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Linear Stability of Rotating Viscoelastic Fluid Saturated Porous Layer

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Abstract

A stability analysis of thermal convection of a saturated rotating, viscoelastic porous layer is carried out by employing normal mode approach to linear stability. The fluid is assumed to be heated from below. To arrive at the closed form solutions, we have adopted method of small oscillations. The modified Darcy law is used to describe the fluid motion. The influence of dimensionless parameters such as relaxation, retardation, Taylor Darcy number, Rayleigh Darcy number on the stability characteristics of the flow is analyzed numerically. The analysis is restricted to long wave approximations.

Keywords: Viscoelastic fluids; thermal convection; porous media; rotation; linear stability.

1. INTRODUCTION

Convection instability is the major driving agent behind most of the weather phenomenon in the atmosphere from ordinary cyclones to hurricanes, thunderstorms and tornadoes. The study of buoyancy driven convection flows through porous media has numerous applications in several geophysical and engineering problems. Hence the study of stability of rotating Viscoelastic porous fluid saturated porous layers has received the attention of many researchers in the recent decades.

Bertola and Cafaro [1] conducted a theoretical analysis on the instability of viscoelastic fluid saturated horizontal porous layer heated from below. The effect of rotation on the overstability of viscoelastic fluid layer was analyzed by Bhatia and Steiner [2] and they found that rotation has a destabilizing influence on the oscillatory

convection problem. But this result is in contrast with an ordinary viscous fluid on which rotation has a stabilizing effect. Chanderasekhar [3] investigated the effect of Coriolis force on thermal instability of a fluid layer using various procedures and found that convection sets in for high Rayleigh number. The onset of buoyancy driven motion in a horizontal porous layer saturated by viscoelastic fluids was analyzed by Kim *et al.* [4]. They found that the onset of convection bifurcation is supercritical and stable. Kumar and Mohan [5] studied the effect of thermal instability of a heterogeneous oldroydian viscoelastic fluid heated from below in porous medium. They derived the sufficient conditions for non-existence of overstability. A linear stability analysis was carried out to study the onset of convection of viscoelastic fluid in a horizontal fluid layer heated underneath by Rajib and Layek [6] and they inferred that relaxation time destabilizes the system while retardation time stabilizes the system.

Ramesh and Arvind [7] analyzed thermal instability of Maxwell viscoelastic fluids in the presence of rotation and variable gravity and found that rotation has a stabilizing effect, while variable gravity field has destabilizing effect on the system. The stability of an incompressible simple viscoelastic fluid heated from below has been discussed by Sokolov and Tanner [9] and they found the stability criteria for the oscillatory mode. Vest and Arpaci [10] analyzed the thermal instability of Maxwell fluid which is heated from below. They confirmed that the elasticity property has a destabilizing effect on the liquid which is heated from below and oscillatory convection sets in for low Rayleigh number. Even though most of the earlier literatures apply Darcy law to characterize Newtonian fluids in porous media, few works are also found on modified Darcy law for basic understanding of convection and heat transfer in a non-Newtonian fluid saturated porous medium. Tan and Masuoka [11] proposed a Darcy Maxwell model by introducing a flow resistance term to overcome the disadvantage encountered in the modified Darcy Maxwell Jeffery model.

Yoon *et al.* [12] investigated the influence of viscoelastic effects of saturated liquids on the onset conditions in connection with oscillatory instabilities at the threshold of stationary convection. They found that the resulting oscillatory instabilities occur at lower values of Darcy-Rayleigh number than the critical value for the stationary convection. The criteria of occurrence of onset conditions of stationary convection on thermal viscoelastic fluids in rotating fluids through porous medium heated from below has been analyzed by Saini *et al.* [8] and they observed that critical Rayleigh-Darcy number for overstability increases with increase in retardation time and Taylor-Darcy number.

There are many investigations available on thermal convection of viscoelastic fluids heated from below, but attention has not been given to study the stability of the viscoelastic fluids. So our aim of this paper is to extend the work of Saini *et al.* [8] to study the stability characteristics of thermal convective rotating viscoelastic fluid saturated porous layer and to investigate the effects of relaxation, retardation, Taylor Darcy number and Rayleigh Darcy number on the flow characteristics using the linear analysis.

2. MATHEMATICAL FORMULATION

We have considered an infinite convective saturated rotating horizontal porous layer of viscoelastic fluid confined between two rigid boundaries of vertical height *d* which is heated from below as shown in Figure 1. The porous medium is homogeneous and isotropic. The porous layer is subject to rotation with uniform angular velocity ω.

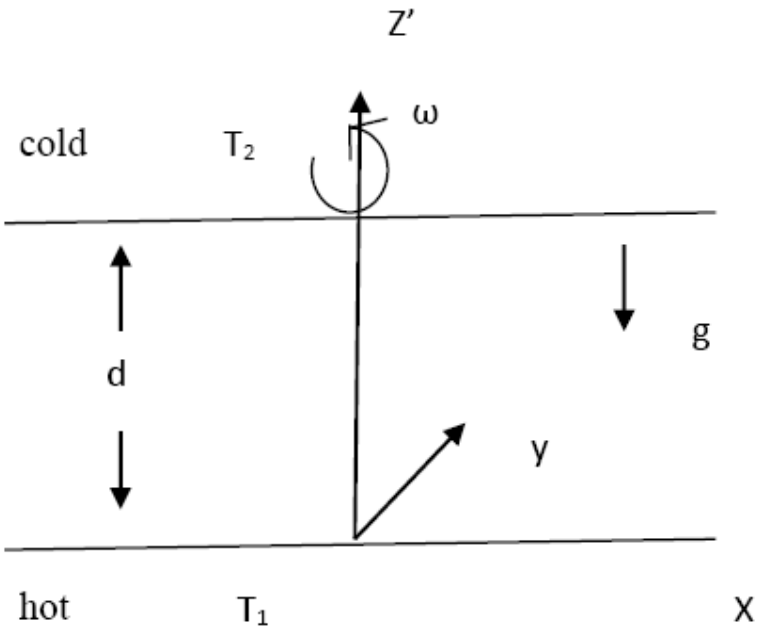

Figure 1: Geometric representation of the problem

Using Boussinesq approximation and modified Darcy Model the governing equations can be written as

$$\nabla.\vec{U} = 0 \tag{1}$$

$$\frac{\mu}{\kappa}\left(1+\bar{\varepsilon}\frac{\partial}{\partial t}\right)\vec{U} = \left(1+\bar{\lambda}\frac{\partial}{\partial t}\right)\left(-\nabla p+\rho g-\frac{2\omega\rho_0\vec{U}}{\phi}\hat{K}\right) \tag{2}$$

$$\frac{\partial T}{\partial t}+\left(\vec{U}.\nabla\right)T = \alpha\nabla^2 T \tag{3}$$

$$\rho = \rho_0[1-\beta(T-T_1)] \tag{4}$$

where $\vec{U}$, κ, $\bar{\varepsilon}$, $\bar{\lambda}$, $p, T, \rho, g, \alpha, \beta, \hat{K}$, ϕ and ω denote velocity vector, permeability, strain retardation time, stress relaxation time, pressure, temperature, density, gravitational acceleration, effective thermal diffusivity, thermal expansion coefficient, unit vector in the z – direction, porosity of porous medium and uniform angular velocity respectively.

At quiescent state, temperature varies across the layer thickness. The basic state of the system is

$$\vec{U}_b = (0,0,0), \quad \rho = \rho_b(z), \quad p = p_b(z), \quad T = T_b(z) \tag{5}$$

Using equation (5) in (1), (2), (3) and (4), we have

$$-\frac{dp_b}{dz} + \rho_b = 0 \tag{6}$$

$$\rho_b = \rho_0[1 - \beta(T_b - T_1)] \tag{7}$$

$$T_b = T_1 - \frac{\Delta T}{d} z \tag{8}$$

where the subscript b denotes the basic state of the system.

To study the stability of the system the perturbation on the basic state is taken in the form

$$\vec{U} = \vec{U}_b + \vec{U}', \; T = T_b + T', \; p = p_b + p' \tag{9}$$

where the prime denotes the perturbation.

The perturbed equations are obtained by substituting equation (9) in (1), (2), (3) and (4).

$$\nabla.\vec{U}' = 0 \tag{10}$$

$$\frac{\mu}{k}\left(1 + \bar{\varepsilon}\frac{\partial}{\partial t}\right)\vec{U}' = \left(1 + \bar{\lambda}\frac{\partial}{\partial t}\right)\left(-\nabla p' + \rho_0 \beta T' g - \frac{2\omega\rho_0\vec{U}'}{\phi}\hat{k}\right) \tag{11}$$

$$\frac{\partial T'}{\partial t} - \frac{\Delta T}{\mathrm{d}} W = \alpha \nabla^2 T' \tag{12}$$

where W is the z component of velocity.

Introducing the non dimensional quantities $\lambda = \frac{\bar{\lambda}\alpha}{d^2}$, $\varepsilon = \frac{\bar{\varepsilon}\alpha}{d^2}$, $\frac{\alpha}{d}$, ΔT, $\frac{\mu\alpha}{d^2}$ *and* $\frac{d^2}{\alpha}$ for relaxation, retardation, velocity, temperature, pressure and time respectively, equations (10) - (12) becomes

$$\nabla.\vec{U} = 0 \tag{13}$$

$$\frac{1}{Da}\left(1 + \varepsilon\frac{\partial}{\partial t}\right)\vec{U} = \left(1 + \lambda\frac{\partial}{\partial t}\right)\left(-\nabla \mathrm{p} - Ta^{1/2}\vec{U}\hat{k} + RaT\hat{k}\right) \tag{14}$$

$$\frac{\partial T}{\partial t} + (\vec{U}.\nabla)T = \nabla^2 T \tag{15}$$

where

$D_a = \frac{k}{d^2}$ (Darcy number)

$T_a = \left(\frac{2\rho_0\omega d^2}{\phi\mu}\right)^2$ (Taylor number) and

$$R_a = \left(\frac{\rho_0 g \beta \Delta T d^3}{\mu \alpha}\right) \text{ (Rayleigh number).}$$

and the corresponding boundary conditions are

$$\left.\begin{aligned} W = 0,\ T = T_1 \ \text{ at } \ \mathrm{z} = 0 \text{ and } \mathrm{z} = \mathrm{d} \\ W = 0,\ T = T_2 \ at\ z = 0\ and\ \ z = d \end{aligned}\right\} \tag{16}$$

3. LINEAR STABILITY ANALYSIS

Eliminating the pressure gradient by taking curl of equation (14) and using the definitions

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad \text{(Horizontal Laplacian operator)}$$

$$R_D = R_a \,.\, D_a \quad \text{(Rayleigh – Darcy number)}$$

$$T_D = T_a \,.\, {D_a}^2 \quad \text{(Taylor – Darcy number)}$$

We get the following linearized perturbation equations

$$\left(1 + \varepsilon \frac{\partial}{\partial t}\right) \nabla^2 W - R_D \left(1 + \lambda \frac{\partial}{\partial t}\right) \nabla^2 T + T_D \left(1 + \lambda \frac{\partial}{\partial t}\right) \frac{\partial^2 W}{\partial z^2} = 0 \tag{17}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right) T = W \tag{18}$$

Applying the normal mode analysis

$$[W(x,y,z,t), T(x,y,z,t)] = [W(z), T(z)] e^{[i(a_x x + a_y y) + \sigma t]}$$

$$a = \left[{a_x}^2 + {a_y}^2\right]^{1/2}$$

where a_x and a_y are horizontal wave numbers, σ is the temporal growth rate of harmonic disturbance and W(z), T(z) are functions of z.

Equations (17) and (18) reduce to the form

$$(1 + \varepsilon\sigma)(\mathrm{D}^2 - \mathrm{a}^2) W + R_D (1 + \sigma\lambda) \alpha^2 T + T_D (1 + \sigma\lambda) \mathrm{D}^2 W = 0 \tag{19}$$

$$(\mathrm{D}^2 - \mathrm{a}^2 - \sigma) T = -W \tag{20}$$

$$(1 + \varepsilon\sigma)(\mathrm{D}^2 - \mathrm{a}^2)(\mathrm{D}^2 - \mathrm{a}^2 - \sigma) T - R_D (1 + \sigma\lambda) \alpha^2 T$$
$$+ T_D (1 + \sigma\lambda) \mathrm{D}^2 (\mathrm{D}^2 - \mathrm{a}^2 - \sigma) T = 0 \tag{21}$$

and the corresponding boundary condition are

$$W = T = \mathrm{D}^2 T = 0 \ \text{ at } \ \mathrm{z} = 0 \text{ and } \mathrm{z} = 1 \tag{22}$$

where D denotes differentiation w.r.t z.

4. EIGEN VALUE AND EIGEN FUNCTION

Assuming that the nondimensional wave number a is small (a <<1), we introduce the series expansions for the solutions of the form

$$\left.\begin{aligned} \sigma &= \sigma_0 + a^2\sigma_1 + a^4\sigma_2 + \cdots \\ T &= T_0 + a^2T_1 + a^4T_2 + \cdots \\ W &= W_0 + a^2W_1 + a^4W_2 + \cdots \end{aligned}\right\} \tag{23}$$

Collecting terms of o (1), the governing equations for zeroth order approximations are given by

$$(1+\varepsilon\sigma_0)D^2(D^2-\sigma_0)T_0 + T_D(1+\lambda\sigma_0)D^2(D^2-\sigma_0)T_0 = 0 \tag{24}$$

$$(1+\varepsilon\sigma_0)D^2W_0 + T_D(1+\lambda\sigma_0)D^2W_0 = 0 \tag{25}$$

The boundary conditions are

$$W_0 = T_0 = \mathrm{D}^2T_0 = 0 \text{ at } \mathrm{z} = 0 \text{ and } \mathrm{z} = 1 \tag{26}$$

The solutions of equation (24) and (25) on applying boundary conditions (26) are

$$\sigma_0 = -n^2\pi^2$$

$$T_0 = \sin(n\pi z)$$

$$W_0 = 0$$

When we take coefficient of o (a^2), the equations becomes

$$(1+\varepsilon\sigma_0)D^2(D^2-\sigma_0)T_1 + T_D(1+\lambda\sigma_0)D^2(D^2-\sigma_0)T_1$$
$$= R_D(1+\lambda\sigma_0)T_0 + [(1+\varepsilon\sigma_0) + T_D(1+\lambda\sigma_0)]D^2(1+\sigma_1)T_0 \tag{27}$$

$$(1+\varepsilon\sigma_0)D^2W_1 + T_D(1+\lambda\sigma_0)D^2W_1 = -R_D(1+\lambda\sigma_0)T_0 \tag{28}$$

and the boundary conditions becomes

$$W_1 = T_1 = \mathrm{D}^2T_1 = 0 \text{ at } \mathrm{z} = 0 \text{ and } \mathrm{z} = 1 \tag{29}$$

The corresponding solutions are

$$\sigma_1 = -\frac{A_5}{A_6}$$

$$T_1 = A_4\sin(n\pi z) + (A_5 + A_6\,\sigma_1)z\,cos(n\pi z)$$

$$W_1 = A_9\sin(n\pi z)$$

Collecting the coefficients of o (a^4), we get the equations

$$(1+\varepsilon\sigma_0)D^2(D^2-\sigma_0)T_2 + T_D(1+\lambda\sigma_0)D^2(D^2-\sigma_0)T_2$$
$$= \{-(1+\varepsilon\sigma_0) + (1+\varepsilon\sigma_0)D^2\sigma_2 - (1+\varepsilon\sigma_0)\sigma_1$$
$$+\varepsilon\sigma_1^2D^2 + R_D\lambda\,\sigma_1 + T_D\lambda\,\sigma_1D^2 + T_D(1+\lambda\sigma_0)D^2\sigma_2$$

$$+\varepsilon\sigma_1 D^2 + T_D\lambda D^2\sigma_1^2\}T_0 + \{(1+\varepsilon\sigma_0)D^2 + (1+\varepsilon\sigma_0)D^2\,\sigma_1$$
$$+R_D(1+\lambda\sigma_0) + T_D(1+\lambda\sigma_0)D^2 + T_D(1+\lambda\sigma_0)D^2\sigma_1\}T_1 \tag{30}$$

$$(1+\varepsilon\sigma_0)D^2W_2 + T_D(1+\lambda\sigma_0)D^2W_2 = -R_D\{(1+\lambda\sigma_0)T_1 + \lambda\sigma_1 T_0\} - \varepsilon D^2\,\sigma_1 W_1 - T_D\lambda\,\sigma_1 D^2 W_1 \tag{31}$$

and the corresponding boundary conditions are

$$W_2 = T_2 = \mathrm{D}^2 T_2 = 0 \text{ at } \mathrm{z} = 0 \text{ and } \mathrm{z} = 1 \tag{32}$$

Hence the solutions are

$$\sigma_2 = -\frac{1}{A_{16}}[A_{14} + A_{15}\,\sigma_1 + A_{17}\sigma_1^2]$$
$$T_2 = A_{13}\sin(n\pi z) + (A_{14} + A_{15}\,\sigma_1 + A_{16}\,\sigma_2 + A_{17}\sigma_1^2)z\,cos(n\pi z)$$
$$W_2 = A_{20}\sin(n\pi z)$$

5. NUMERICAL DISCUSSION

To get physical insight into the problem, the effects of various non-dimensional parameters such as stress relaxation time λ, strain retardation time ε, Taylor Darcy number T_D (rotation parameter), Rayleigh Darcy number R_D, wave number a and parameter n on stability of fluid flow are discussed numerically and plotted in figures (1) – (16). For graphical results we have considered fixed values of parameters $\varepsilon = 0.2$, $T_D = 1.0, \lambda = 0.5$, $R_D = 5.0$ throughout the entire study except for the varied values as shown below.

Figures (1) – (3) depict the effect of parameter n, Taylor Darcy number T_D and Rayleigh Darcy number R_D on temporal growth rate. It is found that increase in n and rotation decreases the growth rate thereby creating a stable mode in the system, whereas increase in R_D shows an increase in magnitude of frequency which is negative, thus making the system stable. As strain retardation time ε increases there is a decrease in the growth rate and when there is an increase in stress relaxation time, the growth rate increases as shown in figures (4) and (5). Thus the effect of increasing strain retardation time and stress relaxation time stabilizes the system.

The influence of Rayleigh Darcy number R_D and stress relaxation time λ with respect to Taylor Darcy number T_D on temporal growth is shown in figures (6) and (7) and it is seen that the system is stable for increasing R_D and λ. Figures (8) and (9) depict the effect of strain retardation time ε and parameter n with respect to Taylor Darcy number T_D on temporal growth. It is observed that as ε and n increase growth rate gets decreased, thereby creating a stable mode in the system.

The effect of stress relaxation time λ on temporal growth with respect to Rayleigh Darcy number R_D is plotted in figure (10) and it shows that as R_D increases, the

temporal growth also increases which depicts that Rayleigh Darcy number increases the region of stability.

Figures (11) – (12) depict that increase in wave number a and stress relaxation time λ, enhances the velocity profile. The influence of Taylor Darcy number T_D, strain retardation time ε is illustrated in figures (12) – (13). It is observed that increase in rotation parameter and strain retardation time causes depreciation in the velocity profile.

Figures (15) – (16) shows the influence of wave number a and parameter n on temperature profile. It is observed that thermal convection increases due to increase in wave number and oscillatory convection sets in whenever parameter n increases.

6. CONCLUSION

A linear stability analysis of thermal convection in a viscoelastic fluid saturated porous medium subject to uniform rotation has been carried out and the effects of stress relaxation time λ, strain retardation time ε, Taylor Darcy number T_D, Rayleigh Darcy number R_D, wave number a and parameter n were analyzed. Using normal mode analysis the perturbed equations were solved. We observe that increasing strain retardation time and stress relaxation time stabilizes the system. Also it is observed that due to increase in rotation parameter and Rayleigh Darcy number R_D the disturbance decay. It can be inferred from this work that the system remains stable when subjected to long waves.

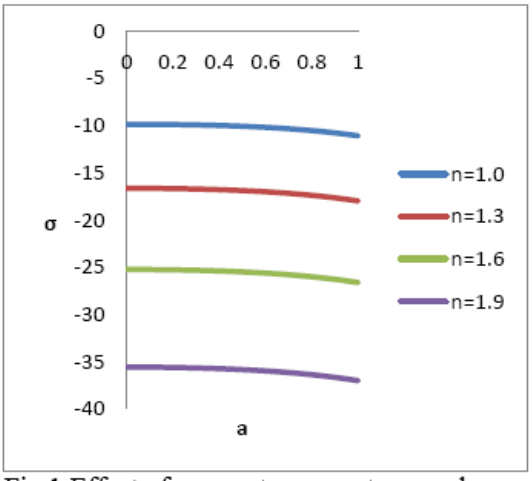

Fig.1 Effect of parameter n on temporal growth rate

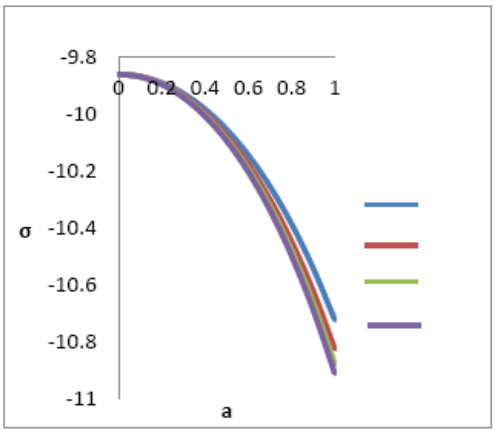

Fig.2 Effect of Taylor Darcy number T_D on temporal growth rate

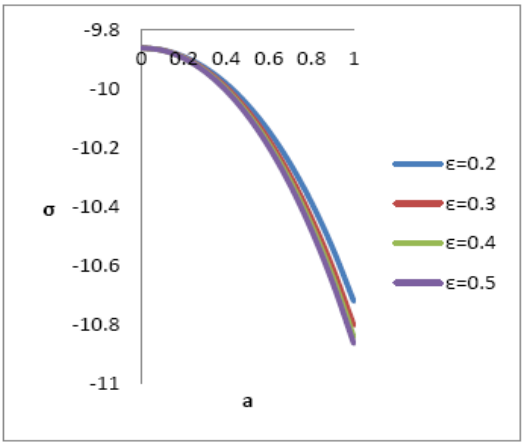

Fig.3 Effect of strain retardation time ε on temporal growth rate

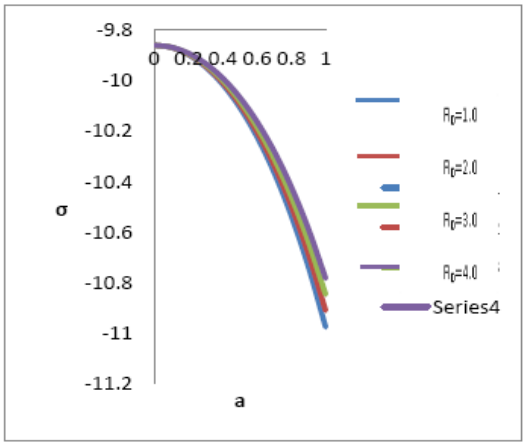

Fig.4 Effect of Rayleigh Darcy number R_D on temproal growth rate

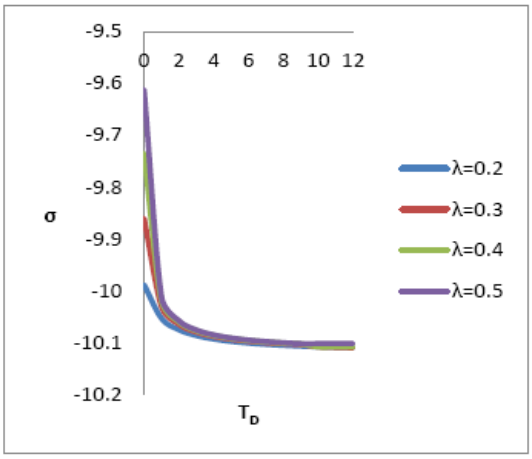

Fig.7 Effect of stress relaxation time λ on temporal growth rate

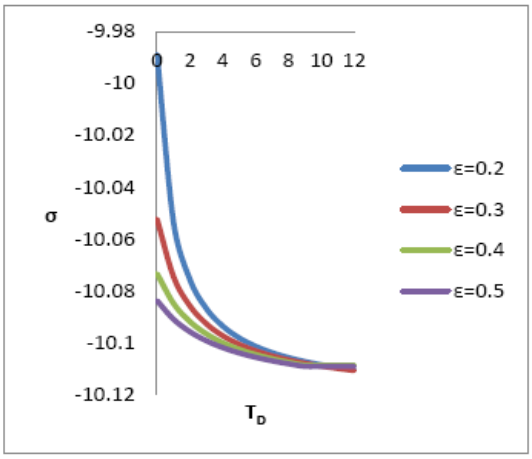

Fig 8 Effect of strain retardation time ε on temporal growth rate

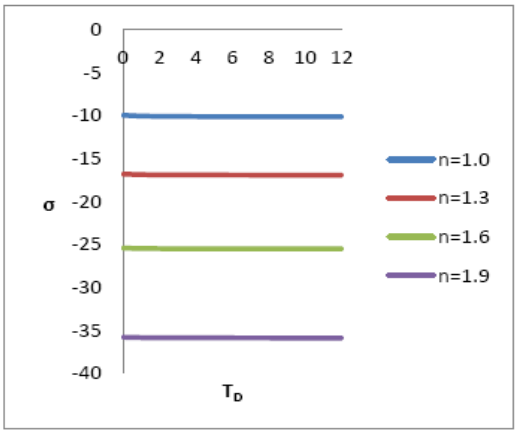

Fig.9 Effect of parameter n on temporal egrowth rate

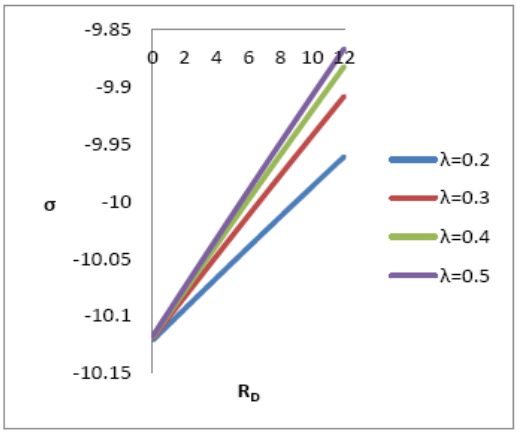

Fig.10 Effect of stress relaxation time λ on temporal growth rate

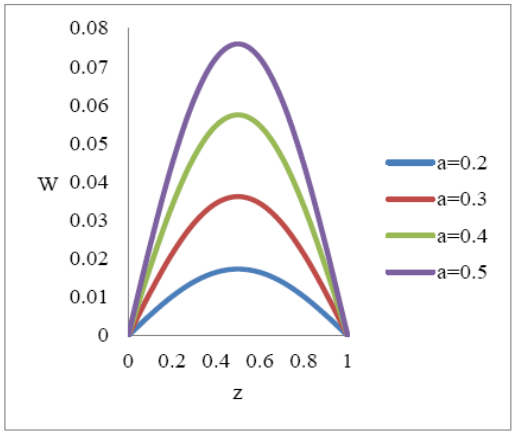

Fig.11 Effect of wave number a on velocity Profile

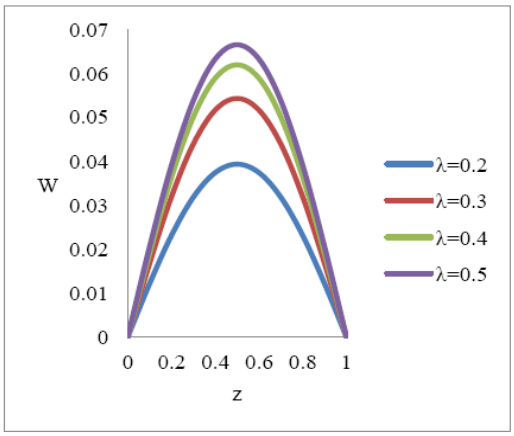

Fig.12 Effect of stress relaxation time λ on Velocity profile

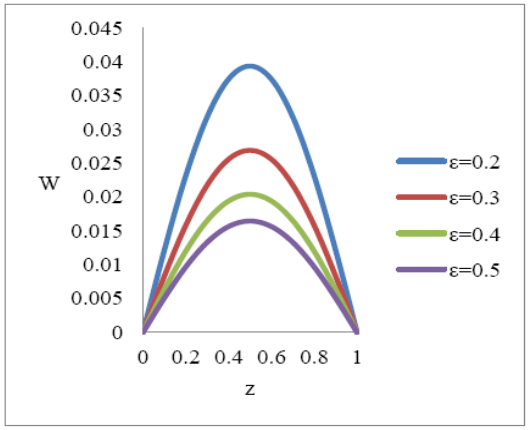

Fig.13 Effect of strain retardation time ε on velocity profile

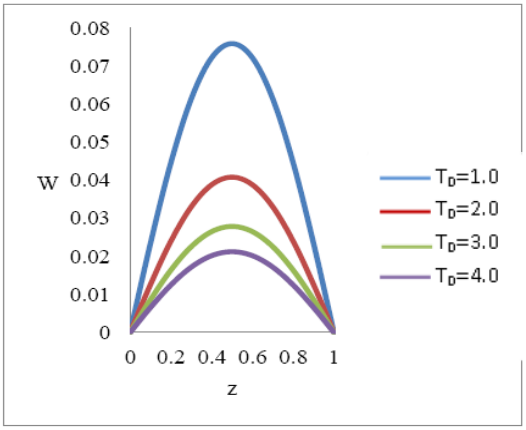

Fig.14 Effect of Taylor Darcy number T_D on velocity profile

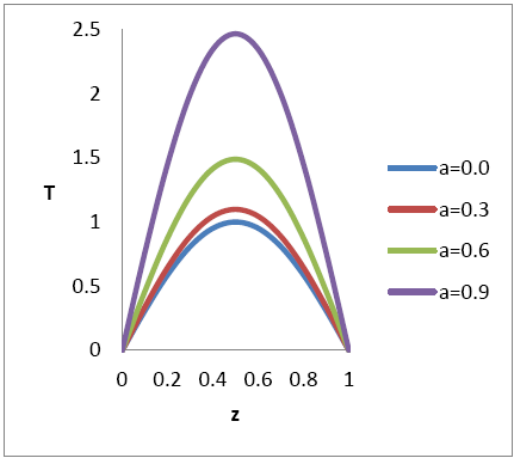

Fig. 15 Effect of wave number a on temperature field

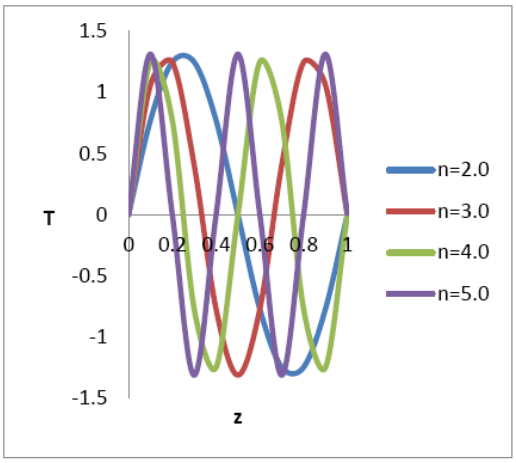

Fig. 16 Effect of parameter n on temperature field

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