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A Generalized Class of Double Sampling Estimators Using Auxiliary Information on an Attribute and an Auxiliary Variable

Shashi Bhushan[1], Praveen Kumar Misra[1] and Sachin Kumar Yadav[2]

[1] *Department of Mathematics and Statistics,*
Dr. Shakuntala Misra National Rehabilitation University, India.

[2] *Department of Statistics, University of Lucknow, India*

Abstract

In the present study, we have proposed a generalized class of double sampling estimators using auxiliary information in both the form variable and attribute. The bias and mean square error of proposed generalized class is obtained has been shown that it attains minimum value under some optimum conditions. It has been shown that many of the well-known double sampling estimators are the member of the proposed class. Also the Bias and MSE table of some proposed estimators is given in the last.

Keywords: Double sampling, auxiliary information, bias, mean square error.

INTRODUCTION

Consider the following notations

Y = Study variable

X = Auxiliary variable

ϕ = Auxiliary attribute

N = Size of population

$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ = Population mean of study variable

$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$ = Population mean of auxiliary variable

$P = \frac{1}{N}\sum_{i=1}^{N} \phi_i$ = Population mean of auxiliary attribute

$S_Y^2 = \frac{1}{N}\sum_{i=1}^{N} (Y_i - \bar{Y})^2$ = Population variance of study variable

$S_X^2 = \frac{1}{N}\sum_{i=1}^{N} (X_i - \bar{X})^2$ =Population variance of auxiliary variable

$S_\phi^2 = \frac{1}{N}\sum_{i=1}^{N} (\phi_i - P)^2$ =Population variance of auxiliary attribute

If the information about the auxiliary variable and attribute is not known then in double sampling scheme these auxiliary characteristics are replaced by the corresponding sample values comes from the large preliminary simple random sample of size n' drawn without replacement from a population of size N in the first phase. Also the characteristic of interest Y and the auxiliary characteristic X and ϕ are observed on the second phase sample of size n drawn from the first phase sample by simple random sample without replacement.

Let

$$\bar{x}' = \frac{\sum_{i=1}^{n'} X_i}{n'} ,\; p' = \frac{1}{n'}\sum_{i=1}^{n'} \phi_i$$

be the sample mean of auxiliary variable and attribute respectively based on the first phase sample of size n'. Also denote the following sample information as

$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$ = Sample mean of study variable, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} X_i$ = Sample mean of auxiliary variable

$p = \frac{1}{n}\sum_{i=1}^{n} \phi_i$ = Sample mean of auxiliary attribute

With this available information we propose a class of estimators for mean value of study variable Y as

$$\hat{\bar{Y}}_g = \bar{y} g(\bar{x}, p) \tag{1.1}$$

Where $g(\bar{x}, p)$ is the function of $\bar{x}$ and p such that

(i) $g(\bar{x}', p') = 1$

(ii) The function $g(\bar{x}, p)$ is continuous and bounded in the closed interval R of real line.

(iii) The first and second order partial derivatives of the function $g(\bar{x}, p)$ are exist and are continuous and bounded in R.

It is to be mentioned that proposed class of estimators is very large. Consider the following member of this class

1. Following Olkin(1958)

$$\hat{\bar{Y}}_g^{(1)} = \bar{y}\left[\alpha\left(\frac{\bar{x}'}{\bar{x}}\right) + (1-\alpha)\left(\frac{p'}{p}\right)\right]$$

2. Following Singh (1967)

$$\hat{\bar{Y}}_g^{(2)} = \bar{y}\left[\alpha\left(\frac{\bar{x}}{\bar{x}'}\right) + (1-\alpha)\left(\frac{p}{p'}\right)\right]$$

3. Following Shukla (1966) and john (1969)

$$\hat{\bar{Y}}_g^{(3)} = \bar{y}\left(\frac{\alpha\bar{x}' + (1-\alpha)p'}{\alpha\bar{x} + (1-\alpha)p}\right)$$

4. Following Sahai et al (1980)

$$\hat{\bar{Y}}_g^{(4)} = \bar{y}\left(\frac{\alpha\bar{x} + (1-\alpha)p}{\alpha\bar{x}' + (1-\alpha)p'}\right)$$

5. Following Mohanty and Pattanaik (1984)

$$\hat{\bar{Y}}_g^{(5)} = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)^{\alpha}\left(\frac{p'}{p}\right)^{1-\alpha}$$

6. Following Mohanty and Pattanaik (1984)

$$\hat{\bar{Y}}_g^{(6)} = \bar{y}\left[\alpha\left(\frac{\bar{x}}{\bar{x}'}\right) + (1-\alpha)\left(\frac{p}{p'}\right)\right]^{-1}$$

7. Following Tuteja and Bahl (1991)

$$\hat{\bar{Y}}_g^{(7)} = \bar{y}\left(\frac{\bar{x}}{\bar{x}'}\right)^{\alpha}\left(\frac{p}{p'}\right)^{1-\alpha}$$

8. Following Tuteja and Bahl (1991)

$$\hat{\bar{Y}}_g^{(8)} = \bar{y}\left[\alpha\left(\frac{\bar{x}'}{\bar{x}}\right) + (1-\alpha)\left(\frac{p'}{p}\right)\right]^{-1}$$

9. Following Naik and Gupta (1996) and Abu Dayyeh (2003) proposed Ratio type estimator by us[2012]

$$\hat{\bar{Y}}_g^{(9)} = \bar{y}\left(\frac{\bar{x}}{\bar{x}'}\right)^{\alpha_1}\left(\frac{p}{p'}\right)^{\alpha_2}$$

10. Following Sahai and Ray (1980)

$$\hat{\bar{Y}}_g^{(10)} = \bar{y}\left(2-\left(\frac{\bar{x}}{\bar{x}'}\right)^{\alpha_1}\right)\left(2-\left(\frac{p}{p'}\right)^{\alpha_2}\right)$$

11. Following Walsh (1970)

$$\hat{\bar{Y}}_g^{(11)} = \bar{y}\frac{\bar{x}}{\left(\bar{x}'+\alpha_1(\bar{x}-\bar{x}')\right)}\cdot\frac{p}{\left(p'+\alpha_1(p-p')\right)}$$

12. Following Srivastava (1971)

$$\hat{\bar{Y}}_g^{(12)} = \bar{y}\exp\left(\alpha_1\log\frac{\bar{x}}{\bar{x}'}+\alpha_2\log\frac{p}{p'}\right)$$

13. Following Srivastava (1971)

$$\hat{\bar{Y}}_g^{(13)} = \bar{y}\exp\left[\alpha_1\left(\frac{\bar{x}}{\bar{x}'}-1\right)+\alpha_2\left(\frac{p}{p'}-1\right)\right]$$

14. Following Srivastava (1971)

$$\hat{\bar{Y}}_g^{(14)} = \bar{y}\left[\alpha\exp\left(\frac{\theta_1}{\alpha}\log\frac{\bar{x}}{\bar{x}'}\right)+(1-\alpha)\exp\left(\frac{\theta_2}{1-\alpha}\log\frac{p}{p'}\right)\right]$$

15. Proposed Estimator by us in our paper

$$\hat{\bar{Y}}_g^{(15)} = \bar{y}\left\{\exp\left(\frac{\bar{x}'-\bar{x}}{\bar{x}'+\bar{x}}\right)\right\}^{\alpha_1}\left\{\exp\left(\frac{p'-p}{p'+p}\right)\right\}^{\alpha_2} \tag{1.2}$$

Many such estimators utilizing the information in both the forms auxiliary variable and attribute can be constructed as the member of this class.

BIAS AND MSE OF THE PROPOSED GENERALIZED CLASS OF DOUBLE SAMPLING ESTIMATORS

To obtain the bias of the proposed class of estimators, we further assume that the third order partial derivatives of $g(\bar{x}, p)$ exist and are bounded and continues. Then taking Taylor's expansion about the point $(\bar{x}', p')$ up to third order terms, we get

$$\hat{\bar{Y}}_g = \bar{y}\left[\begin{array}{l} g(\bar{x}', p') + \frac{1}{1!}\left\{(\bar{x}-\bar{x}')g_{\bar{x}}(\bar{x}', p') + (p-p')g_p(\bar{x}', p')\right\} \\ + \frac{1}{2!}\left\{\begin{array}{l}(\bar{x}-\bar{x}')^2 g_{\bar{x}\bar{x}}(\bar{x}', p') + 2(\bar{x}-\bar{x}')(p-p')g_{\bar{x}p}(\bar{x}', p') \\ +(p-p')^2 g_{pp}(\bar{x}', p')\end{array}\right\} \\ + \frac{1}{3!}\left\{\begin{array}{l}(\bar{x}-\bar{x}')^3 g_{\bar{x}\bar{x}\bar{x}}(\bar{x}'^*, p'^*) + 3(\bar{x}-\bar{x}')^2(p-p')g_{\bar{x}\bar{x}p}(\bar{x}'^*, p'^*) \\ +3(\bar{x}-\bar{x}')(p-p')^2 g_{\bar{x}pp}(\bar{x}'^*, p'^*) + (p-p')^3 g_{ppp}(\bar{x}'^*, p'^*)\end{array}\right\}\end{array}\right] \quad (2.1)$$

where $\bar{x}'^* = \bar{x}' + \theta(\bar{x}-\bar{x}'), p'^* = p' + \theta(p-p'), 0 < \theta < 1$ and $g_a(\bar{x}', p'), g_{ab}(\bar{x}', p')$ and $g_{abc}(\bar{x}', p')$ are the first order, second order and third order partial derivatives of the function $g(\bar{x}, p)$ at the point $(\bar{x}', p')$ about a, a and b and a, b and c respectively in general terms.

Now let us take

$$\bar{y} = \bar{Y} + e_0, \bar{x} = \bar{X} + e_1, p = P + e_2, \bar{x}' = \bar{X} + e_1', p' = P + e_2'$$

$$\text{with } E(e_0) = E(e_1) = E(e_2) = E(e_1') = E(e_2') = 0 \quad (2.2)$$

And the results given in Sukhatme and Sukhathhme (1997)

$$E(e_0^2) = f_n S_Y^2, E(e_1^2) = f_n S_X^2, E(e_2^2) = f_n S_P^2, E(e_1'^2) = f_{n'} S_X^2, E(e_2'^2) = f_{n'} S_P^2$$

$$E(e_0 e_1) = f_n \rho_{YX} S_Y S_X, E(e_0 e_1') = f_{n'} \rho_{YX} S_Y S_X, E(e_0 e_2) = f_n \rho_{YP} S_Y S_P$$

$$E(e_0 e_2') = f_{n'} \rho_{YP} S_Y S_P, E(e_1 e_2) = f_n \rho_{XP} S_X S_P, E(e_1' e_2) = f_{n'} \rho_{XP} S_X S_P$$

$$E(e_1 e_2') = f_{n'} \rho_{XP} S_X S_P, E(e_1' e_2') = f_{n'} \rho_{XP} S_X S_P$$

$$\text{where } f_n = \left(\frac{1}{n} - \frac{1}{N}\right) \quad (2.3)$$

Substituting the values from (2.2) in (2.1) and neglecting the terms of $e_i's(i = 0,1,2)$ having powers greater than two, we get

$$\hat{\bar{Y}}_g = \bar{Y}\left[\begin{array}{l} 1 + \frac{1}{1!}\left\{(e_1 - e_1')g_{\bar{x}}(\bar{x}', p') + (e_2 - e_2')g_p(\bar{x}', p')\right\} \\ + \frac{1}{2!}\left\{\begin{array}{l}(e_1 - e_1')^2 g_{\bar{x}\bar{x}}(\bar{x}', p') + 2(e_1 e_2 - e_1 e_2' - e_1' e_2 + e_1' e_2')g_{\bar{x}p}(\bar{x}', p') \\ +(e_2 - e_2')^2 g_{pp}(\bar{x}', p')\end{array}\right\}\end{array}\right] \quad (2.4)$$

$$+\left[e_0 + \frac{1}{1!}\left\{e_0(e_1 - e_1')g_{\bar{x}}(\bar{x}', p') + e_0(e_2 - e_2')g_p(\bar{x}', p')\right\}\right]$$

On taking expectation and substituting the results from (2.3), we get

$$E(\hat{\hat{\bar{Y}}}_g) = \bar{Y} + \frac{\bar{Y}\left(f_n - f_{n'}\right)}{2}\begin{pmatrix} S_X^2 g_{\bar{x}\bar{x}}(\bar{x}', p') + 2\rho_{XP} S_X S_P g_{\bar{x}p}(\bar{x}', p') \\ + S_P^2 g_{pp}(\bar{x}', p') \end{pmatrix}$$
$$+\left(f_n - f_{n'}\right)\left(\rho_{YX} S_Y S_X g_{\bar{x}}(\bar{x}', p') + \rho_{YP} S_Y S_P g_p(\bar{x}', p')\right)$$

Showing that $\hat{\hat{\bar{Y}}}_g$ is a biased estimator of $\bar{Y}$ and its bias is given by

$$Bias(\hat{\hat{\bar{Y}}}_g) = E(\hat{\hat{\bar{Y}}}_g) - \bar{Y}$$

$$\left.\begin{array}{l} Bias(\hat{\hat{\bar{Y}}}_g) = \dfrac{\bar{Y} f_{nn'}}{2}\left(S_X^2 g_{\bar{x}\bar{x}}(\bar{x}', p') + 2\rho_{XP} S_X S_P g_{\bar{x}p}(\bar{x}', p') + S_P^2 g_{pp}(\bar{x}', p')\right) \\ + f_{nn'}\left(\rho_{YX} S_Y S_X g_{\bar{x}}(\bar{x}', p') + \rho_{YP} S_Y S_P g_p(\bar{x}', p')\right) \end{array}\right\} = B(say) \qquad (2.5)$$

where $f_{nn'} = \left(f_n - f_{n'}\right) = \left(\dfrac{1}{n} - \dfrac{1}{n'}\right)$

The mean square error (MSE) of $\hat{\hat{\bar{Y}}}_g$ is given by using (2.4)

$$MSE(\hat{\hat{\bar{Y}}}_g) = E(\hat{\hat{\bar{Y}}}_g - \bar{Y})^2$$
$$= E\left[e_0 + \bar{Y}\left\{\left(e_1 - e_1'\right) g_{\bar{x}}(\bar{x}', p') + \left(e_2 - e_2'\right) g_p(\bar{x}', p')\right\}\right]^2$$

(To the first order of approximation)

$$= \begin{bmatrix} E(e_0^2) + \bar{Y}^2 \begin{pmatrix} E\left(e_1 - e_1'\right)^2 g_{\bar{x}}^2(\bar{x}', p') + E\left(e_2 - e_2'\right)^2 g_p^2(\bar{x}', p') \\ +2E(e_1 e_2 - e_1 e_2' - e_1' e_2 + e_1' e_2') g_{\bar{x}}(\bar{x}', p') g_p(\bar{x}', p') \end{pmatrix} \\ +2\bar{Y}\left(E\left(e_0 e_1 - e_0 e_1'\right) g_{\bar{x}}(\bar{x}', p') + E\left(e_0 e_2 - e_0 e_2'\right) g_p(\bar{x}', p')\right) \end{bmatrix}$$

Substituting the results from (2.3), we get

$$MSE(\hat{\hat{\bar{Y}}}_g) = \begin{bmatrix} f_n S_Y^2 + f_{nn'} \bar{Y}^2 \begin{pmatrix} S_X^2 g_{\bar{x}}^2(\bar{x}', p') + S_P^2 g_p^2(\bar{x}', p') \\ +2\rho_{XP} S_X S_P g_{\bar{x}}(\bar{x}', p') g_p(\bar{x}', p') \end{pmatrix} \\ +2 f_{nn'} \bar{Y}\left(\rho_{YX} S_Y S_X g_{\bar{x}}(\bar{x}', p') + \rho_{YP} S_Y S_P g_p(\bar{x}', p')\right) \end{bmatrix}$$

$$= f_n S_Y^2 + f_{nn'} M \ \text{(Say)} \qquad (2.6)$$

where

$$M = \bar{Y}^2\left(S_X^2 g_{\bar{x}}^2(\bar{x}', p') + S_P^2 g_p^2(\bar{x}', p') + 2\rho_{XP} S_X S_P g_{\bar{x}}(\bar{x}', p') g_p(\bar{x}', p')\right)$$
$$+2\bar{Y}\left(\rho_{YX} S_Y S_X g_{\bar{x}}(\bar{x}', p') + \rho_{YP} S_Y S_P g_p(\bar{x}', p')\right)$$

This will be minimum when

$$\bar{Y}g_{\bar{x}}(\bar{x}', p') = \frac{(\rho_{YX} - \rho_{YP}\rho_{XP})}{(1-\rho_{XP}^2)}\frac{S_Y}{S_X} \tag{2.7}$$

$$\bar{Y}g_{p}(\bar{x}', p') = \frac{(\rho_{YP} - \rho_{YX}\rho_{XP})}{(1-\rho_{XP}^2)}\frac{S_Y}{S_P} \tag{2.8}$$

And its minimum value under the optimum values of the characteristic scalars is given by

$$\min MSE(\hat{\bar{Y}}_g) = (f_n - f_{nn'}R_{Y.XP}^2)S_Y^2 \tag{2.9}$$

BIAS AND MSE OF THE PROPOSED MEMBER OF THE CLASS

The bias and MSE of some proposed members of the class defined in equation (1.2) are given in the following table

Table 3.1: Derivatives of $g(\bar{x}, p)$ of proposed members of the generalized class

Estimator	$g_{\bar{x}}(\bar{x}', p')$	$g_p(\bar{x}', p')$	$g_{\bar{x}\bar{x}}(\bar{x}', p')$	$g_{pp}(\bar{x}', p')$	$g_{\bar{x}p}(\bar{x}', p')$
$\hat{\bar{Y}}_g^{(1)}$	$-\frac{\alpha}{\bar{x}'}$	$-\frac{(1-\alpha)}{p'}$	$\frac{2\alpha}{\bar{x}'^2}$	$\frac{2(1-\alpha)}{p'^2}$	0
$\hat{\bar{Y}}_g^{(2)}$	$\frac{\alpha}{\bar{x}'}$	$\frac{(1-\alpha)}{p'}$	0	0	0
$\hat{\bar{Y}}_g^{(3)}$	$\frac{-\alpha}{K}$	$\frac{-(1-\alpha)}{K}$	$\frac{2\alpha^2}{K^2}$	$\frac{2(1-\alpha)^2}{K^2}$	$\frac{2\alpha(1-\alpha)}{K^2}$
$\hat{\bar{Y}}_g^{(4)}$	$\frac{\alpha}{K}$	$\frac{(1-\alpha)}{K}$	0	0	0
$\hat{\bar{Y}}_g^{(5)}$	$-\frac{\alpha}{\bar{x}'}$	$-\frac{(1-\alpha)}{p'}$	$\frac{\alpha(\alpha+1)}{\bar{x}'^2}$	$\frac{(1-\alpha)(2-\alpha)}{p'^2}$	$\frac{\alpha(1-\alpha)}{\bar{x}'p'}$
$\hat{\bar{Y}}_g^{(6)}$	$-\frac{\alpha}{\bar{x}'}$	$-\frac{(1-\alpha)}{p'}$	$\frac{2\alpha^2}{\bar{x}'^2}$	$\frac{2(1-\alpha)^2}{p'^2}$	$\frac{2\alpha(1-\alpha)}{\bar{x}'p'}$
$\hat{\bar{Y}}_g^{(7)}$	$\frac{\alpha}{\bar{x}'}$	$\frac{(1-\alpha)}{p'}$	$\frac{\alpha(\alpha-1)}{\bar{x}'^2}$	$\frac{\alpha(\alpha-1)}{p'^2}$	$\frac{\alpha(1-\alpha)}{\bar{x}'p'}$

$\hat{\bar{Y}}_g^{(8)}$	$\frac{\alpha}{\bar{x}'}$	$\frac{(1-\alpha)}{p'}$	$\frac{2\alpha(\alpha-1)}{\bar{x}'^2}$	$\frac{2\alpha(\alpha-1)}{p'^2}$	$\frac{2\alpha(1-\alpha)}{\bar{x}'p'}$
$\hat{\bar{Y}}_g^{(9)}$	$\frac{\alpha_1}{\bar{x}'}$	$\frac{\alpha_2}{p'}$	$\frac{\alpha_1(\alpha_1-1)}{\bar{x}'^2}$	$\frac{\alpha_2(\alpha_2-1)}{p'^2}$	$\frac{\alpha_1\alpha_2}{\bar{x}'p'}$
$\hat{\bar{Y}}_g^{(10)}$	$-\frac{\alpha_1}{\bar{x}'}$	$-\frac{\alpha_2}{p'}$	$-\frac{\alpha_1(\alpha_1-1)}{\bar{x}'^2}$	$-\frac{\alpha_2(\alpha_2-1)}{p'^2}$	$\frac{\alpha_1\alpha_2}{\bar{x}'p'}$
$\hat{\bar{Y}}_g^{(11)}$	$\frac{(1-\alpha_1)}{\bar{x}'}$	$\frac{(1-\alpha_2)}{p'}$	$\frac{2\alpha_1(\alpha_1-1)}{\bar{x}'^2}$	$\frac{2\alpha_2(\alpha_2-1)}{p'^2}$	$\frac{(1-\alpha_1)(1-\alpha_2)}{\bar{x}'p'}$
$\hat{\bar{Y}}_g^{(12)}$	$\frac{\alpha_1}{\bar{x}'}$	$\frac{\alpha_2}{p'}$	$\frac{\alpha_1(\alpha_1-1)}{\bar{x}'^2}$	$\frac{\alpha_2(\alpha_2-1)}{p'^2}$	$\frac{\alpha_1\alpha_2}{\bar{x}'p'}$
$\hat{\bar{Y}}_g^{(13)}$	$\frac{\alpha_1}{\bar{x}'}$	$\frac{\alpha_2}{p'}$	$\frac{\alpha_1^2}{\bar{x}'^2}$	$\frac{\alpha_2^2}{p'^2}$	$\frac{\alpha_1\alpha_2}{\bar{x}'p'}$
$\hat{\bar{Y}}_g^{(14)}$	$\frac{\theta_1}{\bar{x}'}$	$\frac{\theta_2}{p'}$	$\frac{\theta_1}{\bar{x}'^2}\left(\frac{\theta_1}{\alpha}-1\right)$	$\frac{\theta_2}{p'^2}\left(\frac{\theta_2}{1-\alpha}-1\right)$	0
$\hat{\bar{Y}}_g^{(15)}$	$-\frac{\alpha_1}{2\bar{x}'}$	$-\frac{\alpha_2}{2p'}$	$\frac{\alpha_1(\alpha_1+2)}{4\bar{x}'^2}$	$\frac{\alpha_2(\alpha_2+2)}{4p'^2}$	$\frac{\alpha_1\alpha_2}{4\bar{x}'p'}$

Table 3.2: Bias of the proposed members of the generalized class

Estimator	Bias
$\hat{\bar{Y}}_g^{(1)}$	$f_{nn'}\bar{Y}\left[\alpha C_X^2+(1-\alpha)C_P^2-\alpha\rho_{YX}C_YC_X-(1-\alpha)\rho_{YP}C_YC_P\right]$
$\hat{\bar{Y}}_g^{(2)}$	$f_{nn'}\bar{Y}\left[\alpha\rho_{YX}C_YC_X+(1-\alpha)\rho_{YP}C_YC_P\right]$
$\hat{\bar{Y}}_g^{(3)}$	$\frac{f_{nn'}\bar{Y}}{K^2}\left[\alpha^2S_X^2+(1-\alpha)^2S_P^2+2\alpha(1-\alpha)\rho_{XP}S_XS_P-\alpha\rho_{YX}C_YS_XK-(1-\alpha)\rho_{YP}C_YS_PK\right]$
$\hat{\bar{Y}}_g^{(4)}$	$\frac{f_{nn'}}{K}\left[\alpha\rho_{YX}S_YS_X+(1-\alpha)\rho_{YP}S_YS_P\right]$
$\hat{\bar{Y}}_g^{(5)}$	$\frac{f_{nn'}\bar{Y}}{2}\left[\begin{array}{l}\alpha(\alpha+1)C_X^2+(1-\alpha)(2-\alpha)C_P^2+2\alpha(1-\alpha)\rho_{XP}C_XC_P-2\alpha\rho_{YX}C_YC_X\\-2(1-\alpha)\rho_{YP}C_YC_P\end{array}\right]$

$\hat{\bar{Y}}_g^{(6)}$	$f_{nn'}\bar{Y}\left[\alpha^2C_X^2+(1-\alpha)^2C_P^2+2\alpha(1-\alpha)\rho_{XP}C_XC_P-\alpha\rho_{YX}C_YC_X-(1-\alpha)\rho_{YP}C_YC_P\right]$
$\hat{\bar{Y}}_g^{(7)}$	$\frac{f_{nn'}\bar{Y}}{2}\left[\alpha(\alpha-1)C_X^2+\alpha(\alpha-1)C_P^2+2\alpha(1-\alpha)\rho_{XP}C_XC_P+2\alpha\rho_{YX}C_YC_X+2(1-\alpha)\rho_{YP}C_YC_P\right]$
$\hat{\bar{Y}}_g^{(8)}$	$f_{nn'}\bar{Y}\left[\alpha(\alpha-1)C_X^2+\alpha(\alpha-1)C_P^2+2\alpha(1-\alpha)\rho_{XP}C_XC_P+\alpha\rho_{YX}C_YC_X+(1-\alpha)\rho_{YP}C_YC_P\right]$
$\hat{\bar{Y}}_g^{(9)}$	$\frac{f_{nn'}\bar{Y}}{2}\left[\alpha_1(\alpha_1-1)C_X^2+\alpha_2(\alpha_2-1)C_P^2+2\alpha_1\alpha_2\rho_{XP}C_XC_P+2\alpha_1\rho_{YX}C_YC_X+2\alpha_2\rho_{YP}C_YC_P\right]$
$\hat{\bar{Y}}_g^{(10)}$	$\frac{f_{nn'}\bar{Y}}{2}\left[\alpha_1(1-\alpha_1)C_X^2+\alpha_2(1-\alpha_2)C_P^2+2\alpha_1\alpha_2\rho_{XP}C_XC_P-2\alpha_1\rho_{YX}C_YC_X-2\alpha_2\rho_{YP}C_YC_P\right]$
$\hat{\bar{Y}}_g^{(11)}$	$f_{nn'}\bar{Y}\left[\begin{array}{l}\alpha_1(\alpha_1-1)C_X^2+\alpha_2(\alpha_2-1)C_P^2+2(\alpha_1-1)(\alpha_2-1)\rho_{XP}C_XC_P\\-(\alpha_1-1)\rho_{YX}C_YC_X-(\alpha_2-1)\rho_{YP}C_YC_P\end{array}\right]$
$\hat{\bar{Y}}_g^{(12)}$	$\frac{f_{nn'}\bar{Y}}{2}\left[\alpha_1(\alpha_1-1)C_X^2+\alpha_2(\alpha_2-1)C_P^2+2\alpha_1\alpha_2\rho_{XP}C_XC_P+2\alpha_1\rho_{YX}C_YC_X+2\alpha_2\rho_{YP}C_YC_P\right]$
$\hat{\bar{Y}}_g^{(13)}$	$\frac{f_{nn'}\bar{Y}}{2}\left[\alpha_1^2C_X^2+\alpha_2^2C_P^2+2\alpha_1\alpha_2\rho_{XP}C_XC_P+2\alpha_1\rho_{YX}C_YC_X+2\alpha_2\rho_{YP}C_YC_P\right]$
$\hat{\bar{Y}}_g^{(14)}$	$\frac{f_{nn'}\bar{Y}}{2}\left[\theta_1\left(\frac{\theta_1}{\alpha}-1\right)C_X^2+\theta_2\left(\frac{\theta_2}{1-\alpha}-1\right)C_P^2+2\theta_1\rho_{YX}C_YC_X+2\theta_2\rho_{YP}C_YC_P\right]$
$\hat{\bar{Y}}_g^{(15)}$	$f_{nn'}\bar{Y}\left[\alpha_1\frac{C_X^2}{4}+\alpha_2\frac{C_P^2}{4}+\alpha_1^2\frac{C_X^2}{8}+\alpha_2^2\frac{C_P^2}{8}-\alpha_1\frac{\rho_{XY}C_XC_Y}{2}-\alpha_2\frac{\rho_{PY}C_PC_Y}{2}+\alpha_1\alpha_2\frac{\rho_{XP}C_XC_P}{4}\right]$

Table 3.3: MSE of the proposed members of the generalized class

Estimator	**MSE**
$\hat{\bar{Y}}_g^{(1)}$	$\bar{Y}^2\left[f_nC_Y^2+f_{nn'}\left\{\begin{array}{l}\alpha^2C_X^2+(1-\alpha)^2C_P^2+2\alpha(1-\alpha)\rho_{XP}C_XC_P\\-2\alpha\rho_{YX}C_YC_X-2(1-\alpha)\rho_{YP}C_YC_P\end{array}\right\}\right]$
$\hat{\bar{Y}}_g^{(2)}$	$\bar{Y}^2\left[f_nC_Y^2+f_{nn'}\left\{\begin{array}{l}\alpha^2C_X^2+(1-\alpha)^2C_P^2+2\alpha(1-\alpha)\rho_{XP}C_XC_P\\+2\alpha\rho_{YX}C_YC_X+2(1-\alpha)\rho_{YP}C_YC_P\end{array}\right\}\right]$
$\hat{\bar{Y}}_g^{(3)}$	$\frac{\bar{Y}^2}{K^2}\left[f_nC_Y^2+f_{nn'}\left\{\begin{array}{l}\alpha^2S_X^2+(1-\alpha)^2S_P^2+2\alpha(1-\alpha)\rho_{XP}S_XS_P\\-2\alpha\rho_{YX}C_YS_XK-2(1-\alpha)\rho_{YP}C_YS_PK\end{array}\right\}\right]$
$\hat{\bar{Y}}_g^{(4)}$	$\frac{1}{K^2}\left[f_nS_Y^2+f_{nn'}\left\{\begin{array}{l}\bar{Y}^2\left(\alpha^2S_X^2+(1-\alpha)^2S_P^2+2\alpha(1-\alpha)\rho_{XP}S_XS_P\right)\\+2\bar{Y}\left(\alpha\rho_{YX}S_YS_X+(1-\alpha)\rho_{YP}S_YS_P\right)K\end{array}\right\}\right]$

$\hat{\hat{\bar{Y}}}_g^{(5)}$	$\bar{Y}^2\left[f_n C_Y^2 + f_{nn'}\begin{Bmatrix}\alpha^2 C_X^2 + (1-\alpha)^2 C_P^2 + 2\alpha(1-\alpha)\rho_{XP}C_X C_P \\ -2\alpha\rho_{YX}C_Y C_X - 2(1-\alpha)\rho_{YP}C_Y C_P\end{Bmatrix}\right]$
$\hat{\hat{\bar{Y}}}_g^{(6)}$	$\bar{Y}^2\left[f_n C_Y^2 + f_{nn'}\begin{Bmatrix}\alpha^2 C_X^2 + (1-\alpha)^2 C_P^2 + 2\alpha(1-\alpha)\rho_{XP}C_X C_P \\ -2\alpha\rho_{YX}C_Y C_X - 2(1-\alpha)\rho_{YP}C_Y C_P\end{Bmatrix}\right]$
$\hat{\hat{\bar{Y}}}_g^{(7)}$	$\bar{Y}^2\left[f_n C_Y^2 + f_{nn'}\begin{Bmatrix}\alpha^2 C_X^2 + (1-\alpha)^2 C_P^2 + 2\alpha(1-\alpha)\rho_{XP}C_X C_P \\ +2\alpha\rho_{YX}C_Y C_X + 2(1-\alpha)\rho_{YP}C_Y C_P\end{Bmatrix}\right]$
$\hat{\hat{\bar{Y}}}_g^{(8)}$	$\bar{Y}^2\left[f_n C_Y^2 + f_{nn'}\begin{Bmatrix}\alpha^2 C_X^2 + (1-\alpha)^2 C_P^2 + 2\alpha(1-\alpha)\rho_{XP}C_X C_P \\ +2\alpha\rho_{YX}C_Y C_X + 2(1-\alpha)\rho_{YP}C_Y C_P\end{Bmatrix}\right]$
$\hat{\hat{\bar{Y}}}_g^{(9)}$	$\bar{Y}^2\left[f_n C_Y^2 + f_{nn'}\begin{Bmatrix}\alpha_1^2 C_X^2 + \alpha_2^2 C_P^2 + 2\alpha_1\alpha_2\rho_{XP}C_X C_P \\ +2\alpha_1\rho_{YX}C_Y C_X + 2\alpha_2\rho_{YP}C_Y C_P\end{Bmatrix}\right]$
$\hat{\hat{\bar{Y}}}_g^{(10)}$	$\bar{Y}^2\left[f_n C_Y^2 + f_{nn'}\begin{Bmatrix}\alpha_1^2 C_X^2 + \alpha_2^2 C_P^2 + 2\alpha_1\alpha_2\rho_{XP}C_X C_P \\ -2\alpha_1\rho_{YX}C_Y C_X - 2\alpha_2\rho_{YP}C_Y C_P\end{Bmatrix}\right]$
$\hat{\hat{\bar{Y}}}_g^{(11)}$	$\bar{Y}^2\left[f_n C_Y^2 + f_{nn'}\begin{Bmatrix}(\alpha_1-1)^2 C_X^2 + (\alpha_2-1)^2 C_P^2 + 2(\alpha_1-1)(\alpha_2-1)\rho_{XP}C_X C_P \\ -2(\alpha_1-1)\rho_{YX}C_Y C_X - 2(\alpha_2-1)\rho_{YP}C_Y C_P\end{Bmatrix}\right]$
$\hat{\hat{\bar{Y}}}_g^{(12)}$	$\bar{Y}^2\left[f_n C_Y^2 + f_{nn'}\begin{Bmatrix}\alpha_1^2 C_X^2 + \alpha_2^2 C_P^2 + 2\alpha_1\alpha_2\rho_{XP}C_X C_P \\ +2\alpha_1\rho_{YX}C_Y C_X + 2\alpha_2\rho_{YP}C_Y C_P\end{Bmatrix}\right]$
$\hat{\hat{\bar{Y}}}_g^{(13)}$	$\bar{Y}^2\left[f_n C_Y^2 + f_{nn'}\begin{Bmatrix}\alpha_1^2 C_X^2 + \alpha_2^2 C_P^2 + 2\alpha_1\alpha_2\rho_{XP}C_X C_P \\ +2\alpha_1\rho_{YX}C_Y C_X + 2\alpha_2\rho_{YP}C_Y C_P\end{Bmatrix}\right]$
$\hat{\hat{\bar{Y}}}_g^{(14)}$	$\bar{Y}^2\left[f_n C_Y^2 + f_{nn'}\begin{Bmatrix}\theta_1^2 C_X^2 + \theta_2^2 C_P^2 + 2\theta_1\theta_2\rho_{XP}C_X C_P \\ +2\theta_1\rho_{YX}C_Y C_X + 2\theta_2\rho_{YP}C_Y C_P\end{Bmatrix}\right]$
$\hat{\hat{\bar{Y}}}_g^{(15)}$	$\bar{Y}^2\left[f_n C_Y^2 + f_{nn'}\left\{\alpha_1^2\frac{C_X^2}{4} + \alpha_2^2\frac{C_P^2}{4} + \alpha_1\alpha_2\frac{\rho_{XP}C_X C_P}{2} - \alpha_1\rho_{XY}C_X C_Y - \alpha_2\rho_{PY}C_P C_Y\right\}\right]$

CONCLUSION

The above theoretical study establishes the importance of the proposed generalized class of double sampling estimators due to its wider applicability and emphasizes its superiority over the proposed generalized class of estimators in the sense of unbiasedness. Also it attains minimum value of MSE under the optimum values of the characterizing parameters.

$$\min MSE(\hat{\hat{\bar{Y}}}_g^{*}) = f_n(1-R_{Y.XP}^2)S_Y^2 .$$

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