International Journal of Computational Intelligence Research
ISSN 0973-1873 Volume 12, Number 2 (2016), pp. 125-138

http://www.ripublication.com

Neural Network Based Sliding Mode Control for Uncertain Discrete-Time Nonlinear Systems with Time-Varying Delay

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Abstract

The present paper discusses the stability analysis of discrete-time uncertain nonlinear systems with time-varying delay, uncertainties related to parametric uncertainty and unknown nonlinearity. The time-varying delay considered has minimum and maximum bounds. The novelty of Chebyshev Neural Network (CNN) is that it requires much less computation time as compared to multi layer neural network (MLNN) and radial basis function network (RBFN). It is preferred to approximate the unknown nonlinearities. Furthermore, results for robust stabilization of discrete-time uncertain nonlinear systems with time-varying delay are given on the basis of linear matrix inequalities (LMI). The sufficient condition is derived for the asymptotic stability of the defined systems. The proposed controller guarantees the system state trajectory to the designed sliding surface in the presence of uncertainties and time-varying delay. Simulation results illustrate the validity of the proposed approach.

Key-words: Chebyshev Neural Network; Sliding Mode Control; Linear Matrix Inequalities; Lyapunov-Krasovskii Function; Time-varying Delay

1. INTRODUCTION

Time-varying delay often appears in various systems, such as robotic systems, motion control systems, mechanical engineering systems and so on. The time-varying delay in such systems degardes the system performance and are often source of instability. However, in motion control systems the inaccurate modelling and errors caused due to external conditions uncertainties and nonlinearities, deteriorate system performance. Over the past few years, the stability analysis of time-varying delayed systems with uncertainties have been studied and documented in [1]-[6]. The stability analysis of continous time system with time-varying delay using Leibniz-Newton formula and LMI is proposed in [7]. In [8], a new stability condition is derived for discrete time delayed systems, dependent on maximum and minimum delay bounds is also investigated. An LMI based stability analysis for uncertain discrete system with time–varying delay, dependent on upper and lower bounds is proposed in [9]. In [10], a new technique is proposed consisting of time-varying dealy with delay bounds which make the system more robust. In [11], a new stability condition for discrete systems with time-varying delay is proposed using Moon's inequality [12], which is further extended to norm-bounded unceratinties. Among various design techniques for robust control sliding mode control (SMC) is found to be an alternative approach for motion control systems which is insensitive to mismatched uncertainties, nonlinearities and its fast response with asymptotic stability. SMC is considered to be an extension of variable structure control (VSC) [13]. Over past few years, attention has been given to the problem of DT-SMC for time-varying uncertain systems. In [14], a controller is designed using SMC for discrete time systems with uncertainties and bounded disturbances.The robust SMC for discrete systems which include both parametric uncertainties and matched external disturbance, is proposed and investigated in [15]. In [16], sliding mode approach is proposed for control of uncertain time delay system, and sufficient condition for delay independence is derived in terms of LMI which guarantee the reaching condition. By using adaptive algorithms unknown systems parameters and nonlinearities can be estimated. In [17]-[18], robust stabilization of discrete uncertain time-delay systems is designed using adaptive control technique. The approximation of unknown nonlinearities in the presence of time-varying delay and parametric uncertainties for discrete time systems is of interest. Now, it is an well-known fact that neural networks have been extensively used as an approximation tool for unknown nonlinearities [19]. Neural network appears as a powerful tool for control of discrete systems with time-varying delay [20]-[22]. The novality of CNN is that it requires much less computational time as compared to other neural networks like MLNNs, RNNs. The effectiveness of CNN in the stability analysis of unknown discrete-time nonlinear system with constant delay is investigated in [19], using backstepping technique.

This paper considers robust sliding mode controller for uncertain discrete time systems with time-varying delay and unknown nonlinearities. The delay is time-varying with upper and lower bound. The unknown nonlinearities are approximated using CNN. With the help of CNN, weight update law is derived to make this scheme adaptive. An LMI condition is derived for the designed controller reducing the presence of time-varying delay with upper and lower bounds, parametric uncertainties and nonlinearities, which prove the existence of stable sliding surface. Using this condition, and by selecting control law the robust controller is designed to guarentee the reaching condition of the specified sliding surface. The convergence of the states to the origin is also guaranteed. The organization of this paper is as follows. Section 2 elaborates the CNN structure. Preliminaries and problem formulation are presented in Section 3. The controller design is given in Section 4. The stability analysis for the proposed controller is presented in Section 5. The effectiveness of the designed controller is validated by simulation results in Section 6. The paper has concluding remarks in Section 7.

Notations: $\|.\|_F$ implies Frobenius norm, $\|.\|$ denotes Euclidean norm, $tr(.)$ stand for trace of matrix.

2. Chebyshev Neural Network STRUCTURE

An artificial neural network has group of nonlinear elements in it which are interconnected to each other. ANN has evolved as a powerful tool for system identification. It has been studied that an ANN with an hidden layer can approxinmate nonlinear fumctions accurately [23].

Some of the available ANN configurations are multilayer perceptron (MLP), chebyshev neural networks (CNNs), radial basis function network (RBFN) etc. The disadvantage of MLP network is that it requires significant computation for learning as compared to other networks. RBFN is much simpler network as compared to MLP but for effective learning, choosing an appropriate set of RBF centers is a problem. A functional link artificial neural network (FLANN) as proposed by [24] is a single layer network in which the requirement of hidden layer is eliminated by expanding the original input pattern using chebyshev polynomials. The advantage of FLANN is that it requires less computation time and faster convergence rate as compared to MLP and RBFN.

CNN is a functional link feed forward neural network based on chebyshev polynomials. CNN requires less computational time due to the use of complex nonlinear chebyshev polynomials. It has two parts namely, learning and numerical transformation [25]. A finite set of Chebyshev polynomials are used in numerical transformation as a functional expansion (FE) of input pattern. Functional-link neural network based on chebyshev polynomials is the learning part. The higher order chebyshev polynomials is obtained by a recursive formula [25]

$$T_{i+1}(x) = 2xT_i(x) - T_{i-1}(x), \qquad \mathrm{T}_0(x) = 1 \tag{1}$$

where, $T_1(x)$ are Chebyshev polynomials, i is the order of polynomials chosen and x is a scalar quantity. The different choices of $T_1(x)$ are $x \,\&\, 2x$.

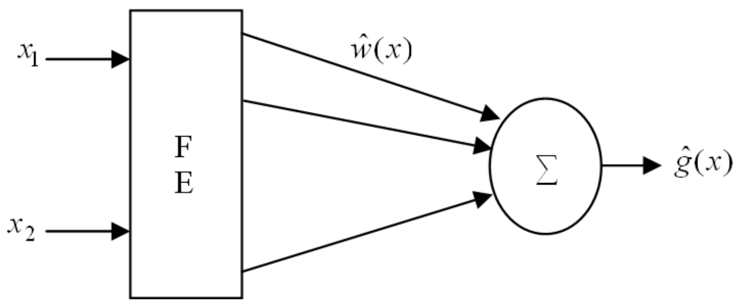

Figure 1: Chebyshev Neural Network[19]

The single layer neural network output is given by

$$\hat{g}(x) = \hat{w}^T \phi \qquad (2)$$

where, w are the weights and ϕ is the basis function of neural network. Based on the approximation property of CNN, there exist ideal weights w, so that the function $g(x)$ to be approximated can be represented as

$$g(x) = w^T \phi + \varepsilon \qquad (3)$$

where, ε is the CNN functional reconstruction error vector and $\|\varepsilon\| \le \varepsilon_N$ is bounded. Approximation of complex nonlinear systems becomes easier as CNN is a single layer neural network.

3. PROBLEM FORMULATION

Consider the following discrete-time system with time-varying delay as in

$$x(k+1) = (A + \Delta A)x(k) + (A_d + \Delta A_d)x(k - h(\mathrm{k})) + B[g(x(k))u(k)] \qquad (4)$$

where, $x(k) \in R^n$ and $u(k) \in R^m$ indicate the state and input vectors respectively. A, A_d and B are known real constant matrices with appropriate dimensions. $g(x(k))$ is unknown nonlinear function of a given system in (4), and $h(\mathrm{k})$is a positive number representing time-varying delay. ΔA and ΔA_d denote real-valued matrix function representing unmatched parametric uncertainties.

Assumption 1: The time-varying delay $h(\mathrm{k})$ satisfy the upper and lower bound on the time delay given by $h_m \le h(k) \le h_M$ where h_m and h_M are constant positive scalar.

Assumption 2: The uncertainties are norm-bounded and satisfy the following assumption [17]:

$$\Delta A = \sum_{i=1}^{p} \alpha_i A_i, \quad |\alpha_i| \le 1 \qquad (5)$$

$$\Delta A_d = \sum_{i=1}^{p} \beta_i A_{di}, \quad |\beta_i| \le 1 \qquad (6)$$

where α_i and β_i are unknown scaling parameters for above given uncertainties. Suppose G , H , G_d and H_d are known real constant matrices with appropriate dimensions,then

$$\Delta A = GDH = [G_1 \ldots\ldots G_p] D [H_1^T \ldots\ldots H_p^T]^T \qquad (7)$$

$$\Delta A_d = G_d D_d H_d = [G_{d1} \ldots\ldots G_{dq}] D_d [H_{d1}^T \ldots\ldots H_{dq}^T]^T \qquad (8)$$

where

$D = blockdiag[\alpha_1 \ldots\ldots\ldots \alpha_p]$

$D_d = blockdiag[\beta_1 \ldots\ldots\ldots \beta_q]$

Assumption 3: In nominal system the pair (A, B) is controllable and rank $(B) = m$

As per this assumption [15], there exist a nonsingular matrix $M \in R^{n \times n}$ such that

$$MB = \begin{bmatrix} 0_{(n-m)xm} \\ B_2 \end{bmatrix} \quad (9)$$

where B_2 is nonsingular. Let us consider the transformation matrix as

$$M = \begin{bmatrix} U_2^T \\ U_1^T \end{bmatrix} \quad (10)$$

where $U_1 \in R^{n \times m}$ and $U_2 \in R^{n \times (n-m)}$ are two sub-blocks of a unitary matrix resulting from singular value decomposition(SVD) of matrix B ,i.e.,

$$B = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma \\ 0_{(n-m)xm} \end{bmatrix} V^T \quad (11)$$

where $\Sigma \in R^{m \times m}$ is a diagonal positive-definite matrix and $V \in R^{m \times m}$ is a unitary matrix [9]. By applying state transformation $z = Mx$, system (4) has the regular form as

$$z(k+1) = (\overline{A} + \Delta \overline{A})\, z(k) + (\overline{A}_d + \Delta \overline{A}_d)\, z(k - h(k)) + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} (g(z(k))\, u(k)) \quad (12)$$

where

$\overline{A} = MAM^{-1}$, $\Delta\overline{A} = M\Delta AM^{-1}$, $\overline{A}d = MA_dM^{-1}$, $\Delta\overline{A}d = M\Delta A_dM^{-1}$ and $\overline{B} = MB = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}$.

System (14) can be rewritten as

$$\begin{aligned} z_1(k+1) &= (\overline{A}11 + \Delta\overline{A}11)z_1(k) + (\overline{A}d11 + \Delta\overline{A}d11)z_1(k-h(k)) \\ &\quad + (\overline{A}12 + \Delta\overline{A}12)z_2(k) + (\overline{A}d12 + \Delta\overline{A}d12)z_2(k-h(k)) \end{aligned} \quad (13)$$

$$\begin{aligned} z_2(k+1) &= (\overline{A}21 + \Delta\overline{A}21)z_1(k) + (\overline{A}d21 + \Delta\overline{A}d21)\, z_1(k-h(k)) \\ &\quad + (\overline{A}22 + \Delta\overline{A}22)\, z_2(k) + (\overline{A}d22 + \Delta\overline{A}d22)\, z_2(k-h(k)) + B_2\,(g(z(k))\; u(k)) \end{aligned} \quad (14)$$

where

$z_1 \in R^{n-m}, z_2 \in R^m, B_2 = \Sigma V^T$, $\overline{A}11 = U_2^T AU_2$, $\overline{A}12 = U_2^T AU_1$, $\overline{A}d11 = U_2^T A_d U_2$, $\overline{A}d12 = U_2^T A_d U_1$, $\Delta\overline{A}11 = U_2^T \Delta AU_2 = U_2^T GDHU_2$, $\Delta\overline{A}12 = U_2^T \Delta AU_1 = U_2^T GDHU_1$, $\Delta\overline{A}d11 = U_2^T \Delta A_d U_2 = U_2^T G_d D_d H_d U_2$, $\Delta\overline{A}d12 = U_2^T \Delta A_d U_1 = U_2^T G_d D_d H_d U_1$.

4. DISCRETE SLIDING MODE CONTROLLER

The design of sliding surface will be the first step in the design of discrete-time SMC control algorithm. The linear sliding surface is defined as:

$$s(k) = \overline{C}z(k) = \begin{bmatrix} C & I_m \end{bmatrix} z(k) = Cz_1(k) + z_2(k) \quad (15)$$

where $\overline{C} \in R^{m \times n}$ and $C \in R^{m \times (n-m)}$ are real matrix of appropriate dimensions. For a system to be asymptotically stable, the sliding surface is defined as follows.

$$s(k) = 0 \quad (16)$$

Substituting $z_2(k) = -Cz_1(k)$ into (15) gives ideal quasi-sliding condition

$$
\begin{aligned}
z_1(k+1) &= (\bar{A}11 + \Delta\bar{A}11 - \bar{A}12C - \Delta\bar{A}12C)\, z_1(k) \\
&\quad + (\bar{A}d11 + \Delta\bar{A}d11 - \bar{A}d12C - \Delta\bar{A}d12C)\, z_1(k - h(k))
\end{aligned} \tag{17}
$$

The sub-system (19) represents sliding motion of discrete-time system (4).
The design of control law will be the second step which guarantee the sliding mode reaching condition of the given linear sliding surface. The control law is

$$
u(k) = (-B_2^{-1} / \hat{g}(z(k))) \begin{bmatrix} qTs\ (k) + \varepsilon T \operatorname{sgn}\ (s(k)) + \bar{C}(\bar{A} - I)z(k) \\ + \overline{CA}d\ z\ (k - h\ (k)\ ) + diag\ [\operatorname{sgn}\ (s_i)]\, f_d\ (k) \end{bmatrix} \tag{18}
$$

where

$$
f_d(k) = \sum_{j=1}^{p} \left| \overline{C}TA_j T^{-1} z(k) \right| + \sum_{j=1}^{q} \left| \overline{C}TA_{dj} T^{-1} z(k - h(k)) \right|
$$

$q > 0, \varepsilon > 0, 1 - qT > 0, T$ is sampling time. q is the approximation rate and ε is the reaching rate.
The complete definition for SMC in discrete-time system and presented a reaching law in the form of equation given below [26].

$$
s(k+1) = (1 - qT)s(k) - \varepsilon T \operatorname{sgn}(s(k)) \tag{19}
$$

This reaching law guarantees the systems states trajectory starting from any initial state, will move monotonically towards the sliding surface and cross it in finite time. After crossing the switching plane for the first time, the trajectory will again crosses the switching plain in every successive sampling period, resulting in a zigzag motion around the switching plane. Therefore, the trajectory of the system (4) forced to be driven by the control law (20) into sliding mode band in finite time and stay on it.

5. STABILITY ANALYSIS

For the given unknown nonlinear system, following assumptions are required for the stability analysis.

Assumption 4: The state delay $h(k)$ is a time-varying delay which is induced by the network transmission. For time-varying delay the upper and lower bounds are taken as h_M, h_m respectively

Assumption 5: The nonlinear function $g(x(k))$ is bounded and unknown.

Assumption 6: The neural network weights w are bounded so that $\| w \| \le w_M$, where w_M is a known bound. $\| \cdot \|_F$ denotes the Frobenius norm, the Frobenius norm is given as, $\|A\|_F^2 = tr\left(A^T A\right)$

Assumption 6: Let $\tilde{g}(x(k)) = G\hat{g}(x(k))$, where $G = G^T$ is a $n \times n$ symmetric matrix, and $\tilde{g}(x(k))$ and $\hat{g}x(k)$ are the n-column vectors.

Theorem 1: Given the system (4) and Assumptions (3-6), control law (20), the weight update law is given by

$$\begin{aligned}\hat{w}(k+1)=\hat{w}(k)&+\left[z^T(k)\begin{bmatrix}-\bar{\bar{A}}^T(P+R)\,G\,\bar{C}\,(\bar{A}-I)+(\bar{A}-I)^T\bar{C}\,(P+R)\,G\,\bar{C}(\bar{A}-I)\\ -(\bar{A}-I)^T\bar{C}\;G^T(P+R)\,\bar{\bar{A}}+(\bar{A}-I)^T\bar{C}\;G^T(P+R)\;\bar{C}\;(\bar{A}-I)\\ +(\bar{A}-I)^T\bar{C}\;G^T(P+R)\;G\;\bar{C}\;(\bar{A}-I)+R\;G\;\bar{C}(\bar{A}-I)+(\bar{A}-I)^T\bar{C}G^TR+\tau Q\end{bmatrix}z(k)\right]^{1/2}\\ &+\left[z^T(k)\begin{bmatrix}-\bar{\bar{A}}^T(P+R)G\bar{C}\bar{A}_d+(\bar{A}-I)^T\bar{C}\,(P+R)G\,\bar{C}\bar{A}_d+(\bar{A}-I)^T\bar{C}\;G^T(P+R)\bar{C}\bar{A}_d\\ +(\bar{A}-I)^T\bar{C}\;G^T\,(P+R)G\;\bar{C}\bar{A}_d-(\bar{A}-I)^T\bar{C}\;G^T\,(P+R)G\bar{\bar{A}}_d+R\;G\;\bar{C}\bar{A}_d\end{bmatrix}z(k-h(k))\right]^{1/2}\\ &+\left[z^T(k-h(k))\begin{bmatrix}-\bar{\bar{A}}_d^T\,(P+R)G\bar{C}(\bar{A}-I)-\bar{A}_d^T\bar{C}G^T(P+R)\bar{\bar{A}}+\bar{A}_d^T\bar{C}G^T(P+R)\bar{C}(\bar{A}-I)\\ +\bar{A}_d^T\bar{C}(P+R)G\;\bar{C}(\bar{A}-I)+\bar{A}_d^T\bar{C}G^T(P+R)G\bar{C}(\bar{A}-I)+\bar{A}_d^T\bar{C}\;G^TR\end{bmatrix}z(k)\right]^{1/2}\\ &+\left[z^T(k-h(k))\begin{bmatrix}-\bar{\bar{A}}_d^T\,(P+R)G\;\bar{C}\bar{A}_d+\bar{A}_d^T\bar{C}\;G^T(P+R)\bar{C}\bar{A}_d+\bar{A}_d^T\bar{C}\;(P+R)G\;\bar{C}\bar{A}_d\\ -\bar{A}_d^T\bar{C}\;G^T(P+R)\,\bar{\bar{A}}_d+\bar{A}_d^T\bar{C}G^T(P+R)\;G\;\bar{C}\bar{A}_d\end{bmatrix}z(k-h(k))\right]^{1/2}\end{aligned} \tag{20}$$

with

$$\left(\begin{aligned}&z^T(k)\begin{bmatrix}-\bar{\bar{A}}^T\;(P+R)+(\bar{A}-I)^T\bar{C}(P+R)+R-\bar{\bar{A}}^T\;PG\\ +(\bar{A}^T\;-I)\bar{C}PG+(\bar{A}^T\;-I)\bar{C}G^T\;P+(\bar{A}-I)^T\;\bar{C}G^T\;PG\end{bmatrix}\mu\\ &+z^T\;(k-h(k))\begin{bmatrix}-\bar{\bar{A}}_d^T\,(P+R)+\bar{A}_d^T\bar{C}(P+R)+\bar{A}_d^T\bar{C}PG\\ +\bar{A}_d^T\bar{C}G^T\;P+\bar{A}_d^T\bar{C}G^T\;PG-\bar{\bar{A}}_d^T PG\end{bmatrix}\mu\end{aligned}\right)\langle 0 \tag{21}$$

$$\left(\begin{aligned}&\mu^T\begin{bmatrix}-(P+R)\bar{\bar{A}}_d+(P+R)\bar{C}\bar{A}_d+PG\bar{C}\bar{A}_d+G^T\;P\bar{C}\bar{A}_d\\ +G^T\;PG\bar{C}\bar{A}_d-G^T\;P\bar{\bar{A}}_d\end{bmatrix}z(k-h(k))\\ &+\mu^T\begin{bmatrix}-(P+R)\bar{\bar{A}}+(P+R)\bar{C}(\bar{A}-I)+R+PG\bar{C}(\bar{A}-I)\\ -G^T\;P\bar{\bar{A}}+G^T\;P\bar{C}(\bar{A}-I)+G^T\;PG\bar{C}(\bar{A}-I)\end{bmatrix}z(k)\\ &+\mu^T\left[\mathrm{P+R+PG\bar{C}}+G^T\;P+G^T\;PG\right]\mu\end{aligned}\right)\langle 0 \tag{22}$$

where $\mu = qTs(k) + \varepsilon T\,\mathrm{sgn}(\,s(k)) + diag\,[\mathrm{sgn}(\,s_i\,)]\,f_d\,(k)$, $\bar{\bar{A}} = \bar{A} + \Delta\bar{A}$, $\tau = h_M - h_m + 1$ and $\bar{\bar{A}}_d = \bar{A}_d + \Delta\bar{A}_d$

Suppose there exist an $n\times n$ positive-definite matrix P , an $n\times n$ nonnegative-definite matrix Q ,an $n\times n$ nonnegative-definite matrix R and $n\times n$ symmetric matrix G such that following LMI holds,

$$H1) = \begin{bmatrix} -\bar{\bar{A}}^T(P+R)G\bar{C}(\bar{A}-I) & -\bar{\bar{A}}^T(P+R)G\overline{C}\overline{A_d} \\ +(\bar{A}-I)^T\bar{C}(P+R)G\bar{C}(\bar{A}-I) & +(\bar{A}-I)^T\bar{C}(P+R)G\overline{CA_d} \\ -(\bar{A}-I)^T\bar{C}G^T(P+R)\bar{\bar{A}}+ & +(\bar{A}-I)^T\bar{C}G^T(P+R)\overline{CA_d} \\ (\bar{A}-I)^T\bar{C}G^T(P+R)\bar{C}(\bar{A}-I) & +(\bar{A}-I)^T\bar{C}G^T(P+R)G\bar{C}\overline{A_d} \\ (\bar{A}-I)^T\bar{C}G^T(P+R)G\bar{C}(\bar{A}-I)+ & -(\bar{A}-I)^T\bar{C}G^T(P+R)G\overline{\overline{A_d}} \\ RG\bar{C}(\bar{A}-I)+(\bar{A}-I)^T\bar{C}G^TR+\tau Q & +RG\bar{C}\overline{A_d} \\ * & -\overline{\overline{A_d}}^T(P+R)G\overline{CA_d}+\overline{A_d}^T\bar{C}G^T(P+R) \\ & \overline{CA_d}+\overline{A_d}^T\bar{C}(P+R)G\overline{CA_d}-\overline{A_d}^T\bar{C}G^T \\ & (P+R)\overline{\overline{A_d}}+\overline{A_d}^T\bar{C}G^T(P+R)G\overline{CA_d} \end{bmatrix} \quad (23)$$

Thus by properly choosing the design parameters and the control gain, the trajectory of the state reaches towards the sliding surface.

Proof:

Choose Lyapunov-Krasovskii function,

$$\Delta V(k) = \Delta V_1(k) + \Delta V_2(k) + \Delta V_3(k) + \Delta V_4(k) + \Delta V_5(k) \quad (24)$$

where,

$$\Delta V_1(k) = z^T(k+1)Pz(k+1) - z^T(k)Pz(k) \quad (25)$$

$$\Delta V_2(k) = \sum_{i=k+1-h(k+1)}^{k} z^T(i)Q\,z(i) - \sum_{i=k-h(k)}^{k-1} z^T(i)Q\,z(i) \quad (26)$$

$$\Delta V_3(k) = \sum_{j=-h_M+2}^{-h_m+1}\sum_{j=k+j}^{k} z^T(i)Q\,z(i) - \sum_{j=-h_M+2}^{-h_m+1}\sum_{j=k+j-1}^{k-1} z^T(i)Q\,z(i) \quad (27)$$

$$\Delta V_4(k) = \sum_{j=-h_M}^{-1}\sum_{j=k+1+j}^{k} \eta^T(i)R\,\eta(i) - \sum_{j=-h_M}^{-1}\sum_{j=k+j}^{k-1} \eta^T(i)R\,\eta(i) \quad (28)$$

where

$$z(k-h(k)) = z(k) - \sum_{i=k-h(k)}^{k-1}\eta(i) \quad , \quad \eta(k) = z(k+1) - z(k) \quad (29)$$

$$\Delta V_5(k) = tr\left(\tilde{w}^T(k+1)\tilde{w}(k+1)\right) - tr\left(\tilde{w}^T(k)\tilde{w}(k)\right) \quad (30)$$

Since P, Q, R are symmetric positive-definite matrices to be determined, $V(k)$ is then positive-definite.

Substituting (12),(18) and (29) in (24) and using Assumption 6 gives,

$$\Delta V(k) = M(k) + z^T(k)\tau Q z(k) + \|\hat{w}(k+1)\|^2 - \|\hat{w}(k)\|^2 + H(k) + I(k) \quad (31)$$

where,

$$M(k)=z^T(k)\begin{bmatrix}(\bar{\bar{A}}+\overline{\overline{A_d}}+\bar{C}(\bar{A}-I))^T(P+h_M R)(\bar{\bar{A}}+\overline{\overline{A_d}}+\bar{C}(\bar{A}-I))-2(\bar{\bar{A}}+\overline{\overline{A_d}}\\ +\bar{C}(\bar{A}-I))^T(P+h_M R)\overline{CA_d}-R\bar{C}\bar{\bar{A}}+\overline{\overline{A_d}}\ -\bar{C}(\bar{A}-I)+\overline{CA_d}-(\bar{\bar{A}}+\overline{\overline{A_d}}\\ +(\bar{A}-I)\bar{C}+\overline{A_d}\bar{C})^T R-P-Q+R\end{bmatrix}z(k)$$

$$+\left[\sum_{i=k-h(k)}^{k-1}\right]\left[2z^T(k)\left[(\overline{CA_d}-\overline{\overline{A_d}})^T(P+h_M R)(\bar{\bar{A}}+\overline{\overline{A_d}}-\bar{C}(\bar{A}-I)-\overline{CA_d})+Q-(\overline{CA_d}-\overline{\overline{A_d}})^T\right]R\right]\eta(i)$$

$$+\left[\sum_{i=k-h(k)}^{k-1}\eta(i)\right]^T\left[(\overline{\overline{A_d}}-\overline{CA_d})^T(P+h_M R)(\overline{\overline{A_d}}-\overline{CA_d})-Q-R\right]\sum_{i=k-h(k)}^{k-1}\eta(i)$$

$$H(k)=z^T(k)\begin{bmatrix}-\bar{\bar{A}}^T(P+R)G\bar{C}(\bar{A}-I)+(\bar{A}-I)^T\bar{C}(P+R)G\bar{C}(\bar{A}-I)-(\bar{A}-I)^T\bar{C}G^T(P+R)\bar{\bar{A}}\\ +(\bar{A}-I)^T\bar{C}G^T(P+R)\bar{C}(\bar{A}-I)+(\bar{A}-I)^T\bar{C}G^T(P+R)G\bar{C}(\bar{A}-I)\\ +RG\bar{C}(\bar{A}-I)+(\bar{A}-I)^T\bar{C}G^T R+\tau Q\end{bmatrix}z(k)$$

$$+z^T(k)\begin{bmatrix}-\bar{\bar{A}}^T(P+R)G\bar{C}\overline{A_d}+(\bar{A}-I)^T\bar{C}(P+R)G\bar{C}\overline{A_d}+(\bar{A}-I)^T\bar{C}G^T(P+R)\overline{CA_d}\\ +(\bar{A}-I)^T\bar{C}G^T(P+R)G\overline{CA_d}-(\bar{A}-I)^T\bar{C}G^T(P+R)G\overline{\overline{A_d}}+RG\overline{CA_d}\end{bmatrix}z(k-h(k))$$

$$+z^T(k-h(k))\begin{bmatrix}-\overline{\overline{A_d}}^T(P+R)G\bar{C}(\bar{A}-I)-\overline{A_d}^T\bar{C}G^T(P+R)\bar{\bar{A}}+\overline{A_d}^T\bar{C}G^T(P+R)\bar{C}(\bar{A}-I)\\ +\overline{A_d}^T\bar{C}(P+R)G\bar{C}(\bar{A}-I)+\overline{A_d}^T\bar{C}G^T(P+R)G\bar{C}(\bar{A}-I)+\overline{A_d}^T\bar{C}G^T R\end{bmatrix}z(k)$$

$$+z^T(k-h(k))\begin{bmatrix}-\overline{\overline{A_d}}^T(P+R)G\overline{CA_d}+\overline{A_d}^T\bar{C}G^T(P+R)\overline{CA_d}+\overline{A_d}^T\bar{C}(P+R)G\overline{CA_d}\\ -\overline{A_d}^T\bar{C}G^T(P+R)\overline{\overline{A_d}}+\overline{A_d}^T\bar{C}G^T(P+R)G\overline{CA_d}\end{bmatrix}z(k-h(k))$$

$$I(k)=z^T(k)\begin{bmatrix}-\bar{\bar{A}}^T(P+R)+(\bar{A}-I)\bar{C}(P+R)+R-\bar{\bar{A}}^T PG\\ +(\bar{A}-I)^T\bar{C}PG+(\bar{A}-I)^T\bar{C}G^T P+(\bar{A}-I)^T\bar{C}G^T PG\end{bmatrix}\mu$$

$$+\mu^{\mathrm{T}}\begin{bmatrix}-(P+R)\bar{\bar{A}}+(P+R)\bar{C}(\bar{A}-I)+R+PG\bar{C}(\bar{A}-I)-G^T P\bar{\bar{A}}\\ +G^T P\bar{C}(\bar{A}-I)+G^T PG\bar{C}(\bar{A}-I)\end{bmatrix}z(k)$$

$$+z^T(k-h(k))\begin{bmatrix}-\overline{\overline{A_d}}(P+R)+\overline{A_d}\bar{C}(P+R)+\overline{A_d}\bar{C}PG\\ +\overline{A_d}^T\bar{C}G^T P+\overline{A_d}^T\bar{C}G^T PG-\overline{\overline{A_d}}PG\end{bmatrix}\mu$$

$$+\mu^{\mathrm{T}}\begin{bmatrix}-(P+R)\overline{\overline{A_d}}+(P+R)\bar{C}\overline{A_d}+PG\bar{C}\overline{A_d}\\ +G^T P\bar{C}\overline{A_d}+G^T PG\bar{C}\overline{A_d}-G^T P\overline{\overline{A_d}}\end{bmatrix}z(k-h(k))$$

$$+\mu^{\mathrm{T}}\left[P+R+PG\bar{C}+G^T P+G^T PG\right]\mu$$

Lemma 1[12]: Assume that $a\in R^{n_a}$, $b\in R^{n_b}$ and $N\in R^{n_a\times n_b}$. Then, for the matrices $\alpha\in R^{n_a\times n_a}$, $\beta\in R^{n_a\times n_b}$ and $\gamma\in R^{n_b\times n_b}$ the following inequality gives

$$-2a^T Nb \le \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} \alpha & \beta - N \\ \beta^T - N^T & \gamma \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \tag{32}$$

where $\begin{bmatrix} \alpha & \beta \\ \beta^T & \gamma \end{bmatrix} \ge 0$

Using Lemma 1 in $M(k)$ and identifying $N = R(\overline{C}\,\overline{A_d} - \overline{\overline{A_d}}),\ a = z(k),\ b = \eta(i)$ gives rise to

$$-[2z^T(k)[(\overline{C}\,\overline{A_d} - \overline{\overline{A_d}})^T R]\eta(i)] \le M(k) \tag{33}$$

Manipulating the terms which are nonquadratic using the following inequality $\sqrt{ab} \le \frac{a+b}{2}$ in equation (31) except term $M(k)$, gives

$$\Delta V(k) \le -2|\hat{w}(k)| \times c - 2a^2 - 2|\hat{w}(k)| \times d - 4|\hat{w}(k)| \times \sqrt{ab} - 4c \times \sqrt{ab} - 4d \times \sqrt{ab} \tag{34}$$

where,

$$a = \left[z^T(k) \begin{bmatrix} -\overline{\overline{A}}^T (P+R)G\overline{C}\,\overline{A_d} + (\overline{A}^T - I)\overline{C}(P+R)G\overline{C}\,\overline{A_d} + (\overline{A}^T - I)\overline{C}G^T(P+R)\overline{C}\,\overline{A_d} \\ +(\overline{A}^T - I)\overline{C}G^T(P+R)\overline{C}\,\overline{A_d} - (\overline{A}^T - I)\overline{C}G^T(P+R)\overline{\overline{A_d}} + RG\overline{C}\,\overline{A_d} \end{bmatrix} z(k-h(k)) \right]^{1/2}$$

$$b = \left[z(k-h(k)) \begin{bmatrix} -\overline{\overline{A_d}}^T (P+R)G\overline{C}(\overline{A}-I) - \overline{A_d}^T \overline{C}G^T(P+R)\overline{\overline{A}} + \overline{A_d}^T \overline{C}G^T(P+R)\overline{C}(\overline{A}-I) \\ +\overline{A_d}^T \overline{C}(P+R)G\overline{C}(\overline{A}-I) + \overline{A_d}^T \overline{C}G^T(P+R)G\overline{C}(\overline{A}-I) + \overline{A_d}^T \overline{C}G^T R \end{bmatrix} z(k) \right]^{1/2}$$

$$c = \left[z^T(k) \begin{bmatrix} -2\overline{\overline{A}}^T (P+R)G\overline{C}(\overline{A}-I) - 2(\overline{A}^T - I)\overline{C}(P+R)G\overline{C}(\overline{A}-I) \\ +(\overline{A}^T - I)\overline{C}G^T(P+R)\overline{A} + 2RG\overline{C}(\overline{A}-I) + \tau Q \end{bmatrix} z(k) \right]^{1/2}$$

$$d = \left[z^T(k-h(k)) \left[-2\overline{\overline{A_d}}^T (P+R)G\overline{C}\,\overline{A_d} + 2\overline{A_d}^T \overline{C}G^T(P+R)\overline{C}\,\overline{A_d} + \overline{A_d}^T \overline{C}G^T(P+R)G\overline{C}\,\overline{A_d} \right] z(k-h(k)) \right]^{1/2}$$

Since the terms in equations (33-34) are negative and satisfying the LMI in (23). The remaining non-negative terms are satisfying the condition (21-22). Therefore the system in (4) is stable with control law (18) and LMI in (23)

6. SIMULATION RESULTS

A numerical example is given to authenticate the effectiveness and performance of the discrete-time nonlinear uncertain system with time-varying delay.

Example 1:

Consider the system proposed in (4). The set of parameters for the given system are [9]

$$A = \begin{bmatrix} 1.0 & -0.4 \\ 0.6 & 0.8 \end{bmatrix},\ A_d = \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & -0.2 \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\Delta A = \begin{bmatrix} 0.02\sin(0.01k\pi) & 0.01\sin(0.01k\pi) \\ 0.01\cos(0.01k\pi) & 0.005\sin(0.01k\pi) \end{bmatrix},$$

$$\Delta A_d = \begin{bmatrix} 0.01\sin(0.01k\pi) & 0.02\cos(0.01k\pi) \\ 0.005\sin(0.01k\pi) & 0.01\cos(0.01k\pi) \end{bmatrix}$$

and $g(x(k)) = \begin{bmatrix} \dfrac{1.4x_1^2(k)}{1+x_1^2(k)} \\ \dfrac{x_1(k)}{1+x_1^2(k)+x_2^2(k)} \end{bmatrix}$

$$G = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, H = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.25 \\ 0.26 & 0 \\ 0 & 0.3 \end{bmatrix}$$

$$G_d = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, H_d = \begin{bmatrix} 0.32 & 0 \\ 0 & 0.14 \\ 0.22 & 0 \\ 0 & 0.25 \end{bmatrix}$$

Consider the minimum and maximum delay dounds of $h(\text{k})$ are h_m =0 and h_M =5.The initial condition of the system states x_1 and x_2 are taken as $[2 \quad 1]^T$. By taking transformation matrix $M = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and applying Matlab LMI Toolbox to solve the LMI in (23), the values of P, Q, $R$and G are obtained as

$$P = 10^8 \begin{bmatrix} 3.3002 & 0 \\ 0 & 3.3002 \end{bmatrix}$$

$$Q = 10^8 \begin{bmatrix} 3.3002 & 0 \\ 0 & 3.3002 \end{bmatrix}$$

$$G = 10^8 \begin{bmatrix} 3.3002 & 0 \\ 0 & 3.3002 \end{bmatrix}$$

$$R = 10^8 \begin{bmatrix} 3.3002 & 0 \\ 0 & 3.3002 \end{bmatrix}$$

Using (16), the linear sliding surface is obtained as $s(k) = [1 \quad 0.6454]x(k) = 0$ .The robust control law that takes the system towards the sliding surface can be choosen as

$$u(k) = (-B_2^{-1} / \hat{g}(x(k)))[\, qTs(k) + \varepsilon T \operatorname{sgn}(s(k)) + \overline{C}(\overline{A} - I)x(k) + \overline{C}\overline{A}d\, x(k-h) + diag\, [\operatorname{sgn}(s)]\, f_d(k)\,]$$

where

$f_d(k) = 0.49|x_1(k)| + 0.36|x_2(k)| + 0.40|x_1(k-2)| + 0.23|x_2(k-2)|$ $q = 2, \varepsilon = 0.2, T = 0.1$

The system trajectories states x_1 and x_2 are shown in Figure 2 and Figure 3. It is observed that system states move toward to zero quickly. The adaptive controller stabilizes the system with time-varying using sliding mode control technique as shown in Figure 4. The resulting sliding surface is demonstrated in Figure 5. It is observed from the simulation results that the trajectories of the system reaches the sliding surface in finite time and the reaching motion satisfies the sliding condition in the presence of parametric uncertainties and nonlinearity. The merit of the proposed simulation results validates the stability and effectiveness of proposed scheme.

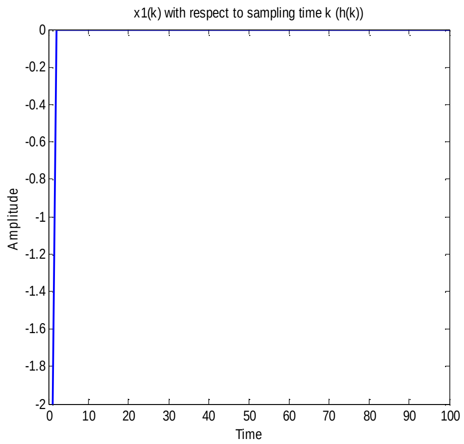

Figure 2. $x_1(k)$ with sampling time(t)

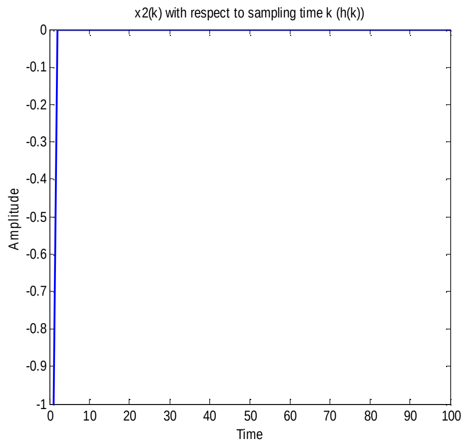

Figure 3: $x_2(k)$ with sampling time(t)

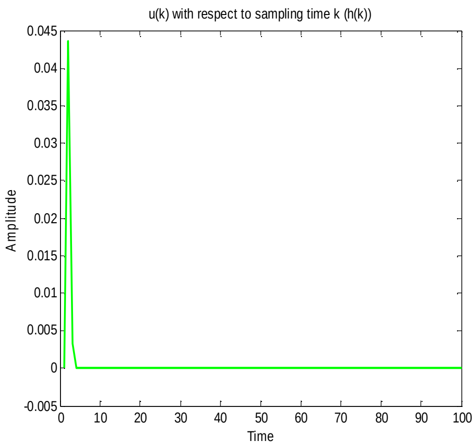

Figure 4: $u(k)$ with sampling time(t)

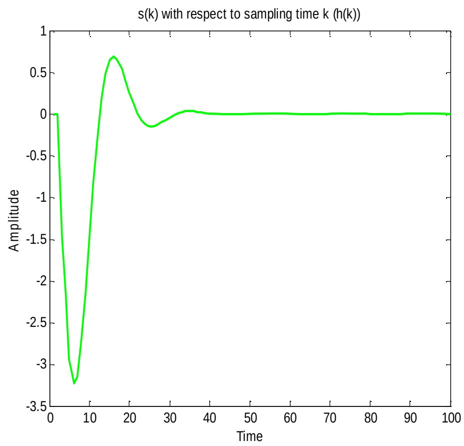

Figure 5: $s(k)$ with sampling time(t)

7. CONCLUSION

In this paper, robust nonlinear discrete-time uncertain sliding mode control for time-varying delay has been studied. The unknown non-linearity of the system has been approximated using CNN. By using LMI, a sufficient condition is derived to guareentee the asymptotic stability of the closed loop system, and the exixtence of the sliding surface dependent on the minimum and maximum delay bounds. CNN based sliding mode controller is proposed which is robust to parametric uncertainty, nonlinearities and time-varying delay. The adaptive sliding mode controller is derived so that the sliding mode reaching condition is satisfied by the motion control. Simulation results validates the effectiveness of the proposed CNN based controller.

REFERENCES

[1] Y.Y. Cao, Y.X. Sun, J. Lam, " Delay-dependent robust H∞ control for uncertain systems with time-varying delays,". IEEE Proc, Control Theory Appl.,1998,145,(3), pp. 338 – 344.

[2] W.H. Chen, Z.H. Guan, X. Lu, " Delay-dependent guaranteed cost control for uncertain discrete-time systems with delay,". IEE Proc. Control Theory Appl., 2003, 150, (4), pp. 412 – 416.

[3] H.J. Gao, C.H. Wang, "Robust L2–L∞ filtering for uncertain systems with multiple time-varying state delays,". IEEE Trans. Circuits Syst., 2003,50, (4), pp. 594-599.

[4] Y. He, M. Wu, J.H. She, G.P. Liu, " Delay-dependent robust stability criteria for uncertain neutral systems with mixed delays,". Syst. Control Lett., 2004,51,(1), pp. 57-65.

[5] Y. He, M. Wu, G.P. Liu, " Parameter-dependent lyapunov functional for stability of time-delay systems with polytopic-type uncertainties,".IEEE Trans. Autom. Control, 2004, 49, (5), pp. 828 – 832.

[6] V. Kharitonov, A.D. Melhor, " On delay-dependent stability conditions,". Syst. Control Lett.,2002,40, (1),pp. 71 – 76.

[7] M. Wu, Y. He, J. She, G. Liu, " Delay-dependent criteria for robust stability of time-varying delay systems,". Automatica,2004,40, (8),pp. 1435 – 1439.

[8] X.G. Liu, R.R. Martin, M. Wu, M.L. Tang, "Delay-dependent robust stabilisation of discrete-time systems with time-varying delay,"IEE Proceedings-Control Theory and Applications,2006, 153,(6), pp. 689 – 702.

[9] M.Yan,Y.Shi,."Robust discrete-time sliding mode control for uncertain systems with time–varying state delay". IET Control Theory Appl.,2008, (2), pp. 662-674.

[10] P.Park, J.W.Ko, " Stability and robust stability for systems with time-varying delay," Automatica, 2007, 43, pp. 1855-1858.

[11] H. Gao, J. Lam, C. Wang, Y. Wang, " Delay-dependent output-feedback stabilization of discrete-time systems with time-varying state delay,". IEE Proc. Control Theory Appl., 2004, 151, (6), pp. 691 – 698.

[12] Y.S.Moon, P.Park, W.H.Kwon and Y.S.Lee., "Delay-dependent robust stabilization of uncertain state-delayed systems". *Int. J. Control*., 2001,74., (14), pp 1447-1455.

[13] V. I. Utkin, "Variable structure systems with sliding modes," *IEEETrans. Automat. Contr.,* 1977, 22, (2), pp. 212–222.

[14] X. Chen and T. Fukuda, " Robust adaptive quasi-sliding mode controller for discrete-time systems," *Systems & Control Letters*, 1998, 35, (5), pp.165-173.

[15] Z.Lin., Y.Xia., P.Shi.,and H.Wu., "Robust sliding mode control for uncertain linear discrete systems independent of time-delay". *International Journal of Innovative Computing, Information and Control*, 2011, 7, (2), pp. 869-880.

[16] Y. Xia and Y. Jia, "Robust sliding-mode control for uncertain time-delay systems: an LMI approach," *IEEE Transactions on Automatic Control*, 2003, 48, (6), pp. 1086–1092.

[17] M. S. Mahmoud, E. Boukas and A. Ismail, “ Robust adaptive control of uncertain discrete-time state-delay systems”, *Computers & Mathematics with Applications,* 2008, 55, (1)2, pp.2887-2908.

[18] Y. Zhang, C. Wen and Y. C. Soh, “Robust adaptive control of uncertain discrete-time systems”, *Automatica*, 1999, 35, (2), pp.321-329.

[19] Vinay Kumar Deolia,Shubhi Purwar and T. N. Sharma. “Stabilization of Unknown Nonlinear Discrete-Time Delay Systems Based on Neural Network” in *Intelligent Control and Automation (ICA), Scientific research publications*, 2012, 3, (4).

[20] T. Zhang, S.S. Ge, C.C. Hang. “Design and performance analysis of a direct adaptive controller for nonlinear systems”. *Automatica*, 1999, 35, pp 1809 – 1817.

[21] S. Chen, S.A. Billings. “Neural networks for nonlinear dynamic system modeling and identification”. *Int. J. Control*, 1992, 56, pp 319 – 346.

[22] S.Jagannathan,F.L.Lewis,“Identification of nonlinear dynamical systems using multilayered neural networks”. *Automatica*, 1996, 36, pp 1707 – 1712.

[23] K. Hornik, M. Stinchcombe, H. White, "Multilayer feedforward networks are universal approximators" *Neural Networks*, 1989, 2, (5), pp. 359-366.

[24] Y. H. Pao, Adaptive Pattern Recognition and Neural Netvorks Reading, MA: Addision-Wesley, 1989.

[25] Jagdish C.Patra and Alex C.Kot, “Non-linear Dynamic System Identification Using Chebyshev Functional Link Artificial Neural Networks’, *IEEE Transaction On Systems,Man and Cyberntics*, 2002, 32, (4), pp 3214-3219.

[26] W. Gao, Y. Wang, and A. Homaifa, “Discrete time variable structure control Systems,” *IEEE Transaction Ind. Electron*., 1995, 42, pp. 117–122.

[27] Goyal, V. , Deolia, V. and Sharma, T., “Robust Sliding Mode Control for Nonlinear Discrete-Time Delayed Systems Based on Neural Network”. International Journal on *Intelligent Control and Automation,* 6, pp. 75-83, 2015.