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Modeling and Algorithm Development for Cattle Feed Mix Formulation

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Abstract

The objective of this paper is to develop the algorithms for optimal feed mix of dairy cows at different stages of livestock. These algorithms have been proposed to formulate feed mix at minimum cost and maximum shelf life for dairy cattle. Different forms of mathematical programming have been used to develop these algorithms such as stochastic and weighted goal programming. These algorithms incorporate nutrient variability of different nutrients present in the feed ingredients which has not been done by the available softwares. These algorithms also minimize the deviations of the objectives of cost minimization and shelf life maximization.

Keywords: Cattles; feed mix; algorithms; cost minimization; shelf-life maximization.

INTRODUCTION

Feed mix is a mixture of different feed ingredients used for animal feeding. A balanced feed mix provides required nutrition to the cattle at different stages of livestock therefore, in dairy industry, formulation and computation of balanced feed mix is of utmost importance. It provides better yields, productivity and nutrient utilization. For achieving these objectives, optimization should be done in such a manner that optimal feed mix can fulfil the nutritional requirements of the animal at different stages of production and livestock. Optimization models are in use for more than a half century at commercial level and for livestock management. A linear programming technique has been developed for defining feed formulation problem [1]. Various animal feed formulation techniques have been reviewed in an article [2]. Another review article has been presented for formulating animal feed mix on the basis of various programming techniques [3]. An LP model has been presented to find the least-cost ration for drought maintenance of dry adult sheep. Results showed a considerable reduction in the feeding cost compared to currently recommended standards [4]. A linear programming model has been developed to find the least cost ration for broilers of age 6 to 10 weeks for the utilization of locally available and non-conventional feed stuff-Duckweed (Lemna paucicostata) [5]. A model has been developed for optimal beef production systems in Ireland in which the objective is to maximize farm gross margin under a constraint set of animal nutritional requirements [6]. A linear model has been developed for the Nigerian Poultry Industry and it has been found that cost was reduced by 9% compared to existing practice of the farm [7]. A nonlinear programming model has been developed for weight gain in sheep [8]. A paper has been presented in which LP, LP with a margin of safety (LPMS) and SP models have been used to formulate poultry rations at least cost, with a given probability level to meet nutrient requirements, set by the NRC in 1984. LP has formulated least cost feed mix but has not the ability to consider nutrient variability. The LPMS and SP models have met poultry nutritional requirements at different confidence levels varying from P 0.5 to 0.90. The SP model has produced lower cost feed mix than LPMS [9]. Linear and stochastic programming techniques have been used for incorporating nutrient variability in animal feed formulation [10]. To consider nutrient variability in least-cost feed formulation model for African catfish SP technique has been used [11]. Goal Programming has been presented as a tool for formulating feed mixes using one hundred and fifty food raw materials. Results by GP showed improvement over those of LP [12]. A handy spreadsheet tool has been developed for the formulation of a daily cow feed mix supported by linear programming and weighted goal programming techniques [13]. A multi criteria programming model has been developed using goal programming technique to find optimized feed blend[14]. A model has been developed using a combination of LP and WGP. In this model, multiple goals have been incorporated for optimization. The method has been tested and concluded to more accurate and useful results in practice by WGP as compared to LP [15].A paper has been presented to formulate ruminant ration under bi-objective criteria [16]. The nutritional requirements of dairy cattle are different at different weights as per NRC recommendations [17].

In this paper, objectives are to develop the algorithms:

- to achieve the nutrient variability included feed mix at lowest possible cost which satisfy the nutritional requirement of the cattles at different weight stages.
- to achieve the nutrient variability included feed mix with maximum shelf life which satisfy the nutritional requirement of the cattles at different weight stages.
- to minimize the deviations of the achieved objectives.

This paper presents the technological intervention to the field of animal nutrition by developing algorithms to achieve the above objectives. Previously, cost minimization and shelf life maximization models have been developed [18, 19] but these models were solved by available software. This technology proposes development of algorithms to achieve these two objectives with inclusion of nutrient variability. The density of nutrient contents of different feed ingredients may change considerably. While formulating the optimal feed mix using algorithm based on linear programming model, this variation cannot be considered and cause over-formulation or under-formulation which results in higher cost, over or under achievement of nutrient requirements, adverse effect on the growth rate of the animals etc. Therefore, to reduce the risk of over and under achievement of nutrients, it is essential to consider this variation while developing the algorithms for animal feed mix formulation. This technology is an attempt to deal with this nutrient variability. These algorithms have been developed to incorporate nutrient variability. In addition to this, one of the algorithm is developed to minimize the deviations of the objectives which is done innovatively first time in the area of feed mix formulation.

Algorithm based on linear programming model is used to achieve cattle feed mix at lowest cost for different stages of livestock. It is also used to maximize the shelf life of cattle feed mix. Shelf life can be increased by reducing water content for different stages of livestock. Algorithm based on stochastic programming model is used to incorporate nutrient variability so that the risk of not meeting the nutrient requirement can be minimized. Practically, feed mix formulation is a complex process. It cannot be confined to the achievement of one objective only. Real life problems need a solution which satisfies multiple conflicting objectives on a priority basis. To overcome this drawback Multi-criteria Goal programming model can be used. GP is used with weights and priorities for prioritizing multiple goals of a single farm holder and also to assign weightage to the goals of same priority level. Priorities have been introduced to achieve multiple target values simultaneously with different preferences. Weights have been introduced to give weightage to different goals in the same priority level.

METHODS

Three algorithms have been developed to achieve the said objectives and a combination of mathematical programming has been used for this purpose. In developing algorithms the objective functions are taken as cost minimization and water content minimization for maximum shelf life. Constraints have been formulated on the basis of nutritional requirements of the animals at different stages of livestock. These algorithms will provide the optimal feed mix with lowest cost and minimum water content to reach different weight class of dairy cattle. Firstly algorithm 1 has been developed to achieve the optimal feed mix with rigid nutrient constraints. Then, algorithm 2 has been developed to incorporate nutritional variability of different nutrients. It minimizes the risk of not meeting the nutrient requirement. Finally, algorithm 3 for developing animal feed mix provides the optimal solution with the satisfaction of the constraints depending on the priorities and weights of goals. These algorithms are capable of considering different priorities and weights associated with the goals and therefore providing more practical results.

Notations used for developing mathematical models:

z objective function, a_{ij} amount of ith nutrient available in the jth feed ingredient, x_j quantity of jth feed ingredient in the feed mix, b_i minimum requirement of ith nutrient , c_j per unit cost of feed ingredient j , i index identifying feed nutrient components with i = 1,2,….m , j index identifying feed ingredients with j =1,2,………n.

Algorithm 1 has been developed for formulating and computing feed mix with lowest possible cost and minimum water content on the basis of linear programming model.

Linear Programming Model for computing feed mix:

$$\begin{aligned} &Min\, z = \sum c_j x_j \\ &s.t. \sum_{j=1}^{n} a_{ij} x_j \geq b_i \\ &\quad x_j \geq 0, b_i \geq 0 \end{aligned} \tag{1}$$

Algorithm for computation of feed mix at minimum cost and water content (Algorithm 1):

Step 1: Define nature of objective function (max. or min.).

Step 2: Input number of decision variables (i.e. j).

Step 3: Input cost coefficients (per unit cost of each feed ingredient (x_j)) c_j ^{s} for j=1 to 16 to formulate objective function.

Step 4: Input technological coefficients a_{ij}^{s} and requirement variables(nutrient requirements) b_i^{s} for i=1 to 5 and j=1 to 16 to formulate the constraints.

Step 5: Formulate the linear mathematical model.

Step 6: Introduce artificial variables to get basis matrix as we are not getting identity matrix as basis matrix.

Step 7: Construct auxiliary LPP $\min\ f_a = \sum a_k \ \ for\, k = 1 to 5$

Step 8: Construct simplex table of phase I with 5 basic variables.

Step 9: Check optimality condition using $z_j - c_j$

(i) $z_j - c_j \le 0 \forall j$ for minimization

(ii) $z_j - c_j \ge 0 \forall j$ for maximization .

Step 10: (i) If optimality condition is satisfied then

(a) stop

(b) write the optimal solution of phase I, go to step 12.

(ii) else

(a) find leaving variable,

(b) entering variable and

(c) the pivot element.

(d) Construct the simplex table.

Step 11: Repeat step 9-10.

Step 12: Construct the simplex table of Phase II.

Step 13: Check optimality condition using $z_j - c_j$

(i) $z_j - c_j \le 0 \forall j$ for minimization

(ii) $z_j - c_j \ge 0 \forall j$ for maximization .

Step 14: (i) If optimality condition is satisfied then,

(a) stop

(b) write the optimal solution(feed mix) of the problem.

(ii) else

(a) find leaving variable

(b) entering variable

(c) the pivot element.

(d) construct the simplex table.

Step 15: Repeat step 13-14.

Flow chart for algorithm 1 is shown in fig. 1.

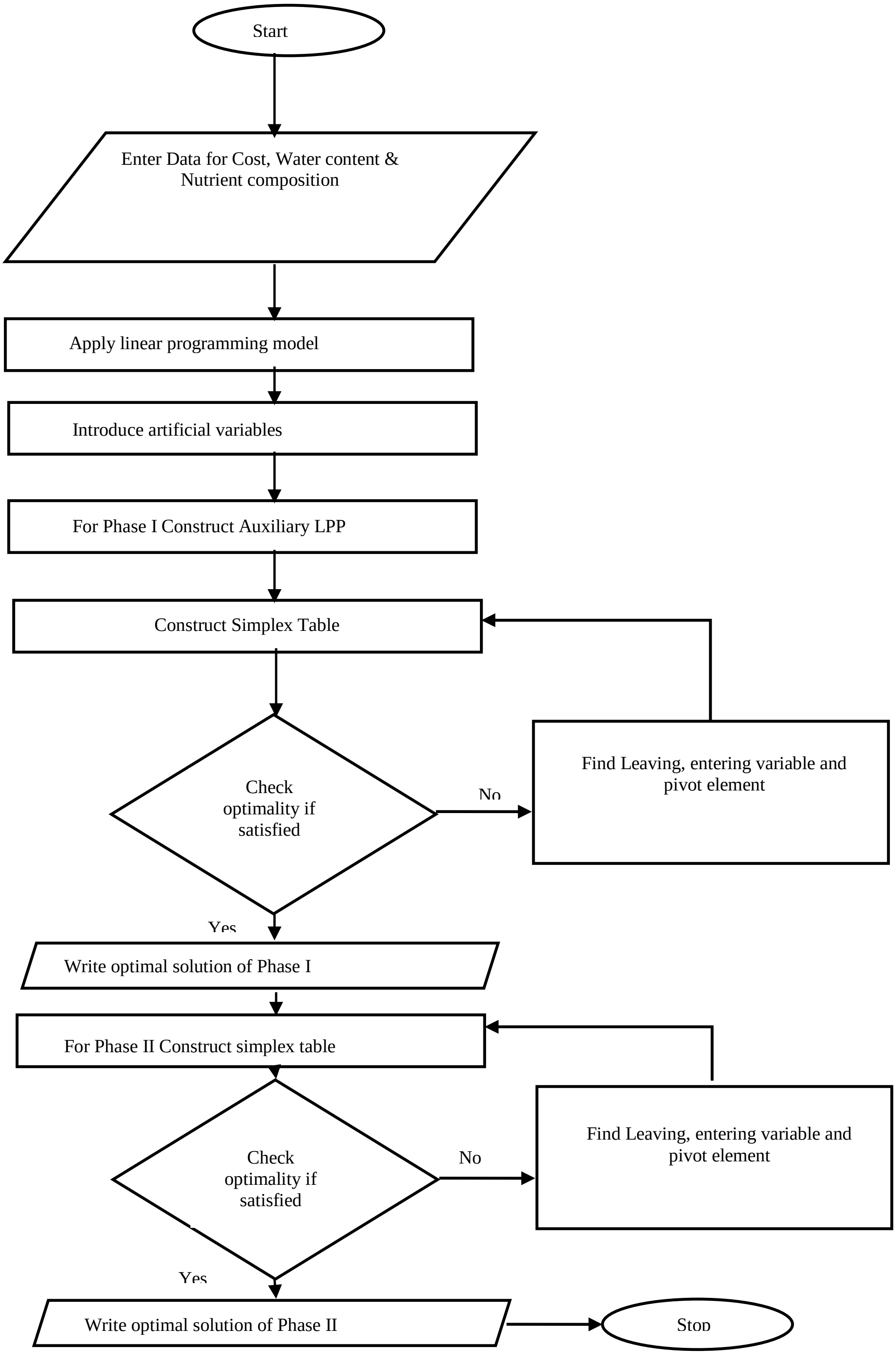

Figure 1: Flow Chart for algorithm 1

Algorithm 1 gives the optimum feed mix with minimum cost and water content but it does not take into account the nutritional variability.

Algorithm 2 has been developed for the same objectives as above but with the incorporation of nutritional variability of different nutrients as it is an important factor in computing feed mix. This variation, if not considered, can affect the growth rate of animal negatively. Therefore, with this algorithm the effect of nutritional variability of different nutrients on the feed mix can be controlled. In the presence of variability, it is possible to determine the probability that the nutrient concentration in the feed mix meets or exceeds the specified requirements in the feed blend. Therefore stochastic programming models are introduced to consider the variability of nutrients present in different feed ingredients. To introduce the variability of nutrient components, nonlinear variance of each nutrient ingredient is added at a desired probability level in the mathematical model. $\sigma_{ij}{}^{2}$ represents variance of nutrient i in ingredient j and it is included with a certain probability level, z represents level of probability and rest of the variables are defined as above .

Stochastic Programming Model for computing feed mix:

$$\begin{aligned} &Min\ \ z = \sum c_j x_j \\ &s.t.\ \sum_{j=1}^{n}\left(a_{ij} - z\left(\sqrt{\sum_{j=1}^{n}\sigma_{ij}{}^{2}}\right)\right)x_j \geq b_i \\ &\quad x_j \geq 0, b_i \geq 0 \end{aligned} \tag{2}$$

Algorithm for computation of feed mix at minimum cost and water content including nutrient variability (Algorithm 2):

Step 1: Define nature of objective function (max. or min.).

Step 2: Input number of decision variables (i.e. j).

Step 3: Input cost coefficients $c_j{}^{s}$ for j=1 to 16 to formulate objective function.

Step 4: Input technological coefficients $a_{ij}{}^{s}$ and requirement variables $b_i{}^{s}$ for i=1 to 5 and j=1 to 16 to formulate the constraints.

Step 5: Input probability level and standard deviation σ_{ij} for i=1 to 5 and j=1 to 16.

Step 6: Calculate z for the given probability level.

Step 7: Formulate the Stochastic mathematical model.

Step 8: Introduce artificial variables to get basis matrix as we are not getting identity matrix as basis matrix.

Step 9: Construct auxiliary LPP $\min\ f_a = \sum a_k\ \ for\ k = 1\ to\ 5$

Step 10: Construct simplex table of phase I with 5 basic variables.

Step 11: Check optimality condition using $z_j - c_j$

(i) $z_j - c_j \le 0 \forall j$ for minimization

(ii) $z_j - c_j \ge 0 \forall j$ for maximization .

Step 12: (i) If optimality condition is satisfied then

(a) stop

(b) write the optimal solution of phase I, go to step 14.

(ii) else

(a) find leaving variable,

(b) entering variable and

(c) the pivot element.

(d) Construct the simplex table.

Step 13: Repeat step 11-12.

Step 14: construct the simplex table of Phase II.

Step 15: Check optimality condition using $z_j - c_j$

(i) $z_j - c_j \le 0 \forall j$ for minimization

(ii) $z_j - c_j \ge 0 \forall j$ for maximization .

Step 16:(i) If optimality condition is satisfied then,

(a) stop

(b) write the optimal solution(feed mix) of the problem.

(ii) else

(a) find leaving variable

(b) entering variable

(c) the pivot element.

(d) construct the simplex table.

Step 17: Repeat step 15-16.

Flow chart for algorithm 2 is shown in fig. 2.

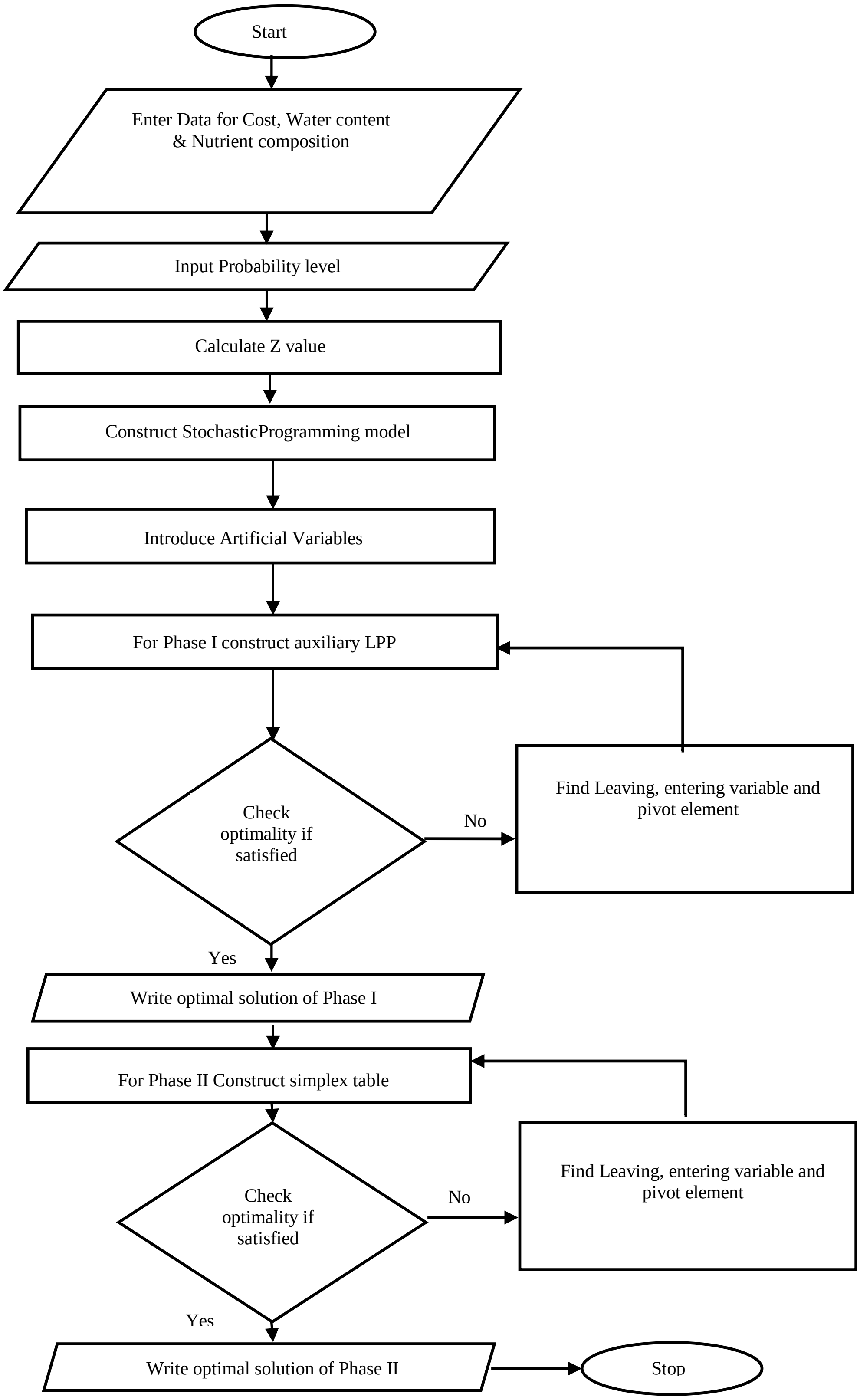

Figure 2: Flow Chart for algorithm 2

Now algorithm has been developed with an emphasis on minimizing the deviations of the above formulated models. Mathematical model has been formulated with the help of weighted goal programming. Two goals are formed for minimizing the deviations for cost and water content minimization. For each goal, the objective functions of stochastic programming model are reconstructed as constraints with deviation variables. Rest of the constraints are same as in stochastic programming model. The objective function of the weighted goal programming model has been defined with the help of priorities, weights and deviation variables, corresponding to the constraints of cost and water content minimization, which is to be minimized as both of the objectives may not be fully satisfied simultaneously.

Weighted Goal Programming Model for determination of feed blend:

$$\begin{aligned} &\min z = P_1 \sum_{k=1}^{2} w_{1k} \frac{d_{1k}{}^{+}}{z_k} + P_2 \sum_{i=1}^{5} w_{2i} \frac{d_{2i}{}^{+} + d_{2i}{}^{-}}{b_i} \\ &s.t. \sum_{j=1}^{16} c_{kj} x_j - d_{1k}{}^{+} = z_k \qquad (3a) \\ &\sum_{j=1}^{n} a_{ij} x_j + d_{2i}{}^{-} - d_{2i}^{+} = b_i \qquad (3b) \\ &x_j \ge 0, b_i \ge 0 \end{aligned} \tag{3}$$

k=1, 2 (corresponding to objectives as goals)

i=1 to 5 (corresponding to constraints as goals)

In the above models, the equations denoted by (3a) are the goals corresponding to the objectives of cost minimization and water content minimization with only over-achievement d_{1k}^{+} . These goals have been given priority P_1 with weights w_{11} and w_{12}. The equations denoted by (3b) are the goals corresponding to the nutritional requirement constraints with over and under achievement d_{2i}^{+} and d_{2i}^{-}. These goals have been given priority P_2 with weights w_{21}, w_{22}, w_{23}, w_{24} and w_{25}.

In developing weighted goal programming model normalization technique has been used to overcome the issue of different units of goals used for developing objective function.

Algorithm for weighted goal programming (Algorithm 3):

Step 1: Set the goals.

Step 2: Set the priorities and weights of the goals.

Step 3: Input goals as constraints.

Step 4: Input hard constraints.

Step 5: Add deviation variables to the goal constraints.

Step 6: Identify the variables to be minimized in the objective function.

Step 7: Write the weighted goal programming model.

Step 8: Identify goals with highest priority.

Step 9: Write the simplex table corresponding to this goal.

Step 10: Apply simplex algorithm.

Step 11: Find the optimal solution of this problem.

Step 12: (i) If alternative optimal solution exists then

a. Find the slack or surplus variables with negative value of $z_j - c_j$
b. Drop those slack or surplus variables from the table.
c. Delete objective function row.
d. Add constraints of next highest priority level.
e. Make solution feasible.
f. Add objective function corresponding to priority.
g. Repeat step 9-12.

(ii) else

a. Stop
b. Write the optimal solution of the problem.

Flow chart for algorithm 3 is shown in figure 3.

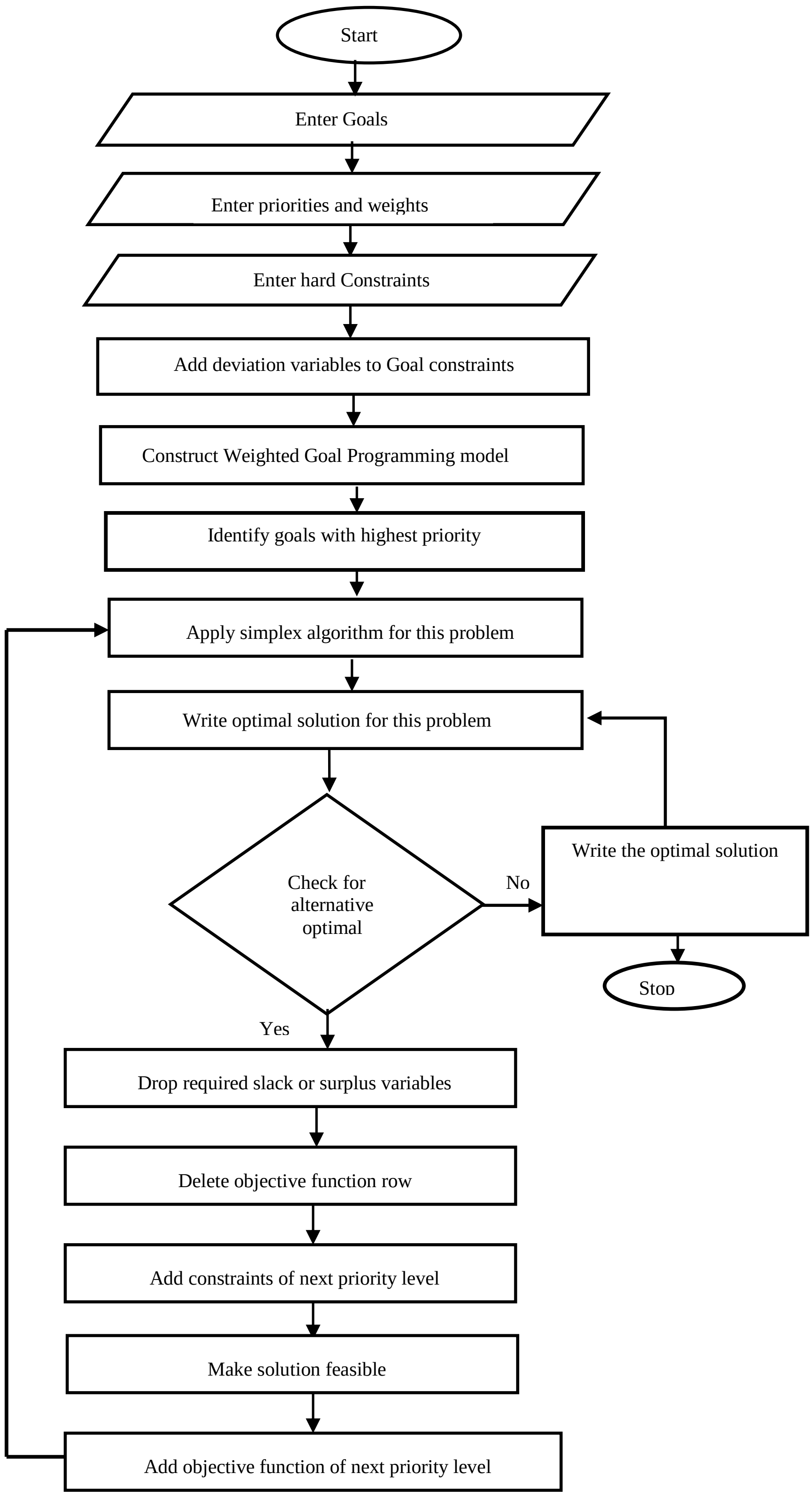

Figure 3: Flow Chart of algorithm 3

RESULTS AND DISCUSSION

Algorithms are developed for computation of optimal feed mix for dairy cattle. Models have been developed by using linear, stochastic and goal programming models and proposes a useful procedure for determination of the optimal livestock feed mix. This paper represents an innovative approach towards introduction of technology and leads to software development in the area of animal feed mix formulation. The paper represents algorithmic approach to bi-criteria model and can be extended to multi-criteria models. Algorithm 1 is providing the optimal feed mix. Algorithm 2 is incorporating nutritional variability of different nutrients to minimize the risk of not meeting the nutrient requirement. Algorithm 3 is providing the optimal solution with the satisfaction of the constraints depending on the priorities and weights of goals. Therefore it can be used to obtain more balanced feed mix which can optimize multiple parameters at the same time. The algorithms have been applied and verified on the data of dairy cattle. Data for composition of feed ingredients with cost, water content and nutrients are given by table 1.

Table 1: Composition of feed ingredients with cost, water content and nutritional composition

Notation	**Feed ingredients**	**Price (Rs./ Kg.)**	**Water content (on the basis of DM)**	**Metabolizable energy(ME) (mj/kg of feed)**	**Crude Protein (CP)(g/kg)**	**NDF (g/kg)**	**DM (g/kg)**	**Ca (g/kg)**	**P (g/kg)**
x_1	Alfalfa hay	14	.11	7.51	163	400	894	15	2.3
x_2	Barley grain	10	.13	10.80	103	189	871	0.7	3.4
x_3	Sugarbeetpulp	15	.11	9.99	83	429	892	13.83	0.89
x_4	Cottonseed meal (high fibre, low oil)	18	.10	9.2	360	330	902	2.62	11
x_5	Soyabean meal(high protein- dehulled)	28	.12	11.98	471	97	881	3.17	6.70
x_6	Sunflower meal(solvent-extracted, dehulled or non-dehulled)	16	.11	8.10	288	400	890	3.92	10.32
x_7	Wheat bran	19	.13	9.57	151	394	870	1.22	9.66
x_8	Maize grain high moisture	23	.35	8.84	62	89	650	0.32	2.01
x_9	Sorghum grain	17	.13	11.80	94	96	874	0.26	2.88
x_{10}	Groundnut meal(solvent-extracted)	25	.11	11.16	489	217	893	1.52	5.54
x_{11}	Rice bran(fibre 11-20%)	10	.10	9.11	115	310	902	0.63	12.45

x_{12}	Oats grain	18	.12	8.70	97	314	879	0.97	3.16
x_{13}	Wheat straw	7	.09	6.19	38	706	910	4.37	0.64
x_{14}	Corn gluten feed	14	.12	10.77	192	350	883	1.41	9.01
x_{15}	Canola meal(solvent-extracted)	24	.10	10.54	351	242	901	6.67	10.45
x_{16}	Cottonseed hulls	11	.10	5.89	46	773	906	1.18	0.91

Table 2: Minimum Nutritional Requirement of different nutrients at different weights of dairy cattle to reach at 600 kg weight

Wt. class/Nutrient	ME(MJ)	CP(g)	DM(g)	Ca(g)	P(g)
200 kg	43.71	533	5000	18	12
300 kg	57.11	671	6670	20	15
450 kg	67.37	749	7870	23	18
600 kg	82.06	879	9580	25	18

Then algorithm 1 is used to solve the model defined by equation (1) for cost minimization and shelf life maximization with rigid nutrient constraints. Optimal value of objective function and feed ingredients according to algorithm 1 has been shown in table III. Graphical results for feed ingredients corresponding to algorithm 1 have been shown in figure 4.

Table 3: Results from algorithm 1

	Feed Ingredients	**Min cost**				**Min Water content**			
		200	**300**	**450**	**600**	**200**	**300**	**450**	**600**
x_1	Alfalfa hay	0.05	0	0	0	0	0	0	0
x_2	Barley grain	1.56	.59	.61	0	0	0	0	0
x_3	Sugarbeetpulp	0	0	0	0	0	0	0	0
x_4	Cottonseed meal (high fibre, low oil)	.66	.5	.4	.16	0	0	0	0
x_{11}	Rice bran(fibre 11-20%)	0	2.46	3.23	5.52	0	0	0	0
x_{12}	Oats grain	0	0	0	0	0	0	0	0
x_{13}	Wheat straw	3.30	3.83	4.46	4.9	3.32	4.69	5.54	6.74
x_{15}	Canola meal(solvent-extracted)	0	0	0	0	2.20	2.66	3.14	3.83
	Objective Function	51.29	66.35	76.87	92.34	.52	.69	.81	.99

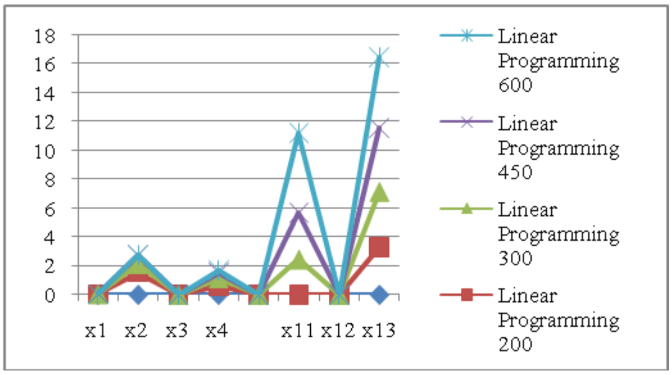

Fig. 4: Optimum values of feed ingredients from algorithm 1

Algorithm 2 is used to solve the model defined by equation 2 for cost minimization and shelf life maximization with variable nutrient concentration. Optimal value of objective function and feed ingredients according to algorithm 2 has been shown in table IV. Graphical results for feed ingredients corresponding to algorithm 2 have been shown in figure 5.

Table 4: Results from algorithm 2

	Feed Ingredients/ weight class	**Min cost**				**Min water content**			
		200	**300**	**450**	**600**	**200**	**300**	**450**	**600**
x_1	Alfalfa hay	0.54	0.48	0.54	0	.34	.22	.23	0
x_2	Barley grain	0	0	0	0.08	0	0	0	0
x_3	Sugarbeetpulp	2.49	3.14	3.69	4.84	1.12	1.34	1.55	1.85
x_4	Cottonseed meal (high fibre, low oil)	0	0	0	0	0	0	0	0
x_{11}	Rice bran(fibre 11-20%)	2.3	3.02	3.59	4.10	0	0	0	0
x_{12}	Oats grain	0	0	0	0	0	0	0	0
x_{13}	Wheat straw	0.28	0.83	1	1.71	1.30	2.17	2.59	3.33
x_{15}	Canola meal(solvent-extracted)	0	0	0	0	2.83	3.71	4.42	5.51
	Objective Function	69.87	89.89	105.80	126.41	.56	.74	.87	1.05

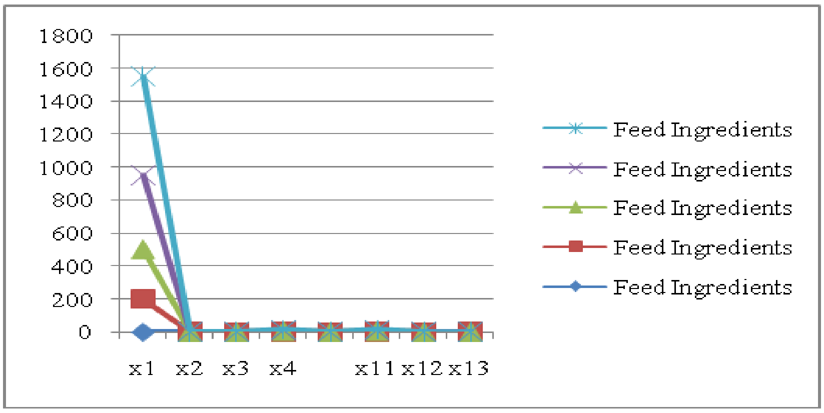

Figure 5: Optimum values of feed ingredients from algorithm 2

Tabular data and figures are depicting that though minimum values of objective functions have been achieved by using algorithm 1, more feed ingredients have been included in the feed mix achieved by algorithm 2. These results are showing that in terms of nutritional variability inclusion algorithm 2 is giving better results. Results from algorithm 3 are controlling over-achievement and under-achievement of values of different nutrients. Priorities are introduced in the model as P_1 and P_2. P_1 is the priority for both of the objectives and P_2 is for all the other constraints.

Results for the numerical data of dairy cattle according to algorithm 3 have been represented in table V. This table provides optimal values of objective function, feed ingredients and deviation variables.

Graphical view of values of deviations and of feed ingredients from algorithm 3 is shown in figure 6 and figure 7.

Table 5: Results obtained from algorithm 3

Variables/weight clas	**200**	**300**	**450**	**600**
$d21^-$	.201	.261	0	1.665
$d23^-$	279.154	388.922	580.528	774.521
$d24^-$	.386	0	0	0
x3	2.987	3.667	4.325	4.901
x4	.187	0	0	0
x9	0	.015	.288	.568
x10	0	.130	.026	.051
x11	2.097	2.908	3.414	3.779
x13	0	.328	.122	.593
x15	.030	0	.016	0
Objective value	2.663	2.652	2.951	4.654

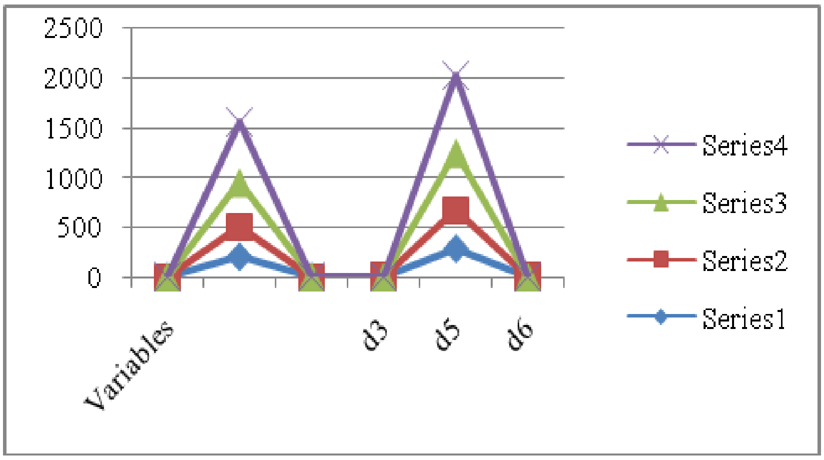

Figure 6: Deviations for weight classes

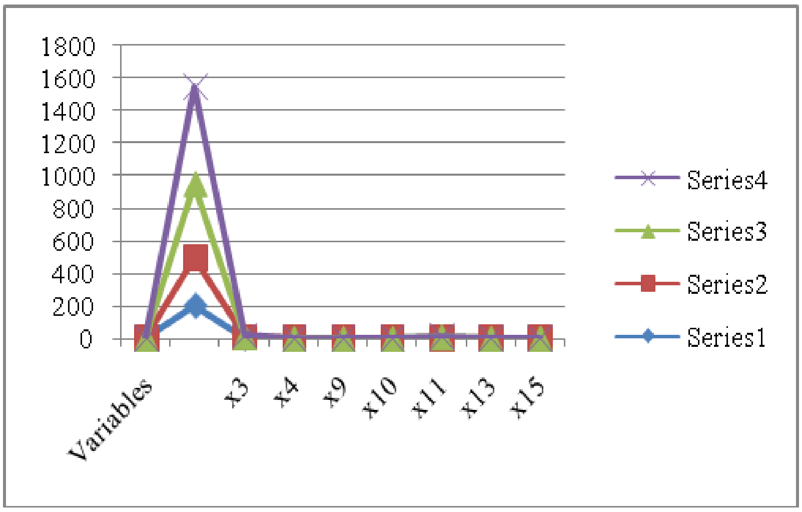

Figure 7: Optimized value for decision variables

Results are showing that maximum deviation is of under achievement of dry matter. Results show underachievement of calcium, ME and Dry matter.

CONCLUSION

In this paper, algorithms have been developed for computation of dairy cattle feed mix with lowest cost and maximum shelf life. Models of linear, stochastic and weighted goal programming with priority functions have been used for developing algorithms. It has shown a step by step approach to refine the results and make them more effective for better results.

REFERENCES

[1] F.V.Waugh. "The minimum cost dairy feed", Journal of Farm Economics, vol. 33, pp. 299-310, 1951.

[2] P. Saxena and M. Chandra. "Animal Diet Formulation: A Review (1950-2010)", CAB Reviews: Perspectives in Agriculture, Veterinary Science, Nutrition and Natural Resources, vol. 6(57), pp. 1-9, 2011.

[3] P. Saxena and N. Khanna. "Animal Diet Formulation: Mathematical Programming Techniques", CAB Reviews, vol. 9(35),pp. 1-12, 2014.

[4] D.T. Vere. "Maintaining sheep during draught with computer formulated rations", Review of Marketing and Agricultural economics, vol. 40(3), pp. 111–122, 1972.

[5] Olorunfemi, O.S. Temitope. "Linear Programming Applications to utilization of duckweed (Lemna paucicostata) in least cost ration formulation for Broiler Finisher", Journal of Applied Science, vol. 6(9), pp. 1909-1914, 2006**.**

[6] P. Crosson, P. O. Kiely, F. P. O. Mara and M. Wallace. "The development of a mathematical model to investigate Irish beef production systems", Agricultural Systems, vol. 89, pp. 349–370, 2006.

[7] V.O. Oladokun and A. Johnson. "Feed formulation problem in Nigerian poultry farms: a mathematical programming approach", American Journal of Scientific and Industrial Research, vol. 3(1), pp. 14–20, 2012.

[8] P. Saxena. "Application of nonlinear programming in the field of animal nutrition: A problem to maximize the weight gain in sheep", National Academy Science Letter, vol. 29 (1-2), pp. 59-64, 2006.

[9] T.H. D'Alfonso, W.B. Roush and J.A. Ventura. "Least cost poultry rations with nutrient variability: A comparison of linear programming with a margin of safety and stochastic programming models", Poultry Science, vol. 71(2), pp. 255–262, 1992.

[10] P.R. Tozer. "Least cost ration formulations for Holstein dairy Heifers by using linear and stochastic programming", Journal of Dairy Science, vol. 83, pp. 443-451, 2000.

[11] I.U. Udo, C.B. Ndome and P.E. Asuquo. "Use of Stochastic programming in least-cost feed formulation for African catfish (Clarias gariepinus) in semi-intensive culture system in Nigeria", Journal of Fisheries and Aquatic Science, vol. 6, pp. 447–55, 2011.

[12] A.M. Anderson and M.D. Earle. "Diet planning in the third world by linear and goal programming", Journal of Operational Research Society, vol. 34(1),pp. 9–16, 1983.

[13] J. Žgajnar, L. Juvančič and S. Kavčič. "Combination of linear and weighted goal programming with penalty function in optimisation of a daily dairy cow ration", Agricultural Economics – Czech, vol. 55(10), pp. 492–500, 2009.

[14] B. Zoran and P. Tunjo. "Optimization of livestock feed blend by use of goal programming", International Journal Production Economics, vol. 130, pp. 218-223, 2011.

[15] J. Prisˇenk J, K. Pazˇek, C. Rozman, J. Turk, M. Janzˇekovicˇ and A. Borec. "Application of weighted goal programming in the optimization of rations for sport horses", Journal of Animal and Feed Sciences, vol. 22, pp. 335–41, 2013.

[16] P. Saxena and N. Khanna. "Formulation and Computation of Animal Feed Mix: Optimization by Combination of Mathematical Programming", Advances in Intelligent Systems and Computing, vol. 337, pp. 621-629, 2015.

[17] National Research Council. Nutrient requirements of Dairy Cattle. 7th Rev. ed. Natl. Acad. Sci., Washington, DC, 2001.

[18] P. Saxena and N. Khanna. "Computation of cattle feed mix by using priority function: Weighted Goal Programming", published in Computing for Sustainable Global Development(9th INDIAcom-2015), pp. 1595-1600, 2nd International Conference On Computing for sustainable global development INDIAcom-2015, BVICAM, New Delhi, March 11-13, 15.

[19] P. Saxena and N. Khanna. "Optimization of Dairy Cattle Feed by Nonlinear Programming", published in Computing for Sustainable Global Development(9th INDIAcom-2015), pp. 1579-1584, 2nd International Conference On Computing for sustainable global development INDIAcom-2015, BVICAM, New Delhi, March 11-13, 15.