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Study of one dimensional transient heat conduction equation using polynomial approximation method

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Abstract

The study of one dimensional unsteady heat conduction equation with and without internal heat generation is studied using polynomial approximation method. The heat conduction equation is of transient type where the temperature varies with respect to time and position. The geometry is assumed as slab and sphere but the heat transfer is one dimensional. The first boundary condition for the sphere is axi-symmetric boundary condition where as the second boundary condition is convection at the periphery and for the slab; one side is supplied with fixed heat flux and other side with convection heat transfer. First the equations and boundary conditions are non-dimensionalised. Then the non-dimensional equation is solved using polynomial approximation method where the non-dimensional temperature is assumed as a second degree polynomial. Using the boundary conditions the polynomials are solved to find out the constants. The results obtained in this computation are plotted in the form of graphs (non-dimensional time Vs non-dimensional temperature) which are compared with the literature and are found to be matched very well. The study of transient heat transfer is carried out in the form of variation of geometry dimensions as well as geometry. It is observed that with increase of time the non-dimensional temperature decreases which signify the increase of heat transfer. It is also observed that with increase of geometric dimension the non-dimensional temperature decreases at a fixed instant of time for particular material.

Keywords: Steady, transient, polynomial approximation method, sphere, slab

I. Introduction

Lumped model analysis can be carried out when the conductive resistance is less than convective resistance i.e. only for low Biot number(0.1). As many engineering application involve higher Biot number, the requirement of modified lumped model is necessary. So the efforts have been done by several authors to improve the lumped model till now. Few of the literatures regarding the improved model have been discussed below.

Devanshu Prasad[1] developed improved lumped model by employing polynomial approximation methods on slab and cylinder under different conditions.

Jian su, Chen An, Alice Cunha da Silva[2] developed improved lumped models for transient combined convective and radiative cooling of multilayer spherical media. Two point Hermite approximation method is used to obtain the average temperature and heat flux in each layer. The plain trapezoidal rule was employed in all layers, except for the innermost layer where the second-order two sided corrected trapezoidal rule is used to obtain average temperatures.

Zheng Tan, Ge Su and Jian Su[3] developed an improved model for combined convective and radiative cooling wall. Two point Hermite approximations for integrals is used to obtain the improved lumped model.

Noorul Haque and Amitesh Paul[4] studied the improved lumped parameter in transient heat conduction.

Amit Prakash and Shahid Mahmood[5] developed modified lumped model for transient heat conduction in a spherical shape for a particular temperature profile. Polynomial approximation method was used and a unique Biot number was obtained.

In the present work an attempt is made to develop a modified Biot number for different temperature profiles for spherical shape and slab, and to analyze a temperature versus time for heat generated in sphere and slab, and for natural convection cooling in spherical shape for different temperature profiles.

II. Theoretical analysis

A. Analysis of temperature variation with time for heat generated sphere:-

The one dimensional heat conduction equation in a sphere with internal heat generation is:

$$\frac{\partial T}{\partial t} = \alpha \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{\bar{q}}{k} \tag{1}$$

Where T is the temperature of sphere, r is the radius of sphere, $\bar{q}$ is the internal heat generated in the sphere, k is the thermal conductivity of the sphere, t is the time, α is thermal diffusivity.

The required boundary conditions for the sphere are:

$$\frac{\partial T}{\partial r} = 0 \quad at\ r = 0 \tag{2}$$

$$-k\frac{\partial T}{\partial r} = \square(T - T_\infty)\ at\ r = R \tag{3}$$

Where T_∞ is the ambient temperature.

Initial condition:

$$T = T_0 \quad at\ t = 0 \tag{4}$$

Dimensionless parameters:

$$\theta = \frac{T - T_\infty}{T_0 - T_\infty}, B = \frac{\square R}{k}, \tau = \frac{\alpha t}{R^2}, x = \frac{r}{R}, Q = \frac{\bar{q}}{k(T - T_\infty)}R^2 \tag{5}$$

Governing equation and boundary conditions are:

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{x^2}\frac{\partial}{\partial x}\left(x^2\frac{\partial \theta}{\partial x}\right) + Q \tag{6}$$

$$\frac{\partial \theta}{\partial x} = 0\ at\ x = 0 \tag{7}$$

$$\frac{\partial \theta}{\partial x} = -B\theta \tag{8}$$

A.1 Solution by polynomial approximation method:

Let the assumed temperature profile be

$$\theta = a_0 + a_1(x^2 - x) + a_2(x^3 - x^2) \tag{9}$$

On differentiating the above equation

$$\frac{\partial \theta}{\partial x} = a_1(2x - 1) + a_2(3x^2 - 2x) \tag{10}$$

On applying first boundary condition, equation (7)

$$a_1 = 0 \tag{11}$$

On applying second boundary condition, equation (8)

$$a_2 = -B\theta \tag{12}$$

$$a_0 = \theta \tag{13}$$

The average temperature of sphere is given by

$$\bar{\theta} = 3\int_0^1 x^2\,\theta dx \tag{14}$$

On solving the above equation

$$\bar{\theta} = \theta\left(1 + \frac{B}{10}\right) \tag{15}$$

$$\frac{\partial\bar{\theta}}{\partial\tau} = \frac{\partial\theta}{\partial\tau}\left(1 + \frac{B}{10}\right) \tag{16}$$

Integrating the governing equation with respect to *x*,

$$\int_0^1 x^2\frac{\partial\theta}{\partial\tau}dx = \int_0^1 \frac{\partial}{\partial x}\left(x^2\frac{\partial\theta}{\partial x}\right)dx + \int_0^1 Qx^2 dx \tag{17}$$

$$\int_0^1 x^2\frac{\partial\theta}{\partial\tau}dx = a_2 + \frac{Q}{3} \tag{18}$$

$$3\int_0^1 x^2\frac{\partial\theta}{\partial\tau}dx = -\,3B\theta + Q \tag{19}$$

$$\frac{\partial\bar{\theta}}{\partial\tau} = -3B\theta + Q \tag{20}$$

$$\frac{\partial\theta}{\partial\tau} = -\frac{3B\theta}{1 + \frac{B}{10}} + \frac{Q}{1 + \frac{B}{10}} \tag{21}$$

Table 1: Dimensionless temperature and modified Biot number for different temperature profiles of a sphere with heat generation

Sl. No.	Temperature Profile	Dimensionless Temperature	X	Y
1	$\theta = a_0(\tau) + a_1(\tau)x + a_2(\tau)x^2$	$\dfrac{e^{-X\tau} + Y}{X}$	$\dfrac{B}{\frac{1}{3} + \frac{B}{15}}$	$\dfrac{Q}{1 + \frac{B}{5}}$
2	$\theta = a_0(\tau) + a_1(\tau)(x^2 - x) + a_2(\tau)(x^3 - x^2)$		$\dfrac{B}{\frac{1}{3} + \frac{B}{30}}$	$\dfrac{Q}{1 + \frac{B}{10}}$
3	$\theta = a_0(\tau) + a_1(\tau)(x^3 - x) + a_2(\tau)x^3$		$\dfrac{3B}{(1 + \frac{B}{6})}$	$\dfrac{Q}{1 + \frac{B}{6}}$

$$\theta = \frac{e^{-x\tau} + Y}{X}$$

Where $X = \frac{B}{\frac{1}{3} + \frac{B}{30}}$, $Y = \frac{Q}{1 + \frac{B}{10}}$

For natural convection cooling in sphere by putting Q = 0 in equation (6), it is solved by similar procedure and θ is obtained as $\theta = e^{-X\tau}$ where $X = \frac{B}{\frac{1}{3}+\frac{B}{30}}$ (22)

For simple lumped model $\theta = e^{-Bi\tau}$ (23)

On comparing equation (22) and equation (23), the modified biot number Bi = $\frac{B}{\frac{1}{3}+\frac{B}{30}}$

Table 2: Dimensionless temperature and modified Biot number for different temperature profiles of natural convection cooling in sphere

Sl. No.	Temperature Profile	Dimensionless Temperature	X
1	$\theta = a_0(\tau) + a_1(\tau)x + a_2(\tau)x^2$	$\theta = e^{-X\tau}$	$\frac{B}{\frac{1}{3} + \frac{B}{15}}$
2	$\theta = a_0(\tau) + a_1(\tau)(x^2 - x) + a_2(\tau)(x^3 - x^2)$		$\frac{B}{\frac{1}{3} + \frac{B}{30}}$
3	$\theta = a_0(\tau) + a_1(\tau)(x^3 - x) + a_2(\tau)x^3$		$\frac{3B}{(1 + \frac{B}{6})}$

Table 3: Dimensionless temperature and modified Biot number for different temperature profiles of a slab with heat generation

Sl. No.	Temperature Profile	Dimensionless Temperature	X	Y
1	$\theta = a_0(\tau) + a_1(\tau)x + a_2(\tau)x^2$	$\theta = \frac{e^{-X\tau} + Y}{X}$	$\frac{B}{1 + \frac{B}{3}}$	$\frac{Q}{1 + \frac{B}{3}}$
2	$\theta = a_0(\tau) + a_1(\tau)(x^3 - x) + a_2(\tau)x^3$		$\frac{B}{1 + \frac{B}{4}}$	$\frac{Q}{1 + \frac{B}{4}}$

III. Result and Discussion

The results are obtained for different profiles at various time intervals. For these reasons the graph has been plotted to analyze the dimensionless temperature with variation of dimensionless time for different approximate temperature profiles in sphere and slab by using modified lumped model(Eq.()).

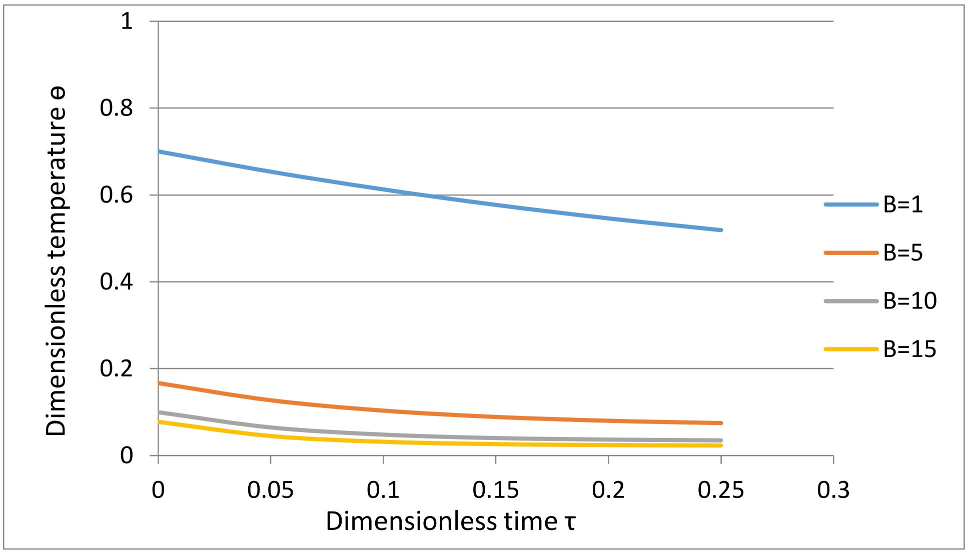

Fig.1 $\boldsymbol{\theta}$ Vs $\boldsymbol{\tau}$ for heat generated in a sphere with temperature profile $\boldsymbol{\theta = a_0(\tau) + a_1(\tau)(x^2 - x) + a_2(\tau)(x^3 - x^2)}$ for different Biot number.

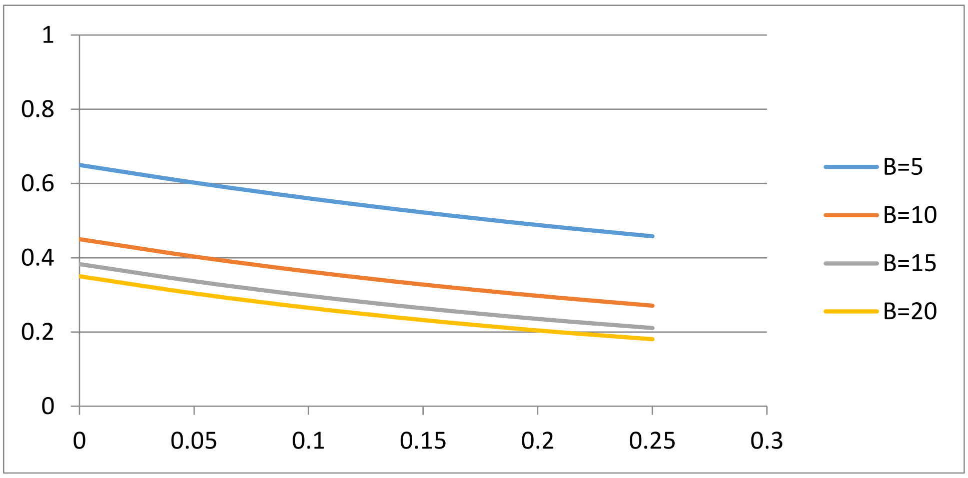

Fig.2 $\boldsymbol{\theta}$ Vs $\boldsymbol{\tau}$ for heat generated in a slab with temperature profile $\boldsymbol{\theta = a_0(\tau) + a_1(\tau)(x^2 - x) + a_2(\tau)(x^3 - x^2)}$ for different Biot number.

Fig1. and Fig.2 shows that at a fixed Biot number, θ gradually decreases with increase of time for both the geometries. For a particular geometry and particular temperature profile when Biot number increases, the magnitude of non dimensional temperature decreases. The magnitude of non-dimensional temperature for sphere is higher than the slab for a fixed time, fixed Biot number and fixed profile.

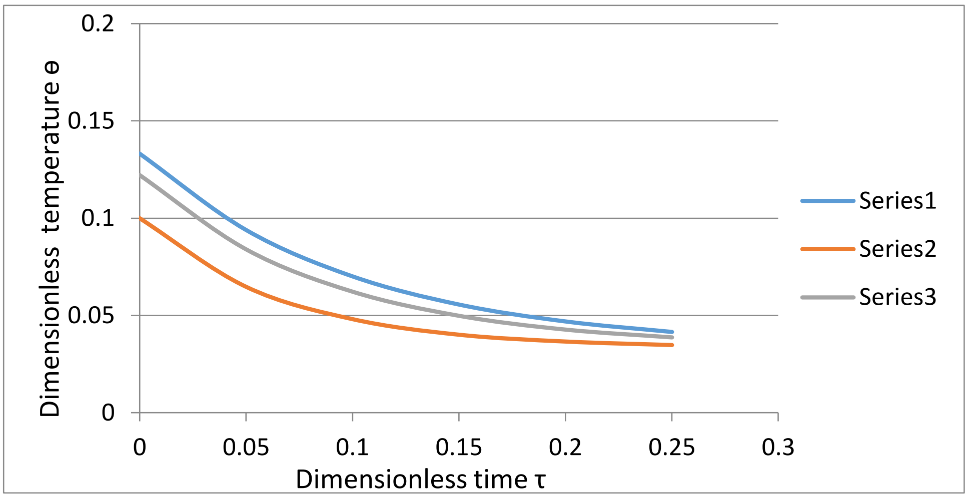

Fig.3 θ Vs τ for heat generated in a sphere for different approximate temperature profiles (B=10)

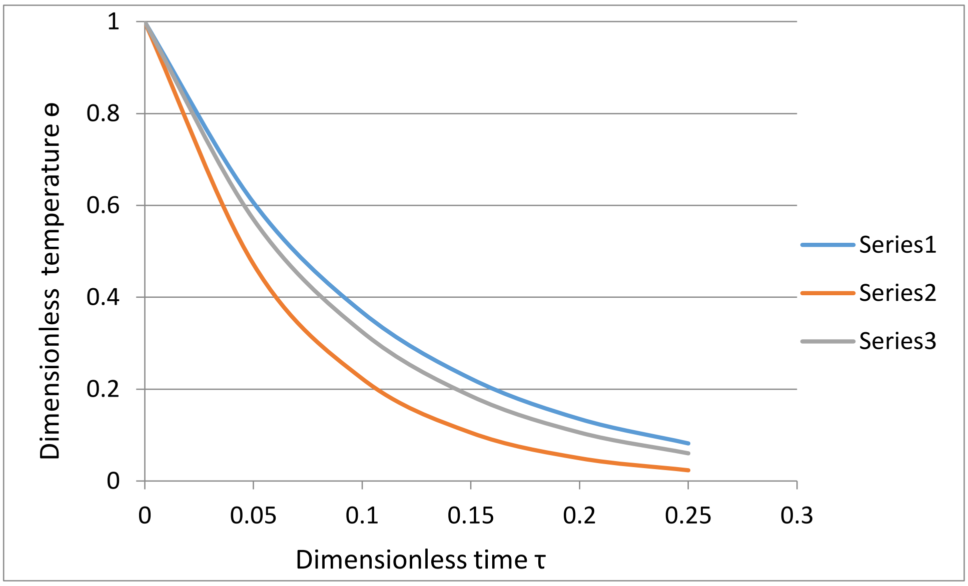

Fig.4 θ Vs τ in a sphere without heat generation for different approximate temperature profiles (B=10)

In fig.3 and fig.4 series 1, series 2 and series 3 represent approximate temperature profiles $\theta = a_0(\tau) + a_1(\tau)x + a_2(\tau)x^2$,$\theta = a_0(\tau) + a_1(\tau)(x^2 - x) + a_2(\tau)(x^3 - x^2)$ and $\theta = a_0(\tau) + a_1(\tau)(x^3 - x) + a_2(\tau)x^3$ respectively.

From the fig.3 and fig.4 it is observed that dimensional temperature is higher in case of second degree polynomial as compared third degree polynomial, for same Biot number.

V. Conclusion

A temperature versus time relationship for heat generated in slab and sphere and natural convection cooling in sphere is obtained by polynomial approximation method for various temperature profiles. It is observed the temperature decreases with time for different temperature profiles. A unique modified biot number is obtained.

IV. References

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