

Design of M-Sequence using LFSR & Analyse Its Performance as a Chip Code in CDMA

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Abstract

In this project m sequences by linear feedback shift register was generated in matlab. The circuit was made in the workspace window and generated the m sequence for the primitive polynomials of degree 3 to 7. After generating the m sequence of the various degrees, auto correlation and cross correlation was performed on the m sequence generated and was calculated the variance and bit error rate of the m sequences.

Keyword: Spread Spectrum CDMA, Orthogonal Property, Chip Code, BER, MATLAB

INTRODUCTION

Telecommunication occurs when the exchange of information between two entities (communication) includes the use of technology.

Communication technology uses channels to transmit information (as electrical signals), either over a physical medium (such as signal cables), or in the form of electromagnetic waves. The word is often used in its plural form, **telecommunications**, because it involves many different technologies.

1.1 CDMA(CODE DIVISION MULTIPLE ACCESS):

Code division multiple access (CDMA) is a channel access method used by various radio communication technologies.

CDMA is a form of multiplexing, which allows numerous signals to occupy a single transmission channel, optimizing the use of available bandwidth. The technology is used in ultra-high-frequency (UHF) cellular telephone systems in the 800-MHz and 1.9-GHz bands.

One of the basic concepts in data communication is the idea of allowing several transmitters to send information simultaneously over a single communication channel. This allows several users to share a bandwidth of frequencies. This concept is called multiplexing. CDMA employs spread-spectrum technology and a special coding scheme (where each transmitter is assigned a code) to allow multiple users to be multiplexed over the same physical channel.

1.2 SPREAD SPECTRUM:

The widest application at this time is its use in military communications systems where spread spectrum serves two functions.

1. The first is that it allows a transmitter to transmit a message to a receiver without the message being detected by a receiver for which it is not intended i.e., the transmission is transparent to an unfriendly receiver. To achieve this transparency the spread spectrum modulation decreases the transmitted power spectral density so that it lies well below the thermal noise level of any unfriendly receiver.
2. The second major application of spread spectrum is found, when, as a matter of fact, it turns out not to be possible to conceal the transmission.

1.3 DIRECT SEQUENCE SPREAD SPECTRUM:

A Direct Sequence (DS) spread spectrum signal is one in which the amplitude of an already modulated signal is amplitude modulated by a very high rate NRZ binary stream of digits. Thus, if the original signal is $s(t)$, where >

$$S(t) = \sqrt{2 P_s} d(t) \cos \omega_0 t \quad \dots \dots \dots <1>$$

(a binary PSK signal), the DS spread spectrum signal is >

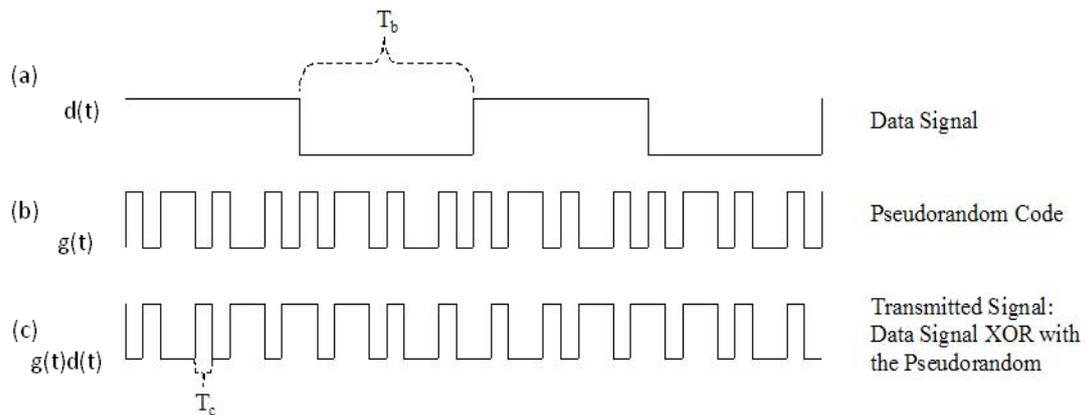
$$V(t) = g(t) s(t) = \sqrt{2 P_s} g(t) d(t) \cos \omega_0 t \quad \dots \dots \dots <2>$$

Where, $g(t)$ is a pseudo-random noise (PN) binary sequence having the values ± 1 .

Here we merely assume that $g(t)$ is a binary sequence as is the data $d(t)$. Th`e

sequence $g(t)$ is generated in a deterministic manner and is repetitive. However, the sequence length before repetition is usually extremely long and to all intents and purposes, and without serious error, we can assume that the sequence is truly random, i.e., there is no correlation at all between the value of a particular bit and the value of any other bits. Furthermore, the bit rate f_c of $g(t)$ is usually much greater than the bit rate f_b of $d(t)$. As a matter of fact the rate of $g(t)$ is usually so much greater than f_b , we say that $g(t)$ “chops the bits of data into chips”, and we call the rate of $g(t)$ the chip rate f_c , retaining the words, bit rate, to represent f_b . [3,4,5]

To see that multiplying the BPSK sequence $s(t)$ by $g(t)$ spreads the spectrum we refer to figure <2> which shows a data sequence $d(t)$., a pseudo-random (often called a pseudo-noise or PN) sequence $g(t)$ and the product sequence $g(t) d(t)$. Note that (as is standard practice) the edges of $g(t)$ and $d(t)$ are aligned, that is, each transition in $d(t)$ coincides with a transition on $g(t)$ [1,4,6].

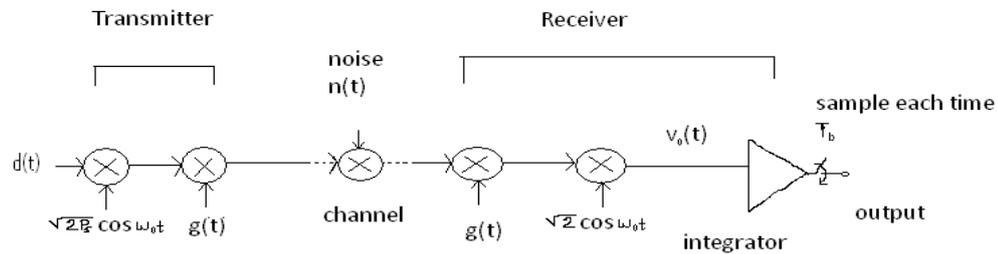


(a) The waveform of the data but stream $d(t)$. (b) The chipping waveform $g(t)$. (c) The waveform of the product $g(t)d(t)$.

Figure <2>

The product sequence is seen to be similar to $g(t)$, indeed if $g(t)$ were truly random, the product sequence would be another random sequence $g'(t)$ having the same chip rate f_c as $g(t)$. Since the bandwidth of the BPSK signal $s(t)$ is nominally $2 f_b$ the bandwidth of the BPSK spread spectrum signal $v(t)$ is $2 f_c$ and the spectrum has been spread by the ratio f_c / f_b . Since the power transmitted by $s(t)$ and $v(t)$ is the same, i.e., P_s , the power spectral density $G_S(f)$ is reduced by the factor f_b / f_c [3,7,8].

To recover the DS spread spectrum signal, the receiver shown in figure <3> first multiplies the incoming signal with the waveform $g(t)$ and then by the carrier $\sqrt{2} \cos \omega_0 t$. The resulting waveform is then integrated for the bit duration and the output of the integrator is sampled, yielding the data $d(k T_b)$. We note that at the receiver it is necessary to regenerate both the sinusoidal carrier of frequency ω_0 and also to regenerate the PN waveform $g(t)$ [11,12].



A BPSK communication system incorporating a spread spectrum technique

Figure<3>

1.4 The probability of a bit being in error (BER):

The probability of a bit being in error (BER) is found using equation, by letting $P_J = (k-1) P_s$:

$$P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\{2 (1 / k-1) (f_c / f_b)\}}$$

Thus in a slotted Code Division Multiple Access (CDMA) system, to ensure a low probability of error, the gain, f_c / f_b , must be adjusted so that $>$

$$f_c / f_b \gg (k-1) / 2$$

In Code Division Multiple Access (CDMA) we assumed that each transmitted signal presented the same power P_s to the receiver[1,3,5].

When an unwanted user's received power is much larger than the received power presented by the desired user, errors can occur. This problem is referred to in the literature as the 'near-far problem', and limits the utility of Direct Sequence (DS) systems to applications where each user's received power is approximated the same.

1.5 CHIPCODE :

In digital communications, a chip is a pulse of a direct-sequence spread spectrum (DSSS) code, such as a pseudo-noise code sequence used in direct-sequence code division multiple access (CDMA) channel access techniques.

In a binary direct-sequence system, each chip is typically a rectangular pulse of +1 or -1 amplitude, which is multiplied by a data sequence (similarly +1 or -1 representing the message bits) and by a carrier waveform to make the transmitted signal. The chips are therefore just the bit sequence out of the code generator; they are called chips to avoid confusing them with message bits[2,3,5].

Spreading sequences are chosen based on their characteristics like autocorrelation, cross correlation properties, etc.

Some of the spreading sequences are listed below

- 1) MAXIMUM LENGTH PSEUDO NOISE (PN) SEQUENCE
- 2) GOLD SEQUENCES
- 3) KASAMI SEQUENCES
- 4) WALSH HADAMARD SEQUENCES

In Code Division Multiple Access (CDMA) system, from equation (a) we get that ,
Each receiver is presented with the same input waveform,

$$v(t) = \sum_{i=1}^k \sqrt{2P_s} g_i(t) d_i(t) \cos (\omega_0 t + \theta_i)$$

Where each signal has the same power P_s to the receiver. Each pseudo-random sequence $g_i(t)$ has the same chip rate f_c and $d_i(t)$ is data transmitted by user i . The data rate for each user is the same, f_b . θ_i is a random phase, statistically independent of the phase of each of the other users. Here, $f_c \gg f_b$.

Here if we use the exact pn-sequence to the receiver then we can get the original sequence $x(t)$ from the sequence $u(t)$.

If the receiver is required to receive each of k users it needs k correlators. For k users there are k receivers which have k *pn-sequences[1,3,5,7].

If the user's pn-sequence and the receiver 1's pn-sequence are not same then the signal become v_{01} and is defined by equation <b & c> =>

$$v_{01} = \sum_{i=1}^k \sqrt{P_s} g_1(t) g_i(t) d_i(t) \cos (\theta_i - \theta_1)$$

$$= \sqrt{P_s} d_1(t) + \sum_{i=2}^k \sqrt{P_s} g_1(t) g_i(t) d_i(t) \cos (\theta_i - \theta_1)$$

Here the product, $P_s g_1(t) g_i(t) d_i(t) \cos (\theta_i - \theta_1)$ is called the MAI(Multiple Access Interference).

If in 1st user we use pn-sequence g_1 but in the 1st receiver we use pn-sequence g_i where $i \neq 1$. Then in 1st receiver we get unnecessary interference (MAI) like =>

$$g_1(t) g_2(t) d_2(t) + g_1(t) g_3(t) d_3(t) + g_1(t) g_4(t) d_4(t) + \dots\dots\dots$$

Here, $g_1(t) g_2(t)$ = cross correlation between $g_1(t)$ and $g_2(t)$.

$g_1(t) g_3(t)$ = cross correlation between $g_1(t)$ and $g_3(t)$. etc.

$g_1^2(t)$ = auto correlation of $g_1(t)$. etc.

The effect of interference can be reduced if

<1>. $g_1^2(t) = 1$ and

<2>. $g_1(t) g_2(t)$ = cross correlation between $g_1(t)$ and $g_2(t)$ = very small.

So the chip codes of Code Division Multiple Access (CDMA) system has to acquire some properties[2,7,8,9].

1.6 M-SEQUENCE:

The Maximal-Length Sequence is a type of Pseudo-Noise Sequence. A Pseudo-Noise Sequence is a Periodic binary sequence with a noise like wave-form. The mathematical base of m-sequences is primarily dependent on the primitive polynomial. To get a m-sequence, first we have to know the primitive polynomial from which the m-sequence is generated.

1.7 Linear Feedback Shift Register (LFSR):

m-sequence is usually generated by Feedback shift register. A Feedback shift register is consists of an ordinary shift register and a logic circuit. The ordinary shift register is made up of m Flip-Flops (two-state memory stages). The logic circuit is interconnected to form a multi loop feedback circuit. When the feedback logic is nothing but Modulo-2 adders, then Feedback shift register is called the linear feedback shift register. The Flip-Flops in the shift register are controlled by a single timing clock. At each pulse of the clock the state of each Flip-Flop is shifted to the next one down the line. During this function the Logic circuit computes the Boolean function (XOR) of the states of the Flip-Flops. The result is then taken as the input of the first Flip-Flop[2,3,4].

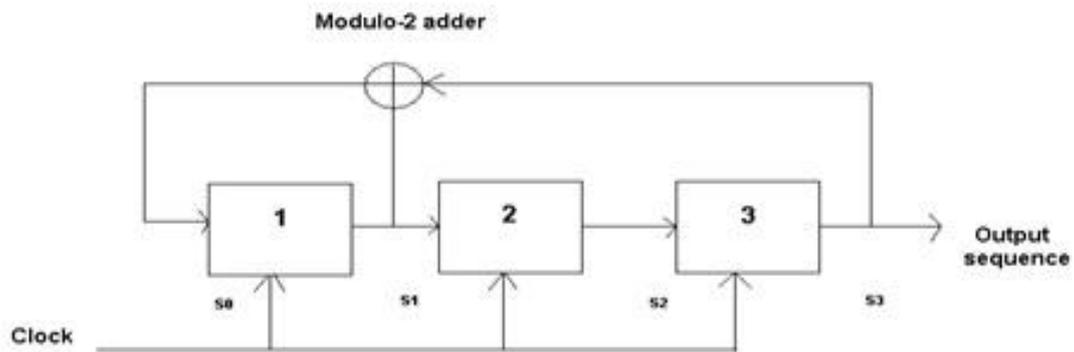
Let, S_j be the state of j^{th} Flip-Flop after the k^{th} clock pulse. Then $S_j(K+1) = S_{j-1}(K)$
 $[K \geq 0, 1 \leq j \leq m]$

[m = the number of Flip-Flops in the shift register]

With a total number of m Flip-Flops the number of possible states of the shift register is 2^m . So the shift register have a period of at most 2^m .

For Linear Feedback shift register 0 state is not present. So its period is at most $2^m - 1$. When its period is exactly $2^m - 1$, then the PN sequence is called a Maximal-Length Sequence[1,3].

Example: Let a primitive polynomial of degree 3 is x^3+x+1 . Then in the Linear Feedback Shift Register (LFSR), there must be 3 normal registers and the tapping would depend on the primitive polynomial's value.



[MLS sequence generator for m=3]

* The above figure is of the Linear Feedback Shift Register(LFSR) from primitive polynomial(x^3+x+1) of degree 3*

First we have to store values in the registers. The value must be at least one non zero (1) element in at least one register. If all the registers contains zero then the output of the Linear Feedback Shift Register (LFSR) also contains zero. This is not permit able because for every clock pulse the output of the Linear Feedback Shift Register (LFSR) will always remain zero[2,4,5].

Let, we store 100 to the Linear Feedback Shift Register (LFSR) then :->

REGISTER CONTENT	STAGE	OUTPUT
100	Initial	0
110	1 st	0
111	2 nd	1
011	3 rd	1
101	4 th	1
010	5 th	0
001	6 th	1
100	7 th	X

Here the m-sequence is the output element of the Linear Feedback Shift Register (LFSR). So, the m-sequence is 0011101 of degree 3.

1.8 Properties of maximal length sequence:

1.8.1 BALANCE PROPERTY : The Number Of “1”s in the sequence is one more than the Number of “0”s.

1.8.2 RUN PROPERTY: All the “Runs” in the sequence of each type (0,1)

One half of the Runs are of Length 1

One quarter of the Runs is of Length 2

One eighth the Runs are of Length 3

And so on as the Fraction represents meaningful number of the Runs.

1.8.3 CORRELATION PROPERTY: The Autocorrelation function of a Maximal-Length Sequence is periodic and binary valued.

If a Linear Time Invariant system’s impulse response is to be measured with MLS, the response can be extracted from the measured system output $y[n]$ by taking its circular Cross-Correlation with the MLS sequence. The Autocorrelation of MLS is 1 for zero lag, and nearly zero ($-1/N$ where N is the sequence length) for all other lags. So the Autocorrelation of MLS approach as the unit impulse functions as MLS sequence increases[7,11,12].

If the impulse response of a system is $h[n]$ and MLS is $s[n]$, then

$$Y[n] = (h * s)[n].$$

Taking the Cross –Correlation with respect to $s[n]$ of both sides,

$$\emptyset_{sy} = h[n] * \emptyset_{ss}$$

and assuming that \emptyset_{ss} is an impulse(valid for long sequences)

$$h[n] = \emptyset_{sy}.$$

Among this properties somewhat is similar to the property of the truly random binary sequence. In a random binary sequence the probability of having symbol 0 or 1 is equally probable.

PROCEDURE(PROJECT DETAILS):

We are studying with m-sequence up to degree 7. We found out each and every different primitive polynomial up to degree 7 and generated different m-sequences from different primitive polynomials. Then calculated their auto correlation results. And further calculated the cross correlation results of different m-sequences

for the same degree. From these results we can verify that whether the higher order m-sequences have orthogonal properties or not.

RESULTS

In order to find the suitability of the m-sequences in Code Division Multiple Access (CDMA) application we have to look into detail into their correlation property.

By working with m-sequences up to degree 7, I point out some conclusion. They are shown below:

1. The auto correlation between any two same m-sequences of same degree is 1 for $\tau = 0$, and $1/7$ for other τ 's.
2. For m-sequence of degree greater than 4 (because for degree 3 & 4 there are only 2 m-sequences for each degree) there are two kind of cross correlation result between any two different m-sequences of same degree. The first kind of cross correlation result contains only three values. The values are -1, -t, t-2.

Where, $t = 2^{m+1/2} + 1$ for odd m (m = degree of the m-sequence)

$t = 2^{m+2/2} + 1$ for even m (m = degree of the m-sequence)

The other kind of cross correlation result varies widely. For m-sequences of degree 3 and 4, the cross correlation result contains these three values only.

I plot the cross correlation and auto correlation result of some particular m-sequences in a graph by mat lab programming to have a final conclusion.

The three primitive polynomials are $x^5 + x^2 + 1$, $x^5 + x^3 + 1$, & $x^5 + x^4 + x^3 + x^2 + 1$ of degree 5. The m-sequences from this primitive polynomials are shown in the excel sheet of degree 5. Here I represent the m-sequence from $x^5 + x^2 + 1$ as g_1 , the m-sequence from $x^5 + x^3 + 1$ as g_2 , and the m-sequence from $x^5 + x^4 + x^3 + x^2 + 1$ as g_3 . I am plotting the auto correlation of g_1 against g_1 in graph 1, the cross correlation between g_1 and g_2 in graph 2, and cross correlation between g_1 and g_3 in graph 3. The graph 3, contains only three values (-1, -t, t-2). So this graph contains good cross correlation properties of m-sequences as compared to graph 2. The graph 2, contains many values as they are varies widely.

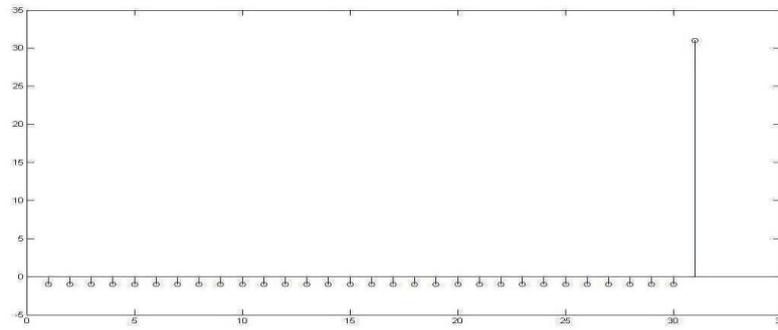


Figure-1(Auto correlation of degree 5 m sequence)

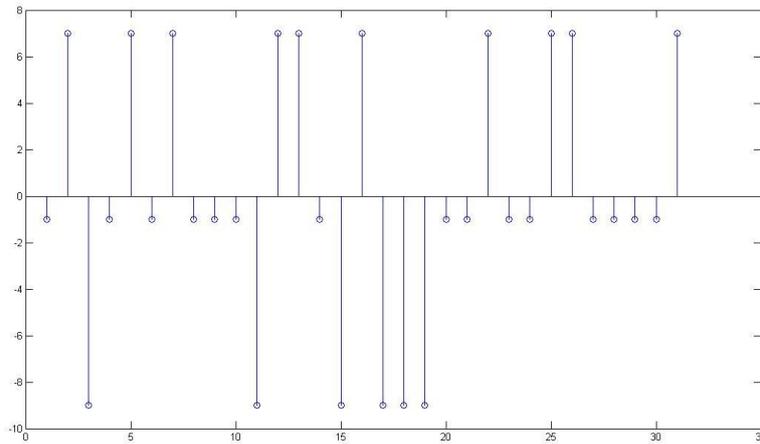


Figure-2(Good cross correlation of degree 5 m sequence)

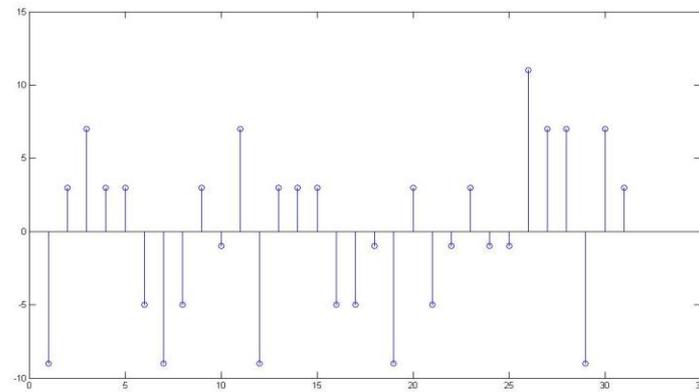


Figure-3(Bad cross correlation of degree 5 m sequence)

CONCLUSION:

1. The cross correlation results of m- sequences are nowhere near zero.
2. There is no symptom of decreasing cross correlation with the increase of higher order.

CONCLUSION

M-sequence as a chip code: If we use m-sequence as a chip code then for every m-sequence there should be one transmitter and receiver in Code Division Multiple Access (CDMA).

For example, there are two m-sequences in degree 3, so there would be just two transmitters and receivers in degree 3.

Let, 1st m-sequence of degree 3 says g1

& 2nd m-sequence of degree 3 says g2.

If transmitter 1 sends data d1, then the multiplied data v1 is ,

$$V1 = d1.g1$$

In CDMA both two receivers receive the signal. In receiver 1, v1 is also multiplied by g1 but in receiver 2, v1 is multiplied by g2.

In receiver 1, $v1.g1 = d1.(g1)^2 = d1$ it's the auto correlation of g1 to g1.

In receiver 2, $v1.g2 = d1.(g1.g2)$ it's the cross correlation of g1 to g2.

In CDMA, the receiver 1 contains d1 because $(g1)^2 = 1$. This is the main data d1.

But in receiver 2, contains the noise signal or the distorted signal because, $g1 \neq g2$.

The cross correlation result contains the noise signal.

To calculate the Bit Error Rate (BER), we have to calculate the variance (σ^2) in the cross correlation result.

The formulae for calculating variance (σ^2) are >

The cross correlation result of the two m-sequence of degree 3 is -5, 3, 3, -1, 3,-1, and -1.

Then the variance (σ^2) is calculated >

$$\sigma = \sqrt{60}$$

$$= 7.83673.$$

Then the Bit Error Rate (P_e) >

$$P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_s}{\sigma^2 (k-1)}} \quad [E_s = \text{the power of the signal} = (\text{the peak of the auto$$

correlation results)²

= 0 when, $k=1$ (here, $k-1=1-1=0$) [k = the number of users]

= 0.103255695 when, $k=2$ (here, $k-1=2-1=1$)

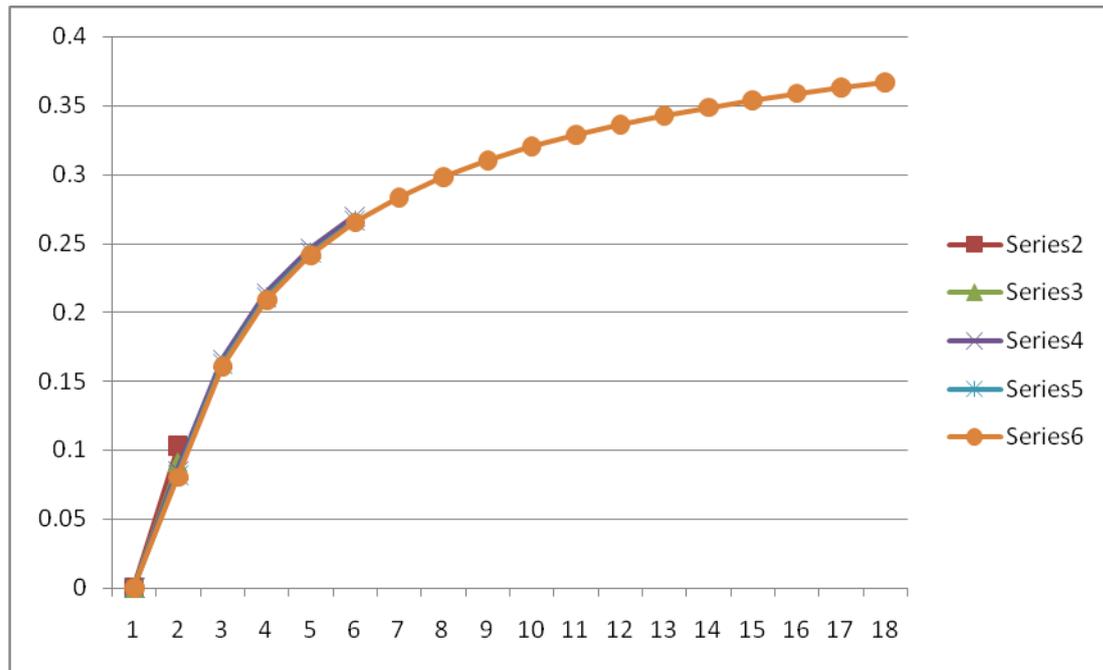


Figure-4 (BER comparison graph of degree 3 to degree 7 m sequences)

I am studying the BER for m-sequence up to 7th degree and placed that result in the excel sheet. I also plotted the result of different BER up to 7th degree into the graph. Finally I noticed that the BER up to 7th degree are much higher. So if we use m-sequence as a chip code we found that the amount of erroneous data is much higher because of the non orthogonality of the m-sequences.

If we use m-sequence as chip codes in CDMA, we see that instead of having good auto correlation properties, the cross correlation properties mismatch the basic requirement of chip code in CDMA because the variety of cross correlation results affects much higher BER in CDMA. A suitable chip code always reduce the BER in CDMA receiver.

So I came to a conclusion that **“m-sequences as a chip code in CDMA are not suitable at all.**

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