

A Study of Crossover Operators for Genetic Algorithms to Solve VRP and its Variants and New Sinusoidal Motion Crossover Operator

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Abstract

Vehicle Routing Problem (VRP) is one of the prominent combinatorial problems and there is a need to address these problems due to increase in logistic activities across the globe. Complexity in solving the problem increases exponentially as the problem size increases. Genetic Algorithm (GA) is widely applied search method to the complex problems such as VRPs. Selection, Crossover and Mutations are the main operators used in GAs. This paper aims at studying different types of Crossover Operators used in Genetic Algorithm applied in solving variety of Vehicle Routing problems. At the end of the paper a new crossover operator 'Sinusoidal Motion Crossover (SMC)' is proposed and demonstrated with two illustrations.

Keywords: Vehicle routing problem, Genetic algorithm, Crossover Operators.

1. INTRODUCTION

Vehicle routing is an important area in the field of Supply chain management. Therefore, solving Vehicle routing problem (VRP) efficiently plays a vital role in effective implementation of supply chain practices for any organization. The general VRP is concerned with delivering goods to a set of customers with known demands through a vehicle route that begins and finishes at the depot with minimum cost. To solve these types of NP-hard problems, it requires sophisticated heuristics such as Genetic Algorithms (GA). A genetic algorithm is one of the prominent global search techniques widely used for combinatorial type problems which is based on imitating

processes observed during natural evolution. Genetic Algorithm provides large set of solutions when compared to other exact algorithms. Operators such as Selection, Crossover and Mutations are vital in deriving solutions using genetic algorithms. Crossover also called as recombination is critical in design and implementation of GAs. Crossover operator creates new individuals termed as offspring or children by combining parts from two or more individuals called as parents.

2. LITERATURE REVIEW

Ombuki et.al (2006) presented a multi-objective genetic algorithm for vehicle routing problem with time windows with the objective of minimizing total number of vehicles and total cost (distance). While solving they utilized Best Cost Route Crossover (BCRC) as crossover operator and constrained route reversal mutation as mutation operator. They concluded that their approach is effective as the solutions obtained are competitive with best known solution in the literature. Honglin and Jijun (2006) considered an improved genetic algorithm for the vehicle routing problem with the objectives to find feasible routes which minimizes the total distance travelled and the number of vehicles used for transportation. To prevent premature convergence and accelerate searching procedure, they utilized PMX crossover, swapping mutation and inversion operators in their improved genetic algorithm. At the end they claim that their proposed algorithm is feasible and robust in solving VRP and it outperforms the published results obtained using hill-climbing, pure GA and hybrid GA.

Kratika et al. (2012) presented a genetic algorithm for the routing and carrier selection problem with the objective to minimize overall cost based on three costs, viz., fixed cost for vehicles in the internal fleet, variable transportation cost of routing every vehicle and a cost of freight charged by external carriers for remaining customers not served by internal fleet. One point crossover and standard simple mutation operators are used as genetic operators for this study. They carried out experiments on standard data sets available from literature and justify that their proposed algorithm is robust in terms of solution quality and running times. Tasan and Gen (2012) considered a genetic algorithm based approach to vehicle routing problem with simultaneous pick-up and deliveries with the objective to determine efficient and effective vehicle routes. They used partial-mapped crossover (PMX) and swap mutation as genetic operators in their study and concluded that their proposed GA based approach performs well.

Chand and Mohanty (2013) proposed master-slave genetic algorithm for real time vehicle routing problem with time windows wherein they have considered simultaneous delivery of products and pick up of wastage materials. They utilized Sub Route Sequence Crossover Method (SRSCM) to generate next generation offspring and used Sub Route Alter Mutation Method (SRAMM) as mutation operator. Authors

here have claimed that their algorithm can decrease the total cost of the route efficiently and perform well on large population instances. Chand and Mohanty (2013) considered the problem of solving vehicle routing problem with proposed non-dominated sorting genetic algorithm and comparison with classical evolutionary algorithms. They discussed different classical evolutionary algorithms and proposed an algorithm NSGA-II (variant-I). In this, they utilized Random Position Crossover Method (RPCM) and Random Incremented Mutation Technique (RIMT), which are problem-specific operators to generate feasible route schedules. At the end, they claimed that their method gives better result as compared to classical algorithms. Chand and Mohanty (2013) considered a multi-objective vehicle routing problem using dominant rank method with the objectives of minimizing the number of vehicles and total cost (distance). They used Sub Route Mapped Crossover Method (SMCM) and Sub Route Exchange Mutation Method (SEMM) as genetic operators. They applied dominant rank method to get Pareto optimal set and they claimed that their method finds optimum solutions effectively.

Puljic and Manger (2013) worked upon comparison of eight evolutionary crossover operators for the vehicle routing problem. The eight evolutionary crossover operators are order crossover, partially mapped crossover, edge recombination crossover, cycle crossover, alternating edges crossover, heuristic greedy crossovers, random crossover and probabilistic crossover. They restricted themselves to investigate relative strengths and weaknesses of various crossover operators and claimed that the results obtained will help in construction of sophisticated algorithms in future. Zhou et al. (2013) considered a multi-objective vehicle routing problem with route balance based on genetic algorithm by simultaneously considering total distance and distance balance of active vehicle fleet as their objectives. They considered tournament selection, one-point crossover, and migrating mutation operators as genetic operators to solve the problem. They considered Solomon's benchmark problems, and concluded that their results are improved in all classes of problems considering the total distance and distance balance and claimed that their suggested approach is sufficient and the average GA performance is good according to the experimental results.

In this paper, an effort has been made to analyze few crossover operators related to VRPs. Some of the crossover operators are explained in brief and rest are listed later. The rest of the paper is organized as follows. In section 2, a brief introduction of variants of vehicle routing problem is given. The section 3 presents the importance of genetic algorithm in solving VRPs along with the steps used in genetic algorithm. The section 4 deals with different types of crossover operators used in solving VRPs and in section 5, new crossover operator is proposed with an illustration. Conclusions and directions for future studies are discussed in section 6.

2. VARIANTS OF VEHICLE ROUTING PROBLEM

Vehicle routing problem is not a new concept in operations management, but due to the advancement in information technology and developing infrastructure more variations have risen in this problem in order to satisfy growing demand across the world. There are several types of VRPs which can be categorized based on their nature and complexity. For the sake of understanding, VRPs' few variants are defined here.

The general VRP was first introduced by Danzig and Ramser [4] in 1959 by the name truck dispatching problem which is a generalization of the Travelling Salesman Problem (TSP), which is concerned with the determination of the shortest route which passes through each of n given points once with the assumption that each pair of points are linked with each other.

Capacitated vehicle routing problem (CVRP) [13] is the basic version of the VRP, which attempts to find number of routes for m number of vehicles with Q units homogenous capacity to minimize total transportation cost of routes while satisfying the delivery demands of n number of customer nodes. Each route originates and terminates at the depot and every single customer will be visited exactly once by specified vehicle.

In Vehicle Routing Problem with Simultaneous pick-up and delivery (VRPSPD) [13], a number of routes are found for v number of vehicles with C units' homogenous capacity, which minimize the total transportation cost of all the routes while satisfying the pick-up and delivery demands of the m number of customer nodes simultaneously.

The Vehicle Routing Problem with Time Windows (VRPTW) [9] is an extension of the VRP wherein a time window is associated with each customer in which each customer provides a time frame within which a particular service or task must be completed, such as loading or unloading of goods by considering vehicle capacity constraints.

There are many variants listed in the literature such as vehicle routing problem with backhauls, vehicle routing with split-loads, multi-depot vehicle routing problem, vehicle routing problem with pickup and delivery, etc. Readers are further directed to refer literature for detailed study of variants of VRPs.

3. IMPORTANCE OF GENETIC ALGORITHM IN SOLVING VRPS

Vehicle routing problem and its variants belong to family of NP-hard problems. Solving such problems will take enormous amount of computation time while using exact methods. Due to this complex nature of the problem, solutions using heuristics will be helpful in order to address larger solution space in less computational time. Genetic Algorithm is one of the efficient metaheuristics applicable to a large set of

combinatorial problems. It is one of the widely used and powerful optimization methods based on the process of natural selection and evolution. Crossover operator and mutation operators are used for transformation to form new individuals. During encoding integer representations of chromosomes are used to in genetic algorithm to solve VRP. The process of genetic algorithm is show in Fig.1.

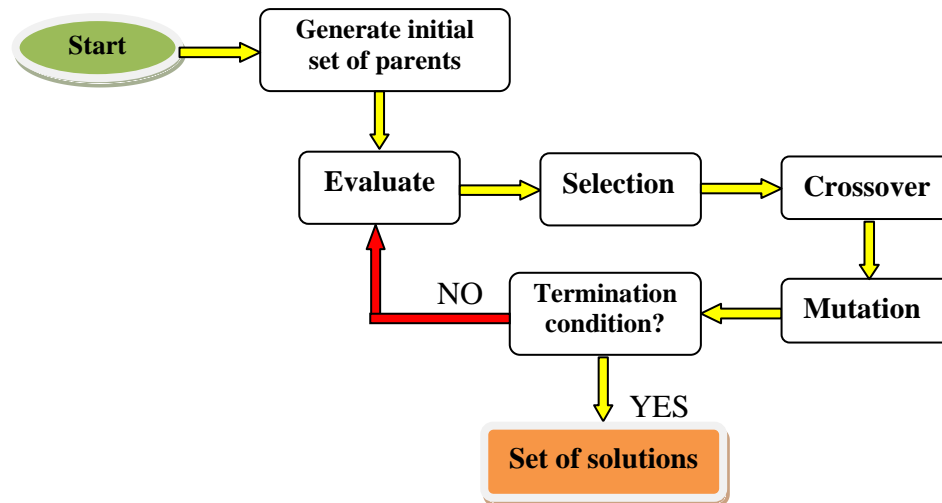


Fig.1 Process of genetic algorithm

4. TYPES OF CROSSOVER OPERATORS USED FOR VRPS

Crossover is one of the prominent operators used in genetic algorithms. Crossover process is vital in generating new chromosomes by combing two or more parent chromosomes with the hope that they create new and efficient chromosomes. Crossover occurs after selection of pairs of parent chromosomes and helps in exchanging information between parents to create children. During crossover the parent chromosomes are taken in pair and their genes are exchanged in certain order to obtain off spring. These offspring become next generation parent chromosomes [10] [11]. It is performed by exchanging alleles between two selected parent chromosomes in order to explore new solution space. In this paper, an attempt is made to study different crossover operators used to solve variants of VRPs. Various crossover operators used in literature are listed and explained below. The list is not inclusive of all the type of crossovers available in the literature covers the prominent crossover types. The objective of this study is to understand prominent crossover operators and to propose a new one.

1. One - Point Crossover
2. Two - Point Crossover

3. Order Crossover (OX)
4. Partially Mapped Crossover (PMX)
5. Cycle Crossover (CX)

4.1 One – Point Crossover:

Probably one of the simplest crossover operators and elementary crossover operators is one-point crossover. In this method, a cut point is selected randomly. Then an offspring for each of the parent chromosomes is constructed by following a sequence of steps as given below. For the purpose of understanding, two parent chromosomes are shown in Fig 2. The randomly selected cut point is 5 which is in between the genes 6 and 8 of the parent 1 as well as between genes 3 and 2 of the parent 2 as shown in Fig 2. The sequence of nodes of a chromosome/offspring after suffixing the first node at the end represents a route for the vehicle.

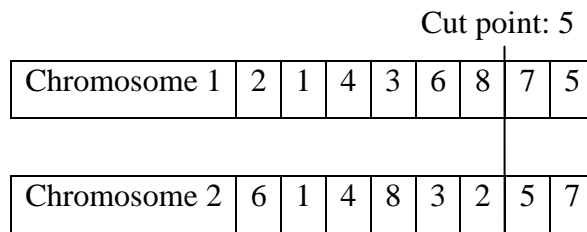


Fig.2 Parent chromosomes with cut pint at the gene position 5

Each cut point is having equal chance of occurrence. The new offspring will inherit genes from first parent till cut point and inherit non repeating genes from the beginning of second parent [11] [14].

Steps for One-Point Crossover:

- i) Select a cut point randomly between any two genes (For convenience purpose cut point is mentioned as an empty rectangular box throughout this paper) as shown in Fig.2a. Hence, two substrings are generated before and after cut point in each of the parent chromosomes.
- ii) Copy first substring from Parent 1 and insert as it is in the Offspring 1.
- iii) Then, copy one by one gene from the Parent 2 and insert them in the Offspring 1 by omitting the repeated values to avoid duplicity.
- iv) The roles of parents are interchanged in order to get offspring 2.

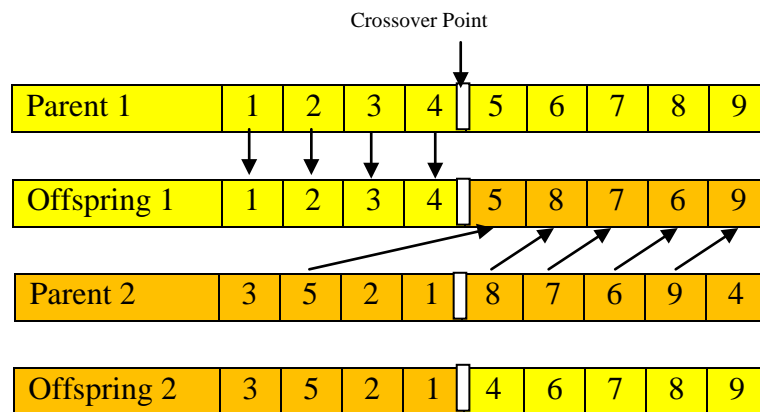


Fig. 2a Diagram representation of One-Point Crossover

4.2 Two - Point Crossover

Two – point crossover is similar to that of one point crossover. Here, instead of one cut point, two cut points are randomly selected at the same positions in two parents and two offspring are generated from them.

Steps for Two-Point Crossover

- i) Select two cut points randomly between genes of each parent.
- ii) Select the substring between the two cut points from Parent 2 and copy it to the Offspring 1 as shown in Fig.3a.
- iii) Remaining values are copied from first parent and placed as it is in Offspring 1 omitting repeated values. (In the Fig.3a genes 1 and 8 have already been placed in the middle of the Offspring 1. Therefore they are omitted by being copied to it.)

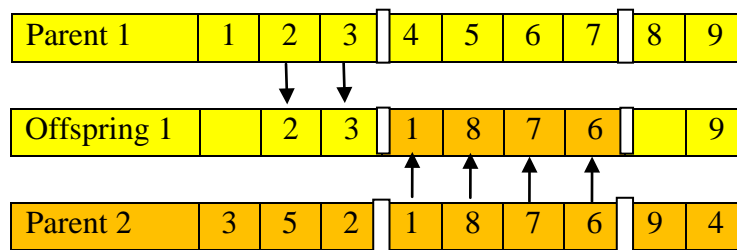


Fig.3a Diagram representation of Two-Point Crossover

- iv) Empty spaces in the Offspring 1 are filled with genes which are not appeared in first parent but in the order in which they appear in second parent after second cut point of it as shown in Fig.3b (Since, gene 4

appears after second cut point of the Parent 2, it is placed first in the Offspring 1 followed by gene 5.)

- v) Similarly, Offspring 2 can be generated by interchanging the roles of parents as shown, which is represented as Offspring 2 in Fig.3b.

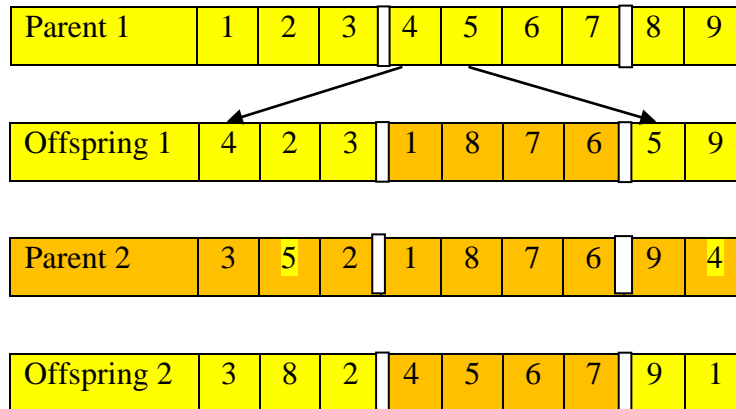


Fig.3b Diagram representation of Two-Point Crossover

4.3 Order Crossover (OX)

Order crossover is similar to that of Two Point crossover, but in the case of Order Crossover, a part from first parent is copied to offspring chromosome and the remaining values are placed in the offspring by the order by which they appear in second parent [12].

Steps for Order Crossover

- i) Select two cut points randomly between the genes of each parent.
- ii) The consecutive alleles between two cut points (5 4 6 7) from Parent 1 are copied to the Offspring 1 as shown in Fig.4a.
- iii) Then, the rest of the genes are copied in the order in which they appear in the Parent 2 starting after the second cut point (exclude the value if it is already inserted) till all positions are filled.
- iv) In order to get the Offspring 2, follow the same procedure by exchanging the positions of the two parents Parent 1 and Parent 2 as shown in Fig.4b.

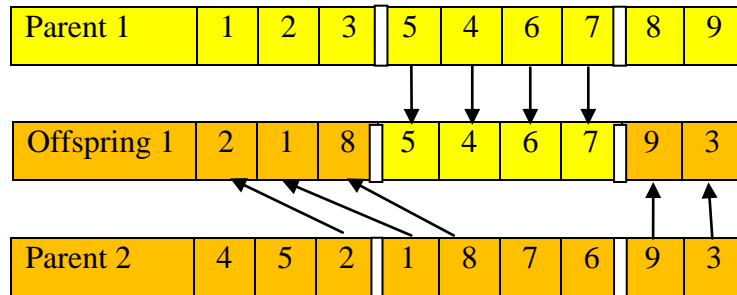


Fig.4a Diagram representation of Order Crossover to generate first offspring

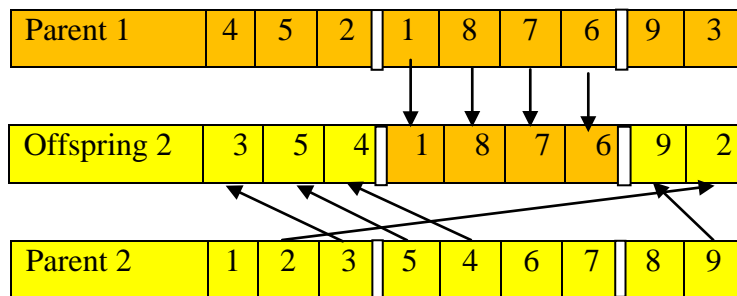


Fig.4b Diagram representation of Order Crossover to generate second offspring

4.4 Partially Mapped Crossover (PMX)

PMX is also known as partially matched crossover and it is similar to that of two – point crossover. In this method, two crossover points are chosen among two parents and the genes between two crossover points are exchanged, while remaining genes are filled by partial mapping [12].

Steps for Partially Mapped Crossover

- i) Select two cut points randomly, excluding the first gene and the last gene in each of the chromosomes.
- ii) Exchange the genes between crossover points to create two new offspring as shown in Fig.5a.
- iii) Next, define a matching section between two cut points of the two offspring and the alleles are mapped (matched) i.e., $5 \leftrightarrow 1$; $4 \leftrightarrow 8$; $6 \leftrightarrow 7$; $7 \leftrightarrow 6$.
- iv) Copy the genes from the Parent 1 to Offspring 1 in the same position, where they appear in the Parent 1 omitting repeated values as shown in Fig.5b.

- v) Then, rest of the values are inserted using partial mapping i.e., 4 is replaced with 8, 5 is replaced with 1 as shown in Fig.5b based on mapping explained in step iii.
- vi) Carry out an analogous procedure to create a new Offspring 2.

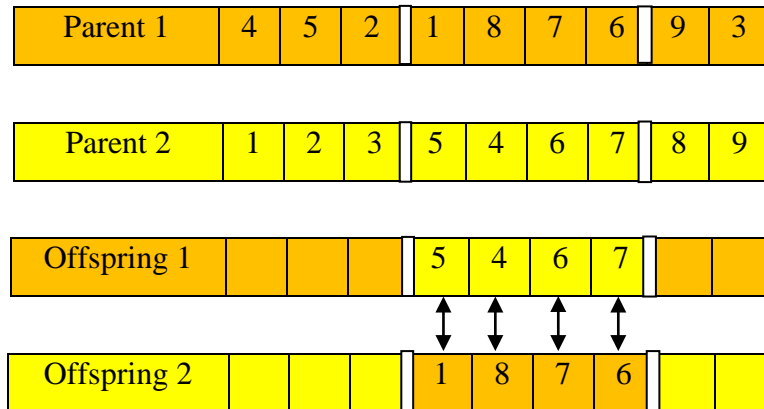


Fig.5a Diagrammatic representation of preliminary stage of PMX.

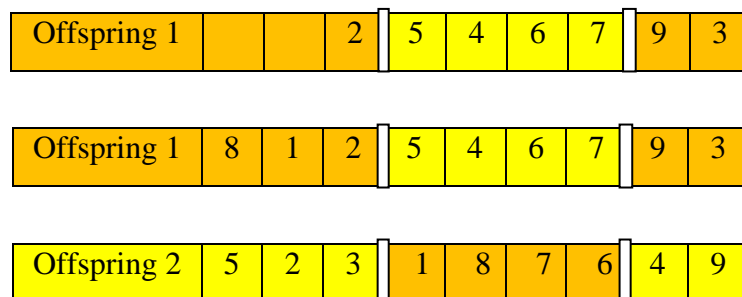


Fig.5b Diagrammatic representation of completed offspring using PMX.

4.5 Cycle Crossover (CX):

In cycle crossover, a gene from one parent will be copied into an offspring, but it should inherit the position of the other parent [12].

Steps for Cycle Crossover

- Start a cycle from first gene of first parent to first gene of second parent as shown in Fig 6a.
- Identify the gene in the first position of the second parent and move to the corresponding gene in the first parent.

- iii) Vertically move from the current gene of the first parent to the gene in the second parent.
- iv) Check whether the gene in the second parent is same as the first gene of the first parent.
If Yes, go to step vi; If not, go to step v.
- v) Move to the gene in the first parent corresponding to the current gene in the second parent and go to step iii.
- vi) Repeat similar steps to obtain the second offspring
- vii) Copy the genes present in the cycle of the first parent to the corresponding positions of the first offspring as shown in Fig 6b.
- viii) Copy the genes present in the cycle of the second parent to the corresponding positions of the second offspring as shown in Fig 6b.
- ix) Copy the remaining genes of the second parent to their corresponding positions of the first offspring as shown in Fig 6c.
- x) Copy the remaining genes of the first parent to their corresponding positions of the second offspring as shown in Fig 6c.
- xi) The current sequence of genes in each of the offspring forms the final corresponding offspring.

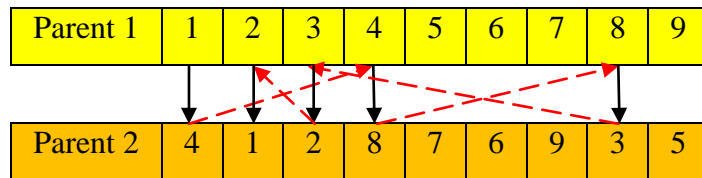


Figure 6a: Diagrammatic representation of formation of cycle.

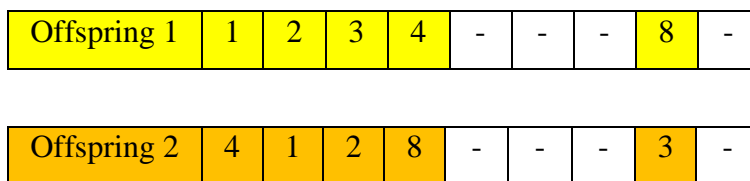


Figure 6b: Diagrammatic representation of formation initial stage of cycle crossover.

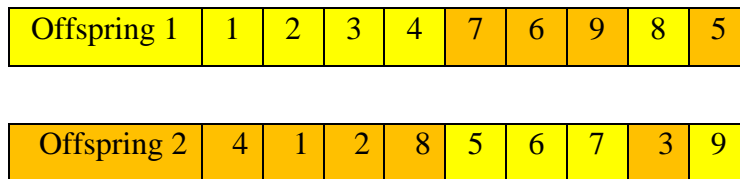


Figure 6c: Diagrammatic representation of completion of offsprings by cycle crossover.

5. PROPOSAL OF A NEW CROSSOVER OPERATOR

In this paper, a new crossover operator is introduced, namely Sinusoidal Motion Crossover. The name is analogous with the sinusoidal motion of waves in relative manner. Using this operator one can generate two offspring simultaneously. Advantage of this method is it provides more randomness in the chromosomes generated. Consider two parents P_1 and P_2 which have 10 vertices indicating 10 cities. Steps for the proposed method are listed below.

Steps for Sinusoidal Motion Crossover:

- i) Select the first gene from the first parent to start with to initialize the crossover.
- ii) Treat the selected gene as the first gene of the first parent (In this instance, first gene from P_1 (6) is considered. Therefore, 6 will be inserted in the first gene position of the offspring 1).
- iii) Then, move to the gene position of the second parent corresponding to the current gene in the first parent (gene value 2).
- iv) Is the current gene in the second parent a repeat gene in the offspring 1?
If no, go to Step v; otherwise, go to Step vi.
- v) Copy the gene at the current position of the second parent as the gene in the first available position from left of the offspring 1 and go to Step vii.
- vi) Copy the repeated gene of the second parent to the first available position from left of the offspring 2.
- vii) Is the current gene position of the second parent is the last gene position in it?
If yes, go to Step xii; otherwise go to Step viii.
- viii) Move to the gene position of the first parent corresponding to the current gene in the second parent.
- ix) Is the current gene in the first parent a repeat gene in the offspring 1?
If no, go to step x; otherwise, go to step xi.
- x) Copy the gene at the current position of the first parent as the gene in the first available position from left of the offspring 1 and go to Step iii.
- xi) Copy the repeated gene of the first parent to the first available position from left of the offspring 2 and go to Step iii.

xii) The current sequence of genes in each of the offspring forms the final corresponding offspring (offspring 1 and offspring 2).

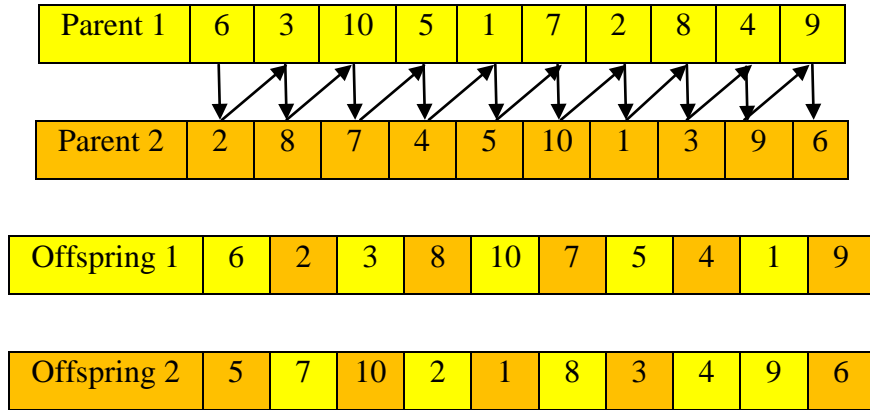


Figure 7: Diagrammatic representation of sinusoidal crossover

Illustrations:

The working of proposed crossover is demonstrated by means of illustrations by taking the data provided in literature [7]. This crossover method is applied to Vehicle Routing Problem with Simultaneous Delivery and Pickups, which is one the variants of VRP. The data is utilized to calculate the total distance travelled by a set of vehicles from Table 1 and Table 2.

Table 1 Distance Matrix

Dij	D	1	2	3	4	5	6	7	8
D	0	40	60	75	90	200	100	160	80
1	40	0	65	40	100	50	75	110	100
2	60	65	0	75	100	100	75	75	75
3	75	40	75	0	100	50	90	90	150
4	90	100	100	100	0	100	75	75	100
5	200	50	100	50	100	0	70	90	75
6	100	75	75	90	75	70	0	70	100
7	160	110	75	90	75	90	70	0	100
8	80	100	75	150	100	75	100	100	0

Table 2 Data Related to Delivery Demand and Pickup Demand

Customer i	1	2	3	4	5	6	7	8
Delivery Demands	2	1.5	4.5	3	1.5	4	2.5	3
Pick-up Demands	3	1	2	2	3	4	1.5	3

Illustration 1

The offspring from the parent chromosomes using sinusoidal crossover method are shown in Fig.8.

Consider first offspring as shown in Fig 8. The construction of the route should be such that it satisfies the delivery and pickup demands mentioned in Table 2. Total capacity of each vehicle is assumed to be 10 tons and the vehicle will leave the depot with 80% of the capacity of the delivery goods. For the offspring 1, the routes are constructed starting from its left such that the utilized capacity is not exceeding the maximum capacity of the vehicle.

The number of vehicles after satisfying the delivery and pickup demand is found out to be three for this instance as shown in Table 3. The distance travelled by each vehicle is calculated using the data available in Table 1.

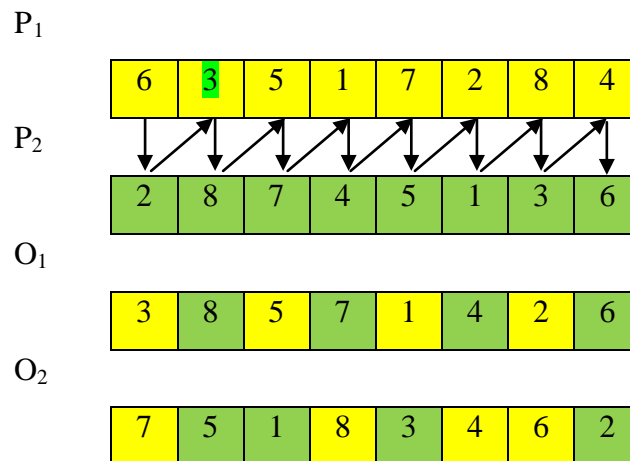
**Fig 8:** Diagrammatic representation of Illustration 1 for sinusoidal crossover

Table 3 Construction of Routes

Sl. NO	Routes	Distance
1	0 – 3 – 8 – 0	75+150+80 = 305
2	0 – 5 – 7 – 1 – 0	200+90+110+40 = 440
3	0 – 4 – 2 – 6 – 0	90+100+75+100 = 365

Total distance for three vehicles = 1110.

Illustration 2

Similar procedure is carried out analogues to illustration 1 and the number of vehicles and total distance travelled are demonstrated below. The crossover is demonstrated in Fig.9. The construction of routes is shown in Table 4.

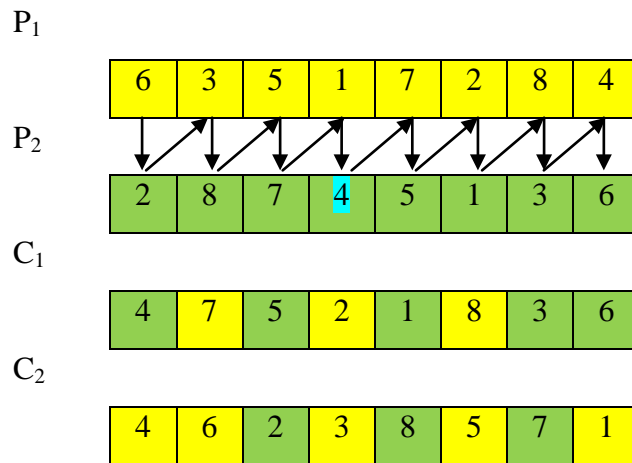


Fig 9: Diagrammatic representation of Illustration 2 for sinusoidal crossover

Table 4 Construction of Routes

Sl. NO	Routes	Distance
1	0-4-7-5-0	90+75+90+200=465
2	0-2-1-8-0	60+65+100+80=305
3	0-3-6-0	75+90+100=265

Total distance for three vehicles = 1035

6. CONCLUSION

The VRP and its variants fall into combinatorial and NP-hard category. So, the usage of heuristics like GA is inevitable. Crossover is the main operator in deciding about the effective implementation of GA to VRPs. In this paper, an attempt has been made to study various prominent crossover operators available in the literature and a new crossover operator, viz. Sinusoidal Motion Crossover (SMC) has been introduced. The proposed method is illustrated by the help of data available in the literature on VRPSDP problems. There were many crossover operators available in the literature, but all of them may not contribute the problem under study. Therefore, this article is limited to study few crossover operators.

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