokes equations and the equation of energy, considered in primitive variables, are first transformed to stream function-vorticity ($\psi-\omega$)

formulation. These are later non-dimensionalized using a set of seven normalizing parameters, viz., X , Y , U , V , ψ , ω and θ . The resulting normalized governing equations are:

$$U \frac{\partial \omega}{\partial X} + V \frac{\partial \omega}{\partial Y} = -\text{Ri}_w^* \frac{\partial \theta}{\partial Y} + \frac{1}{\text{Re}_w} \left(\frac{\partial^2 \omega}{\partial X^2} + \frac{\partial^2 \omega}{\partial Y^2} \right) \quad (1)$$

$$\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = -\omega \quad (2)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{\text{Pe}_w} \left[\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right] \quad (3)$$

In order to discretize the computational domain, a hybrid grid system is used in horizontal (y) direction and axial (x) direction. The temperature boundary condition is obtained through appropriate energy balance. For example, the energy balance for a typical element pertaining to any of the five heat sources (other than their ends) is

$$q_{\text{cond},x,\text{in}} + q_v(\Delta x_{\text{hs}} t) = q_{\text{cond},x,\text{out}} + q_{\text{conv}} + q_{\text{rad}} \quad (4)$$

Substitution of appropriate expressions for various terms in the above equation and subsequent simplification results in:

$$\frac{\partial^2 \theta}{\partial X^2} + \gamma \left(\frac{\partial \theta}{\partial Y} \right)_{Y=0} + A_{g1} A_{g2} - \frac{\varepsilon}{1 - \varepsilon} \gamma N_{\text{RF}} \left[\left(\frac{T}{T_\infty} \right)^4 - J_i' \right] = 0 \quad (5)$$

Here, γ is the thermal conductance parameter, defined as $[k_f W / (k_s t)]$, N_{RF} is the radiation-flow interaction parameter, defined as $[\sigma T_\infty^4 / (k_f \Delta T_{\text{ref}}) / w]$, A_{g1} and A_{g2} are the two non-dimensional geometric ratios, defined as $[w/t]$ and $[w/L_h]$, respectively. The governing equations for the non-dimensional temperature at the remaining portions of the left and right walls are obtained by using a similar treatment.

Solution Methodology and Range of Parameters

The governing non-dimensional Eqs. (1)-(3) are transformed into algebraic equations using the finite volume method. The resulting algebraic equations along with discretized boundary conditions are solved by the Gauss-Seidel iterative solver. A relaxation parameter 0.3 is used on vorticity (ω) and stream function (ψ), while for temperature (θ), full relaxation with relaxation parameter unity is used.

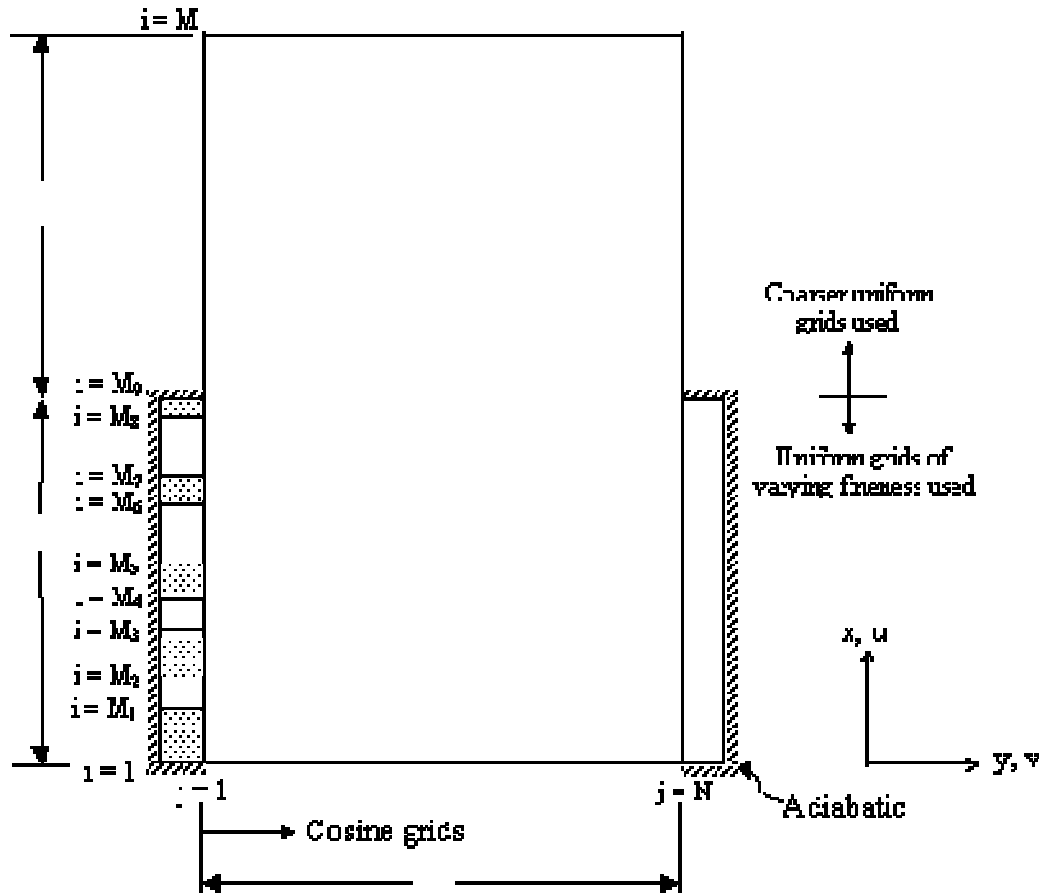


Fig. 1 Schematic showing the channel geometry along with the computational domain employed.

To terminate the iterations, convergence criteria (δ) of 1×10^{-4} , 5×10^{-4} and 1×10^{-4} are imposed on stream function, vorticity and temperature, respectively.

The height of the channel (L) is taken to be 0.2 m and the wall thickness (t) is taken to be 1.5 mm, while the heights of the five non-identical discrete heat sources are taken to be $L_{h1} = 3$ cm, $L_{h2} = 2.5$ cm, $L_{h3} = 2$ cm, $L_{h4} = 1.5$ cm and $L_{h5} = 1$ cm. The range for AR is taken to be from 4 to 20. The surface emissivity (ϵ) of the channel is chosen to vary between 0.05 and 0.85. With regard to thermal conductivity (k_s), in W/m K, of the material of the channel, $0.25 \leq k_s \leq 1$ is chosen. The modified Richardson number (Ri_w^*) is varied between 0.01 and 250, with the lower limit ($Ri_w^* = 0.01$) signifying the asymptotic forced convection limit, the upper limit ($Ri_w^* = 250$) implying the asymptotic free convection limit, while $Ri_w^* = 1$ indicates pure mixed convection regime.

Findings of Simulation Studies

Before taking up the various parametric studies, an exhaustive grid sensitivity analysis is carried out in four phases in all the regimes of mixed convection to fix up an appropriate grid system to discretize the computational domain. In summary, the optimum grid system for the present problem has 261 grids ($M = 261$) in x direction, 81 grids ($N = 81$) in y direction, 201 grids ($M_9 = 201$) along the channel region with, respectively, 12 and 8 grids/cm in the heat source and non-heat source portions of the channel.

Dependence of local temperature distribution on surface emissivity

The variation of local temperature profiles for the left and right walls of the channel, for three typical values of surface emissivity, viz., $\varepsilon = 0.05$, 0.45 and 0.85, is depicted in Figs. 2(a) and 2(b), respectively. The study is carried out for fixed values of input as shown. It can be seen from Fig. 2(a) that, for a given ε , the temperature of the left wall starts increasing rather sharply from the channel entry, reaches a local maximum somewhere at the top end of the bottommost heat source. It again starts decreasing in the immediately succeeding non-heat source portion. After reaching a local minimum, it again starts rising as one moves through the second heat source and a similar behavior, as noticed in the first heat source portion, is observed. The trend continues in the rest of the wall and one can see five local maxima and four local minima in the temperature profile. It is further noted that the local temperature decreases with increasing surface emissivity owing to enhanced radiative heat dissipation from the wall. The above effect of ε is found to be more pronounced as ε increases from 0.05 to 0.45 compared to that noticed between $\varepsilon = 0.45$ and 0.85. In the example considered here, T_{Lmax} is dropping down by 17.29% as ε changes from 0.05 to 0.45, while it comes down by another 5.77% with ε changing to 0.85 subsequently.

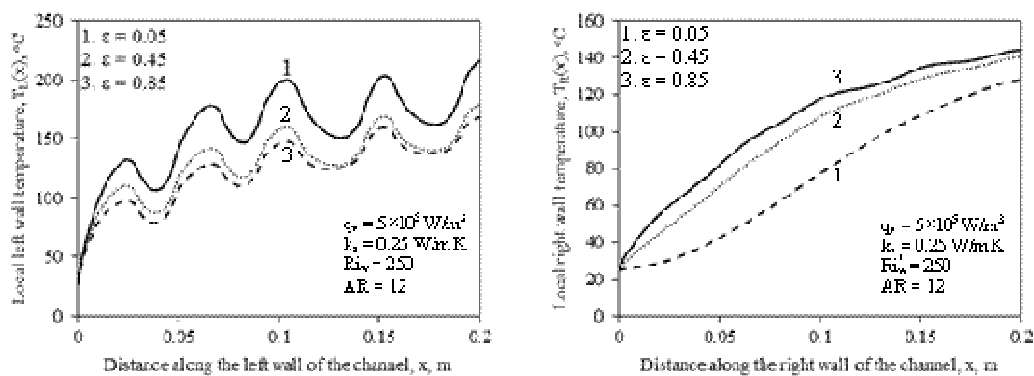


Fig. 2 (a, b) Local temperature profiles along left and right walls for different surface emissivities.

It is observed from Fig. 2(b) that the local right wall temperature, for a given ε , increases as one moves from channel entry. The local temperature, here, is seen to increase with ε , quite appreciably between $\varepsilon = 0.05$ and 0.45 and rather mildly between $\varepsilon = 0.45$ and 0.85 . In the present example, at the channel exit, $T_R(X)$ is seen to be increasing by 9.59% and 2.8%, respectively, as ε is increased between 0.05 and 0.85 in equal steps of 0.40.

Dependence of maximum channel temperature on surface emissivity in different regimes of mixed convection

Figure 3 depicts the effect of surface emissivity (ε) in controlling the maximum temperature of the channel (T_{\max}) in various regimes of mixed convection. Five of the possible surface coatings with emissivity equal to 0.05, 0.25, 0.45, 0.65 and 0.85, along with four values of Ri_w^* , namely 250, 25, 1 and 0.01, are chosen for this study. The remaining input parameters are held fixed as shown in Fig. 3. It is observed that T_{\max} decreases with increasing ε in all the regimes of mixed convection. The degree of decrease in T_{\max} is more pronounced in free convection dominant regime than in forced convection dominant regime. For example, in the present study, T_{\max} comes down by 22.06% and 7.76%, as ε increases from 0.05 to 0.85, for $Ri_w^* = 250$ and 0.01, respectively.

It is also perceived that T_{\max} is decreasing, for any surface coating applied, as the flow regime transits from free convection to forced convection dominant regime owing to increased rate of heat dissipation by forced convection. In the present example, for a moderately emitting surface ($\varepsilon = 0.45$), T_{\max} decreases by 43.87%, 17.99% and 36.45% with a change in Ri_w^* from 250 to 25, 25 to 1 and 1 to 0.01, respectively. Thus, one can control the peak channel temperature in any regime of convection by proper selection of surface coating or alternately with any given surface by appropriate choice of induced fluid velocity.

Study on contributions of mixed convection and radiation in channel heat dissipation with surface emissivity in different regimes of mixed convection

It is necessary to dwell into the individual contributions by mixed convection and radiation for various values of emissivity (ε) and in different regimes of mixed convection. Figure 4 is plotted for a fixed input as shown. Five typical values of surface emissivity and three different values of Ri_w^* , are used as shown in Fig. 4. As ε increases, a decrease in the contribution from convection is observed with a proportionate increase in the role of radiation. In the present example, for $Ri_w^* = 0.01$, contribution from radiation increases from 1.11% to 16.96% with increase in ε from 0.05 to 0.85. A similar exercise in the asymptotic free convection limit increases radiative dissipation from 3.85% to 33.79%. It can also be noted that, for a particular ε , as one moves from forced convection to free convection dominant regime, the contribution of convection decreases with an appropriate increase in radiation contribution. For a good emitter ($\varepsilon = 0.85$), the heat dissipation by mixed convection is decreasing from 83.12% to 69.59% as Ri_w^* is changing from 0.01 to 1. It is further

dropping to 66.21% with a subsequent rise in Ri_w^* to 250. By the same token, one can observe a mirror-image increase in the role of radiation due to the same changes in the limits of Ri_w^* .

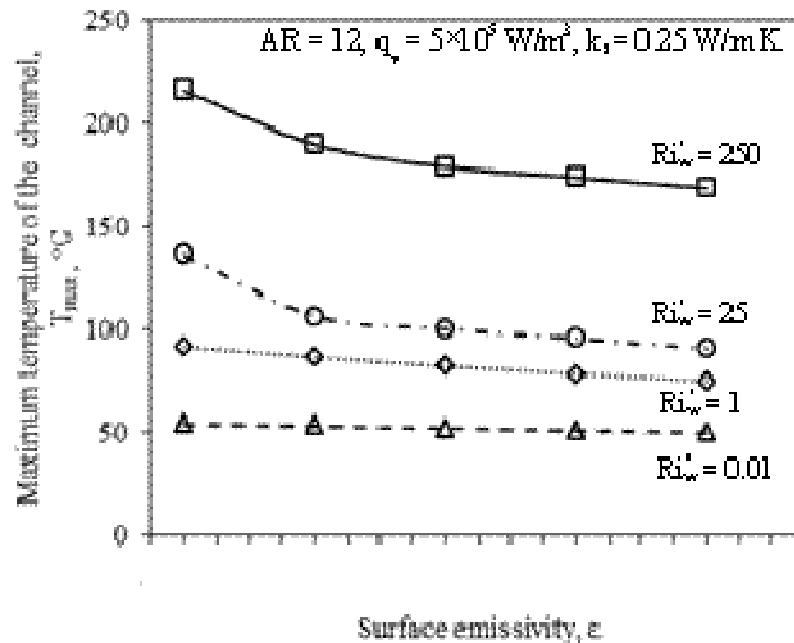


Fig. 3 Variation of maximum temperature with surface emissivity in different dissipation for different surface emissivities.

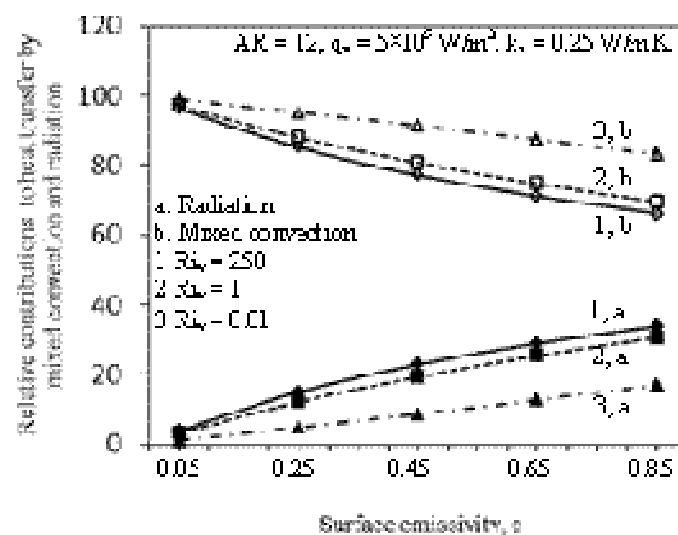


Fig. 4 Relative roles of mixed convection and surface radiation in channel heat regimes of convection.

Concluding Remarks

The present paper attempted a numerical solution to the problem of combined conduction-mixed convection-radiation from the geometry of a vertical parallel-plate channel heated discretely and non-identically. A computer code making use of finite volume method coupled with Gauss-Seidel solver is written. The local left and right wall temperatures, respectively, decrease and increase with increase in emissivity in any given regime of mixed convection. The peak channel temperature could be satisfactorily controlled in any given regime of convection, holding other parameters unaltered, by just altering the surface coating from a good reflector to a good emitter. Studies are performed to look into relative contributions of mixed convection and radiation in channel heat dissipation. It is seen that radiation plays a significant role in deciding the channel heat dissipation and thus the design of the cooling system.

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