

## **Mixed Convection with Internal Conduction and Surface Radiation from a Discretely and Non-Identically Heated Vertical Electronic Board**

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### **Abstract**

The present article aims at reporting the salient results of a numerical study performed on the problem of conjugate mixed convection with surface radiation from a vertical electronic board flush-mounted with five non-identical discrete heat sources arranged in the descending order of their heights from the leading to the trailing edge of the board. Air is considered as the cooling medium and is assumed to be radiatively transparent. The pertinent governing equations concerning fluid flow and heat transfer, without boundary layer approximations, are solved making use of finite volume based finite difference method and an exclusive computer program in C++ has been written for the above purpose. The effect of characteristic velocity of the fluid, thermal conductivity of the board material and surface emissivity on local board temperature distribution, peak board temperature and relative contributions of mixed convection and surface radiation in total heat dissipation has been elucidated.

**Keywords:** Mixed Convection, Internal Conduction, Finite Volume Method, Electronic Board and Radiation.

### **Introduction**

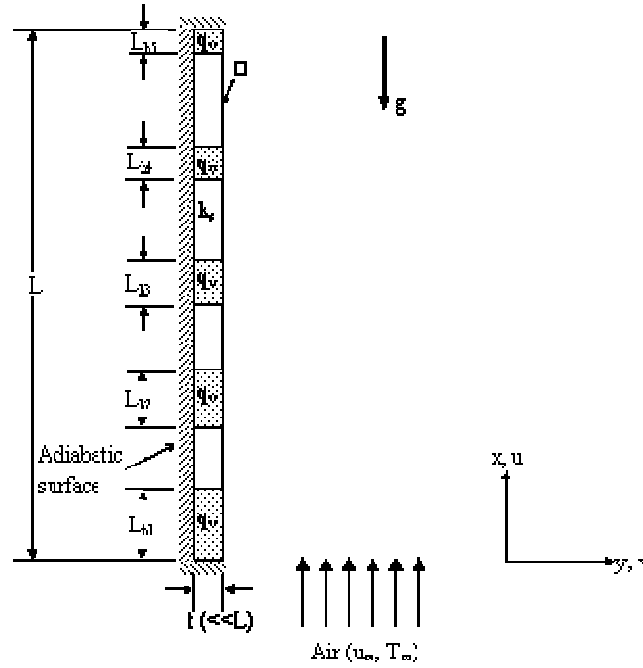
Starting from Blasius [1], who has given exact solution of the fluid flow pertaining to laminar forced convection over a flat plate, many of the researchers have worked on horizontal or vertical plate geometry that very closely simulates an electronic board. Pohlhausen [2] provided an analytical solution to heat transfer for the above problem. Ostrach [3] has provided both fluid flow and heat transfer results of laminar free convection over a flat plate. Results pertaining to the effect of buoyancy on forced

convection fluid flow and heat transfer have been reported by Sparrow and Gregg [4]. Concerning mixed convection, Lloyd and Sparrow [5] studied combined forced and free convection involving vertical surfaces. Results of radiation effect on mixed convection along an isothermal vertical plate have been provided by Hossain and Takhar [6]. Jahangeer et al. [7] studied the problem of conjugate heat transfer from a rectangular fuel element of a nuclear reactor dissipating heat into an upward moving liquid sodium stream, with boundary layer approximations. Sawant and Gururaja Rao [8] numerically investigated the problem of combined conduction-mixed convection-surface radiation from a uniformly heated vertical plate. Steady, mixed convection laminar boundary layer flow of incompressible nanofluids along a vertical plate with temperature dependent heat source/sink has been numerically investigated by Rana and Bhargava [9].

A thorough review of literature reveals that the problem of conjugate mixed convection with interplay of radiation from a discretely heated vertical plate has not been attempted in requisite detail. In view of the above, the present study has been performed on a vertical electronic board with five non-identical flush-mounted discrete heat sources for exploring fluid flow as well as heat transfer.

### **Description of the Problem and Its Formulation**

The problem geometry consists of a vertical electronic board with five flush-mounted non-identical discrete heat sources as shown in Fig. 1, along with system of coordinates. Height and thickness of the board are  $L$  and  $t$ , respectively. Five heat sources of heights  $L_{h1}$ ,  $L_{h2}$ ,  $L_{h3}$ ,  $L_{h4}$  and  $L_{h5}$  are mounted in the descending order of their heights from the leading to the trailing edge of the board, respectively. The thermal conductivity of the board is  $k_s$  W/m K, while  $\varepsilon$  is its surface emissivity. Air is taken as the cooling agent and is radiatively transparent. The thermo physical properties of air are assumed to be constant with only density varying as per the Boussinesq approximation. The board is adiabatic on all its exterior surfaces (bottom, left and top surfaces). Thus, the heat generated in the heat sources is first conducted along the board and is subsequently dissipated to air by combined free and forced convection and radiation from its right surface. Air is impressed vertically upwards from the bottom of the board at a characteristic velocity  $u_\infty$  and temperature  $T_\infty$ .



**Fig. 1** Schematic of the problem geometry considered for the present study.

The governing equations pertaining to fluid flow and heat transfer, which are first considered in their primitive variable form, are converted into vorticity-stream function ( $\omega$ - $\psi$ ) formulation and are later normalized. The final set of non-dimensional governing equations is:

$$U \frac{\partial \omega}{\partial X} + V \frac{\partial \omega}{\partial Y} = -Ri_L^* \frac{\partial \theta}{\partial Y} + \frac{1}{Re_L} \left( \frac{\partial^2 \omega}{\partial X^2} + \frac{\partial^2 \omega}{\partial Y^2} \right) \quad (1)$$

$$\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = -\omega \quad (2)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Pe_L} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (3)$$

The computational domain is extended in axial (X) and transverse (Y) directions and a size of  $2L \times L$  is decided for it. The computational domain is discretized by using a semi-cosine function in the horizontal (Y) direction, while uniform grids of varying fineness are employed in the vertical (X) direction. The temperatures along the board are obtained from the traditional energy balance. For example, the energy balance on a typical interior element pertaining to any of the five heat sources (excluding their ends) is:

$$q_{cond,x,in} + q_v t \Delta x_{hs} = q_{cond,x,out} + q_{conv} + q_{rad} \quad (4)$$

Upon substitution of appropriate expressions for various terms, normalization and subsequent simplification, the above equation leads to:

$$\frac{\partial^2 \theta}{\partial X^2} + \gamma \left( \frac{\partial \theta}{\partial Y} \right)_{Y=0} + A_{g1} A_{g2} - \varepsilon \gamma N_{RF} = 0 \quad (5)$$

Here,  $\gamma$ ,  $N_{RF}$  and  $A_{g1}$  and  $A_{g2}$  are thermal conductance parameter, radiation-flow interaction parameter and non-dimensional geometric ratios, respectively. By adopting a similar procedure one can obtain the governing equations for the remaining portions of the board as well.

### Method of Solution and Range of Parameters

The non-dimensional governing Eqs. (1)-(3) pertaining to fluid flow and heat transfer are converted into algebraic equations using the finite volume method and are subsequently solved for  $\psi$ ,  $\omega$  and  $\theta$  using the Gauss-Siedel iterative solver. An under relaxation parameter 0.5, is imposed on stream function and vorticity, while full relaxation is used for temperature. Strict convergence criteria,  $1 \times 10^{-4}$ ,  $5 \times 10^{-4}$  and  $1 \times 10^{-4}$ , are imposed on  $\psi$ ,  $\omega$  and  $\theta$ , respectively. An exclusive computer code in C++ is written to solve the problem.

The height (L) and the thickness (t) of the board are taken to be 20 cm and 1.5 mm, respectively. The heights of the five heat sources are  $L_{h1} = 3$  cm,  $L_{h2} = 2.5$  cm,  $L_{h3} = 2$  cm,  $L_{h4} = 1.5$  cm and  $L_{h5} = 1$  cm. The range chosen for thermal conductivity ( $k_s$ ) and surface emissivity ( $\varepsilon$ ) are 0.25-1 W/m K and 0.05-0.85, respectively. The modified Richardson number ( $Ri_L^*$ ) is varied in between 0.01-250, where  $Ri_L^* = 250$  signifies the asymptotic free convection limit,  $Ri_L^* = 0.01$  represents the asymptotic forced convection limit with  $Ri_L^* = 1$  indicating pure mixed convection regime. All the studies are performed assuming air (cooling agent) to be entering at a free stream temperature ( $T_\infty$ ) 25°C.

### Simulation Studies and Findings

#### Study of local board temperature distribution in different regimes of mixed convection

Figure 2 shows the nature of variation of local board temperature profiles in various regimes of mixed convection. Five values of  $Ri_L^*$ , namely 0.01, 0.1, 1, 25 and 250, are considered for the current study. The remaining input parameters are kept constant as shown. In the entire regime of mixed convection, as one moves vertically upwards along the board, the local board temperature  $[T(x)]$  is increasing quite sharply reaching its first local maximum somewhere nearer to the top end of the bottom-most heat source. The temperature is then decreasing along the immediately accompanying non-heat source portion to a local minimum. Further, it increases along the second heat source with a second local maximum observed nearer to the top end of the second heat source. The trend continues with three more local maxima noticed as one further move along the board with two local minima sandwiched in between the each pair of the maxima. The third local maximum turns out to be the maximum

temperature ( $T_{\max}$ ) of the board itself and it is typically observed a little after the board center. Further, at any location along the board,  $T(x)$  decreases as the flow transits from the free convection dominant regime to forced convection dominant regime in view of increased convective heat dissipation. In the present example, the third local peak temperature gets decreased by 52.12% with  $Ri_L^*$  changing from 250 to 0.01.

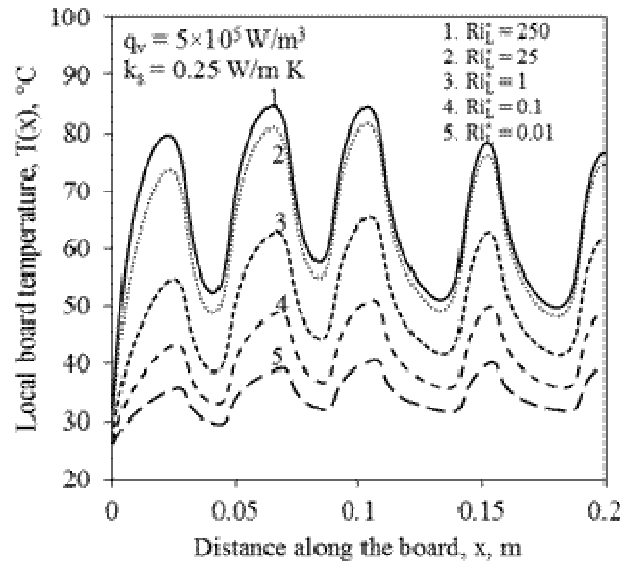
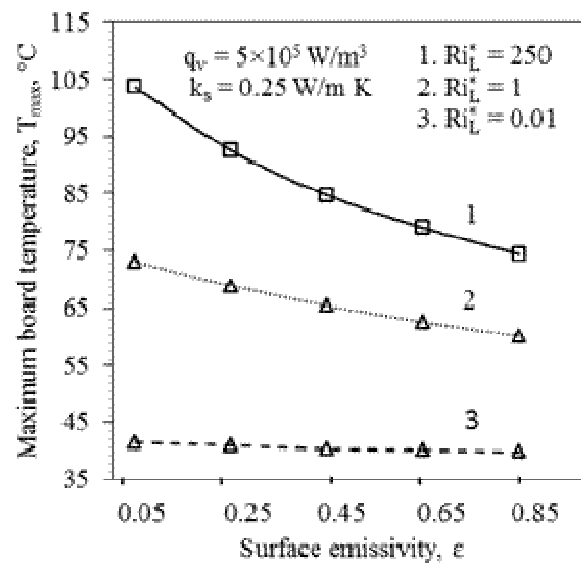


Fig. 2 Local temperature profiles in different convection regimes.

### Variation of peak board temperature with surface emissivity in different convection regimes

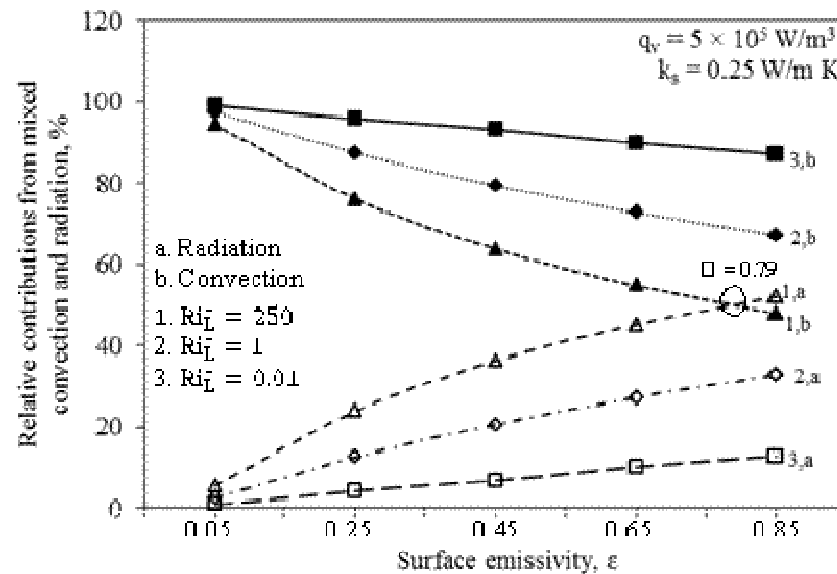
Figure 3 depicts the nature of variation in peak board temperature ( $T_{\max}$ ) with surface emissivity ( $\varepsilon$ ) in different regimes of mixed convection. The results are obtained for the constant input comprising  $q_v = 5 \times 10^5 \text{ W/m}^3$  and  $k_s = 0.25 \text{ W/m K}$ . It is obvious from the figure that  $T_{\max}$  is decreasing with increase in  $\varepsilon$  in the entire regime of mixed convection due to increased radiative heat dissipation. However, the decrease in  $T_{\max}$  with increasing  $\varepsilon$  is more noticeable in free convection dominant regime compared to that in forced convection dominant regime. In the case considered here, with rise in  $\varepsilon$  from 0.05 to 0.85, a 28.18% drop in  $T_{\max}$  is observed in the regime pertaining to  $Ri_L^* = 250$ , while for the regime  $Ri_L^* = 0.01$ , the drop in  $T_{\max}$  is just 3.99% for the same increment in  $\varepsilon$ . Further, for a given  $\varepsilon$ , as the flow transits from free convection to forced convection dominant regime, one can observe a huge drop in  $T_{\max}$ . Here, in the current study, for  $\varepsilon = 0.05$ , with a change in  $Ri_L^*$  from 250 to 0.01,  $T_{\max}$  is dropping down by 60.01%. Thus, one can control the maximum board temperature either by applying a proper surface coating or by an appropriate selection of induced fluid velocity.



**Fig. 3** Peak board temperature variation with surface emissivity in different convection regimes.

### Study to separate out the roles of mixed convection and surface radiation in heat dissipation

The total heat generated within the board due to the presence of discrete heat sources is getting dissipated by means of mixed convection and surface radiation. In order to know the relative contributions from each mode with reference to surface emissivity ( $\epsilon$ ) in different convection regimes, Fig. 4 is plotted by holding the rest of the input parameters constant, as shown. The curves are drawn for three distinct regimes of mixed convection, namely  $Ri_L^* = 250$ , 1 and 0.01. It is obvious from the figure that the relative contribution in heat dissipation by mixed convection is decreasing with increase in surface emissivity in all the regimes of mixed convection accompanying with increasing contribution from radiation. The above is more pronounced in the free convection dominant regime, while it is not that substantial in forced convection dominant regime. In the current study, with increase in  $\epsilon$  from 0.05 to 0.85, contribution from radiation increases from 6.06% to 52.35% with an equivalent decrease in the contribution from mixed convection for  $Ri_L^* = 250$ . Here, for  $\epsilon \approx 0.79$ , one can observe equal amount of heat dissipation by mixed convection and surface radiation, and thereafter, radiation starts overriding convection. Even for  $Ri_L^* = 0.01$ , radiation contribution is getting increased by 11.95% when  $\epsilon$  is increased between the same limits as above. Thus, one should take radiation into the account in estimating the pumping power requirement. The figure also reveals that, for a given  $\epsilon$ , the role of convection is increasing with the flow transiting from free to forced convection bringing down the role of radiation. For example, for  $\epsilon = 0.85$ , a change in  $Ri_L^*$  from 250 to 0.01 increases convective heat dissipation from 47.77% to 87.11%, while radiation suffers a mirror-image decrease in its role.



**Fig. 4** Roles of mixed convection and radiation in board heat dissipation for various surface emissivities in different regimes of mixed convection.

### Concluding Remarks

Numerical studies on mixed convection with internal conduction and surface radiation from a vertical electronic board flush mounted with five non-identical discrete heat sources have been performed in the present paper. The above study concludes that the local board temperature distribution is significantly influenced with the regime of mixed convection. Further, one can control the peak board temperature by just transforming the surface coating from a good reflector to a good emitter. The results pertaining to the relative roles of mixed convection and radiation in heat dissipation elucidated the role of radiation.

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