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# Magnetic Field Across a Multiferroic Film by an incident Plane Harmonic EM Wave

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**Abstract**

Multiferroics are attracting much attention due to their special properties, principally that of coexistence of ferroelectric and ferromagnetic order. Such materials therefore, have the potential of controlling magnetism in devices by application of electric fields. In this paper, a mutiferroic film in free space is considered whose thickness is of the order of a micron or more, so that a continuum theory based on Maxwell's equations is applicable. Assuming isotropy, the constitutive mathematical models for the electric polarization **P** and magnetization **M** are presented to study the electromagnetic field intensity **H** across a film of the material subjected to time harmonic plane electromagnetic (EM) wave of applied electric field intensity $E_0$. Using representative data for the material constants appearing in the formulation, the transmitted value of $|\mathbf{H}|$ is found to be six order of magnitude and higher in comparison to $E_0$.

**Keywords:** Multiferroics, Maxwell's equations, polarization, magnetization, hysteresis, electromagnetic (EM) waves.

## 1. INTRODUCTION

Multiferroics are certain metal oxide and pervoskite materials possessing the property of coexistence of spontaneous hysteretic ferromagnetic (sometimes antiferromagnetic) and ferroelectric order. This special property has the potential of obtaining efficient route to control of magnetism by electric fields resulting in energy efficiency, increase in power output and miniaturization of devices, overcoming the scaling limitation present in the centuries old mechanism of controlling magnetism by current running through wire. Besides the two basic properties, multiferoics may possess ferroelasticity and some other properties like ferrotoroidicity and piezoelectricity. Commonly cited examples of multiferroics are $Bi\,Fe\,O_3$, $Bi\,Mn\,O_3$, $Pb\,V\,O_3$ etc.; but the multiferroic properties can also be created in binary composites constituted of ferromagnetic and ferroelectric particles such as $Ba\,Ti\,O_3$ and $Co\,Fe_2\,O_4$, using a binding polymer.

The mechanisms for multiferroic properties of crystals are complex. Certain models based on lone-pair, geometric, charge ordering and spin driven effects have been proposed for the mutiferroic properties. Vast literature exists on the study of models of these types on molecular scales. Reviews containing large number of references published during the past years can readily be located in Wiki and other pages on multiferroics in the internet cloud. The review by Liu and Yang [6] is a new addition to the series of reviews.

With a view of application on a larger micro and above scale, a continuum theory of multiferroics exhibiting ferromagnetic, ferroeletric and magnetoelectric properties is presented in Bose [1]. This theory is based on adoption of the Maxwell's equations for EM field **E** and **H**, taking in to account possible elastic motion of the material (Panofsky and Phillips [7]) due to electromagnetic forces acting on the material. The multiferroics being dielectric, the electric polarization **P** and magnetization **M** are hysteretically generated at each point of the medium, depending on **E** and **H**. The polarization **P** in general, is constituted of a ferroelectric part $\mathbf{P}^f$ and a magnetoelectric part $\mathbf{P}^{me}$. The former can be phenomenologically modeled by the Landau-Devonshire theory (Jaynes [4], Bell and Cross [2], Coondoo [3], Lallart [5]), in which $\mathbf{P}^f$ is expressed as the gradient of the polarization energy density function $\mathcal{E}_\mathcal{P}$ expanded to sixth degree polynomial consisting of a linear combination of groups of symmetric functions of the components of $\mathbf{P}^f$. This results in iterative equations for the components of $\mathbf{P}^f$ in terms of the components of **E** and magnetic induction **B**, if there is motion of the medium. If the medium is isotropic, as in the case of random orientation of crystals, or in a binary composite, an iterative solution for $\mathbf{P}^f$ of odd powered fifth degree is obtained. The magnetoelectric polarization $\mathbf{P}^{me}$, on the other hand, is modeled by a qudratic in **H**. The magnetization **M**, like **P** in general, consists of ferromagnetic part $\mathrm{M}^h$ and magnetoelectric part $\mathrm{M}^{me}$. Theoretically, $\mathrm{M}^h$ can be modeled by the Landau-Lifshitz-Gilbert equation (Rado and Suhl [8]) or by the more recently given simpler models of Safanov [9]. All the relevant equations are presented in Bose [1], where a formula for the Kelvinic body force per unit volume acting at a point due to the polarization and magnetization fields is also given.

In this paper, the forgoing theoretical approach is applied to study the transmitted magnetic field across a plane multiferroic film when a plane EM wave of given electric intensity $E_0$ and given GHz frequency, is incident at an angle $\theta$ on the film. For simplicity, it is assumed that the film lies in free space and isotropic in regard to ferroelectric behaviour of **P** . The magnetic part $\mathrm{M}^h$ in this planar case is shown to have a linear relation to the magnetic field **H** with hysteretic damping coefficient. With these assumptions , it is shown that the contribution of the magnetoelectric parts of **P** and **M** remain passive, and the transmitted magnetic field intensity **H** is given by simple expressions. The expressions are computed for some reasonable data for the multiferroic material. As is the standard practice, the study is carried out for two cases: when the incident **E** field is parallel to the film and when it lies in a plane perpendicular to the film.

## 2. GOVERNING EQUATIONS OF ELECTROMAGNETIC WAVE TRANSMISSION

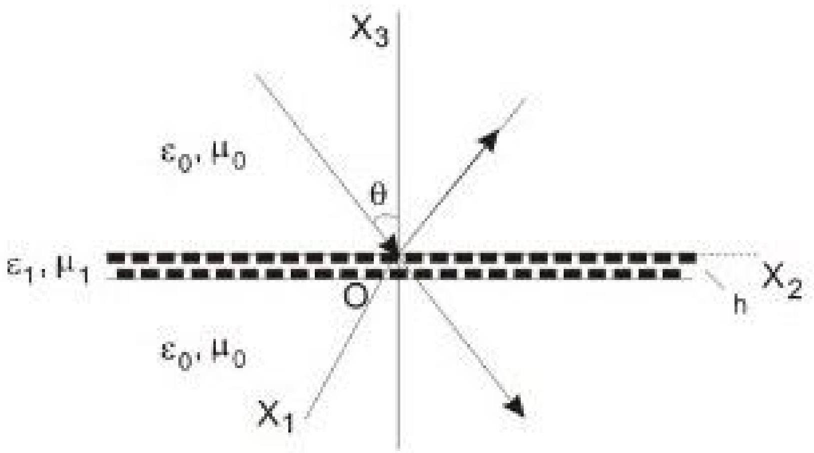


**Figure 1:** Schematic of transmission of EM waves through a multiferroic thin staratum.

Let a plane thin stratum of thickness $h$ of the mutiferroic be parallel to the $X_1X_2$ - plane with the $X_3$ - axis normal to it, as shown in figure 1. The multiferroic stratum is assumed to lie in free space having electric permittivity $\epsilon_0$ and magnetic susceptibility $\mu_0$ in standard notation. The incident electromagnetic wave at angle of incidence $\theta$ is partially reflected back in the domain $x_3 > 0$, transmits through the stratum $-h < x_3 < 0$ and emerges from the the lower half-space $x_3 < -h$.

In the two free half-spaces $x_3 > 0$ and $x_3 < -h$, the Maxwell's equations hold. In standard notations (Stratton [8], p. 268), the equations are

$$\nabla \cdot \mathbf{E} = 0, \quad \nabla \cdot \mathbf{H} = 0 \tag{1a}$$

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}, \quad \nabla \times \mathbf{H} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \tag{1b}$$

where $\mathbf{E}$ and $\mathbf{H}$ are respectively the electric and magnetic field intensities at a point $\mathbf{x}$ at time $t$. If by differentiation with respect to $t$, $\mathbf{H}$ is eliminated, and the vector identity $\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$ is used, then the vector $\mathbf{E}$ is easily found to satisfy the wave equation

$$\nabla^2 \mathbf{E} = \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} \tag{2}$$

where $c = 1/\sqrt{\mu_0 \epsilon_0}$ equals the velocity of light in vacuum. Similarly, it can be shown that $\mathbf{H}$ also satisfies a wave equation of the form (2).

In the multiferroic strip $-h < x_3 < 0$, in addition to the electric and magnetic field intensities $\mathbf{E}$ and $\mathbf{H}$, the electric polarization vector $\mathbf{P}$ and the magnetization vector $\mathbf{M}$ are generated in the dielectric medium. Moreover, the induced electromagnetic force can generate deformation and motion of the medium. An account of the modification of the governing equations is given in Bose [2], pp. 132-138. The Maxwell's equations, taking in to account the motion of the dielectric medium, according to Panofsky and Phillips ([9], p. 164), take the general form

$$\nabla \cdot \mathbf{D} = 0, \quad \nabla \cdot \mathbf{B} = 0 \tag{3a}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times (\mathbf{B} - \mu_0 \, \mathbf{P} \times \dot{\mathbf{u}}) = \mu_0 \frac{\partial \mathbf{D}}{\partial t} \tag{3a}$$

wher $\mathbf{D}$ is the electric displacement vector and $\mathbf{B}$ the induction vector repectively, in standard symbols. The two vectors $\mathbf{D}$ and $\mathbf{B}$ are respectively related to $\mathbf{E}$ and $\mathbf{H}$ by the equations

$$\mathbf{D} = \epsilon_0 \, \mathbf{E} + \mathbf{P}, \quad \mathbf{B} = \mu_0 \, (\mathbf{H} + \mathbf{M}) \tag{4}$$

and in Eq.$(3b)$, $\dot{\mathbf{u}}$ represents the velocity of motion of the medium at the point $\mathbf{x}$.

There has been intensive research on the dependence of $\mathbf{P}$ and $\mathbf{M}$ on $\mathbf{E}$ and $\mathbf{H}$ in multiferroic materials during recent years. The findings of these investigations is summarized in Bose [2]. In general, it is propounded that $\mathbf{P}$ is composed of ferroelectric and magnetoelectric effects, and similarly ferromagnetic and magnetoelectric effects constitute $\mathbf{M}$. For the simplest case, if the multiferroic is considered to be isotropic, then the polarization $\mathbf{P}$ can then be written as

$$\mathbf{P} = \epsilon_0\, \chi_e\, (\mathbf{E} + \dot{u} \times \mathbf{B})\, [1 - \alpha_{11}\, (\epsilon_0 \chi_e)^3\, |\mathbf{E} + \dot{\mathbf{u}} \times \mathbf{B}|^2$$
$$+ (3\epsilon_0\, \chi_e\, \alpha_{11}^2 - \alpha_{111})\, (\epsilon_0\, \chi_e)^5 |\mathbf{E} + \dot{\mathbf{u}} \times \mathbf{B}|^4] + A^{me}\, \mathbf{H} \tag{5}$$

where the first term of the expression arises from the ferroelectric contribution following the Landau-Devonshire theory (Jaynes [9], Bell and Cross [10])and the second term due to linear magnetoelectric effect in the isotropic case (Bose [2]). In Eq. (5), $\chi_e$ represents the nondimensional electric susceptibility and $\alpha_{11}, \alpha_{111} > 0$ are certain constants for the multiferroic material. The ferromagnetic dependence of the magnetization $\mathbf{M}^h$ on $\mathbf{H}$ has on the other hand, different competitive models due to the complexity of phenomenon. The suggested ones in the literature are that of Landau, Lifschitz and Gilbert and that of Safanov (Bose [2]). In the particular case of time harmonic plane electromagnetic wave however, since $\mathbf{E}$ are $\mathbf{B}$ are mutually perpendicular following the first equation of $(3b)$, the vector $\mathbf{M}^h$ and $\mathbf{H}$ must be coplanar, implying that

$$\mathbf{M}^h = \mu_0\, \chi_m\, \mathbf{H} \tag{6}$$

where $\chi_m$ is the nondimensional (volume) magnetic susceptibility, a scalar because of assumed isotropy. Moreover, since multiferroics possess magnetic relaxation, $\chi_m$ must be regarded complex, so that the ferromagnetic part $\mathbf{B}^h$ of $\mathbf{B}$ is of the form

$$\mathbf{B}^h = \mu_0\, (1 + \chi_m)\, \mathbf{H} =: \mu_0\, (\kappa_m\, e^{-i\,\omega\,\phi})\, \mathbf{H} \tag{7}$$

where $\kappa_m := |1 + \chi_m|$ and and $-\omega\,\phi := \arg\,(1 + \chi_m)$, meaning that $\kappa_m$ is the relative permeability of the medium and $\phi$ is the time lag of the following magnetic intensity. It will be assumed that both $\kappa_m$ and $\phi$ are constants for the material. The magnetoelectric contribution of the polarization $\mathbf{P}$ on $\mathbf{M}$ , like that of $\mathbf{H}$ on $\mathbf{P}$ in the isotropic case can be shown to be given by the linear form (Bose [2]), $\mathbf{M}^{me} = C^{me}\, \mathbf{E}$ where $C^{me}$ is a material constant. It therefore follows that the total induction $\mathbf{B}$ is given by the equation

$$\mathbf{B} = \mu_0\, (\kappa_m\, e^{-i\omega\phi})\, \mathbf{H} + C^{me}\, \mathbf{E} \tag{8}$$

Eqs. (5) and (8) give us the required constitutive equations for the isotropic multiferroic material for plane electromagnetic wave propagation.

The field equations for the material are obtained by inserting the expression for $\mathbf{D}$ given in Eq. (4) in the second equation of $(3b)$ and eliminating $\mathbf{B}$ that appears in the first equation. The governing equation for $\mathbf{E}$ is thus obtained as

$$\nabla^2\mathbf{E} - \nabla\left(\nabla\cdot\mathbf{E}\right) = \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} + \mu_0\frac{\partial^2\mathbf{P}}{\partial t^2} + \mu_0\,\nabla\times\frac{\partial}{\partial t}\left(\mathbf{P}\times\dot{u}\right) \tag{9}$$

The corresponding magnetic field $\mathbf{H}$ is given by the equation

$$\frac{\partial\mathbf{H}}{\partial t} = -\frac{1}{\mu_0\,\kappa_m}\,e^{i\omega\phi}\left(\nabla\times\mathbf{E} + C^{me}\,\frac{\partial\mathbf{E}}{\partial t}\right) \tag{10}$$

obtained from the first equation of $(3b)$, and using the constitutive equation (8).

## 3. PLANE WAVE REPRESENTATIONS

The incident time harmonic plane wave as shown in figure 1, can have arbitrary polarization. For theoretical purpose, it can be resolved in to two polarized cases. Case $(a)$, when the $\mathbf{E}$ field vector is parallel to the multiferroic film, conveniently taken parallel to the $X_1$ - axis, and Case $(b)$, when $\mathbf{E}$ is parallel to the plane of incidence, perpendicular to the $X_1$ - axis, so that the $\mathbf{H}$ field vector is parallel to the $X_1$ - axis. The time factor in both the cases will be taken as $e^{-i\omega t}$. It will be assumed that the magnitude of the electric field $|\mathbf{E}_0| =: E_0$ of the incident wave is given.

$Case\ (a)$: Since $\mathbf{E}$ is parallel to $OX_1$ it can be represented in the free domains $x_3 > 0$ and $x_3 < -h$ as

$$\mathbf{E} = E_1(x_2,\, x_3)\, e^{-i\omega t}\mathbf{e}_1 \tag{11}$$

The first equation of $(1b)$ then yields the magnetic field $\mathbf{H}$ in the form

$$\mathbf{H} = \left[H_2(x_2,\, x_3)\,\mathbf{e}_2 + H_3(x_2,\, x_3)\,\mathbf{e}_3\right] e^{-i\omega t} \tag{12}$$

where

$$H_2 = \frac{1}{i\,\omega\mu_0}\frac{\partial E_1}{\partial x_3}, \quad H_3 = -\frac{1}{i\,\omega\mu_0}\frac{\partial E_1}{\partial x_2} \tag{13}$$

The component $E_1$ in Eq. (11) satisfies in virtue of Eq. (2), the Helmholtz's equation

$$(\nabla^2 + \gamma_0^2)\,E_1 = 0 \tag{14}$$

where $\gamma_0^2 = \omega^2/c^2$. If the wave travels in the direction of the $X_2$ -axis, the solution of Eq. (14) is of the form

$$E_1 = \left(A\,e^{i\,\lambda_0 x_3} + B\,e^{-i\,\lambda_0 x_3}\right) e^{i\,k x_2} \tag{15}$$

where $k$ is the wave number and $\lambda_0 = \sqrt{\gamma_0^2 - k^2} = \sqrt{\omega^2/c^2 - k^2}$. The constants $A$ and $B$ are wave amplitudes. The corresponding magnetic field components then become

$$H_2 = \frac{\lambda_0}{\omega\,\mu_0}\left(A\,e^{i\,\lambda_0 x_3} - B\,e^{-i\,\lambda_0 x_3}\right)e^{i\,kx_2} \tag{16a}$$

$$H_3 = -\frac{k}{\omega\,\mu_0}\left(A\,e^{i\,\lambda_0 x_3} + B\,e^{-i\,\lambda_0 x_3}\right)e^{i\,kx_2} \tag{16b}$$

In the multiferroic domain, $0 < x_3 < -h$, Eqs. (9), (10) and (5) hold. If the mechanical response of the film is rigid or like that of an inextensible membrane, then $\dot{\mathbf{u}}$ vanishes. The proof for the latter case is given in section 5 to follow. Assuming the mechanical response to be either of the two kinds just stated, it is assumed that $\dot{\mathbf{u}} = 0$. Moreover, $\mathbf{E}$ has the form (11), so that $\nabla \cdot \mathbf{E} = \frac{\partial E_1}{\partial x_1}\,e^{-i\,\omega t} = 0$. Thus, it follows from Eq. (90) that $\mathbf{P}$ is parallel to $\mathbf{E}$ or to $OX_1$, and consequently $\mathbf{H}$ is parallel to the $X_2X_3$ - plane as the two fields must be orthogonal. For these constraints on $\mathbf{E}$ and $\mathbf{H}$, the constants $A^{me}$ and $C^{me}$ appearing in Eqs. (5) and (10) must vanish, implying that the magnetoelectric effect must remain passive. Hence for the present case $(a)$, Eq. (9) subject to Eq. (5) becomes

$$(\nabla^2 + \gamma_1^2)\,E_1 = 0 \tag{17}$$

where

$$\gamma_1^2 = \gamma_0^2\,\{1 + \chi_e[1 - \alpha_{11}\,(\epsilon_0\,\chi_e)^3\,|E_1|^2 + (3\,\epsilon_0\chi_e\,\alpha_{11}^2 - \alpha_{111})\,(\epsilon_0\chi_e)^5\,|E_1|^4\} \tag{18}$$

Focusing on plane wave transmission through the multiferroic film, the quasilinear equation (17) is linearized by replacing $|E_1|$ in Eq. (18) by the magnitude of the incident wave $E_0$. The corresponding magnetic field $\mathbf{H}$ from Eq. (10) is then given by

$$H_2 = \frac{1}{i\,\omega\mu_0\kappa_m}\frac{\partial E_1}{\partial x_3}\,e^{i\,\omega\phi}, \quad H_3 = -\frac{1}{i\,\omega\mu_0\kappa_m}\frac{\partial E_1}{\partial x_2}\,e^{i\,\omega\phi} \tag{19}$$

The propagating solution of the linearized equation (17) is then of the form

$$E_1 = A_1\,e^{i\,\lambda_1 x_3} + B_1\,e^{-i\lambda_1 x_3})\,e^{ikx_2} \tag{20}$$

where $\lambda_1 = \sqrt{\gamma_1^2 - k^2}$. in which $\gamma_1^2$ is rendered a constant by replacing $|E_1|$ by $E_0$. Using Eq. (9) the corresponding magnetic field components $H_2$ and $H_3$ are given by

$$H_2 = \frac{\lambda_1}{\omega\mu_0\kappa_m}\left(A_1\,e^{i\lambda_1 x_3} - B_1\,e^{-i\lambda_1 x_3}\right)e^{ikx_2}\,e^{i\,\omega\phi} \tag{21a}$$

$$H_3 = -\frac{k}{\omega\mu_0\kappa_m}\left(A_1\,e^{i\lambda_1 x_3} + B_1\,e^{-i\lambda_1 x_3}\right)e^{ikx_2}\,e^{i\,\omega\phi} \tag{21b}$$

$Case(b)$: In this case $\mathbf{H}$ is considered parallel to $OX_1$, and $\mathbf{E}$ parallel to the $X_2X_3$ - plane in the incident and transmitted domains $x_3 > 0$ and $x_3 < -h$. Hence,

$$\mathbf{H} = H_1(x_2,\, x_3)\, e^{-i\,\omega t}\, \mathbf{e}_1 \tag{22}$$

and

$$\mathbf{E} = [E_2(x_2,\, x_3)\, \mathbf{e}_2 + E_3(x_2,\, x_3)\, \mathbf{e}_3]\, e^{-i\,\omega t} \tag{23}$$

where, from the second equation of $(1b)$

$$E_2 = -\frac{1}{i\,\omega\epsilon_0}\,\frac{\partial H_1}{\partial x_3}, \quad H_3 = \frac{1}{i\,\omega\epsilon_0}\,\frac{\partial H_1}{\partial x_2} \tag{24}$$

As in Case $(a)$, $H_1$ satisfies the Helmholtz's equation $(\nabla^2 + \gamma_0^2)\, H_1 = 0$. Its plane wave solution is of the form

$$H_1 = \left(C\, e^{i\lambda_0 x_3} + D\, e^{-i\lambda_0 x_3}\right) e^{ikx_2} \tag{25}$$

and so from Eq. (24)

$$E_2 = -\frac{\lambda_0}{\omega\epsilon_0}\left(Ce^{i\lambda_0 x_3} - De^{-i\lambda_0 x_3}\right) e^{ikx_2} \tag{26$a$}$$

$$E_3 = \frac{k}{\omega\epsilon_0}\left(Ce^{i\lambda_0 x_3} + De^{-i\lambda_0 x_3}\right) e^{ikx_2} \tag{26$b$}$$

where $C$ and $D$ are constant amplitudes.

In the multiferroic domain $-h < x_3 < 0$, the $\mathbf{E}$ and $\mathbf{H}$ fields have again two and one components $(E_2,\, E_3)$ and $H_1$ respectively. Following Eqs. (9) and (5), $\mathbf{P}$ is parallel to $\mathbf{E}$, so that the magnetoelectric effect remains passive. Hence, following the first equation of $(3a)$, $\nabla \cdot \mathbf{E} = 0$. Also, $\dot{\mathbf{u}} = 0$ as shown in section 5. Hence, the Eq. (9) for $\mathbf{E}$ reduces to a quasilinear Helmholtz's equation as in Case $(a)$ that can again be linearized as $(\nabla^2 + \gamma_1^2)\, \mathbf{E} = 0$, replacing $|\mathbf{E}|$ by $E_0$ the magnitude of the incident electric field in the expression for $\gamma_1^2$. The plane wave solution for $E_2$ and $E_3$ are then

$$E_2 = -\frac{\lambda_1}{\omega\epsilon_0}\left(C_1 e^{i\lambda_1 x_3} - D_1 e^{-i\lambda_1 x_3}\right) e^{ikx_2} \tag{27$a$}$$

$$E_3 = \frac{k}{\omega\epsilon_0}\left(C_1 e^{i\lambda_1 x_3} + D_1 e^{-i\lambda_1 x_3}\right) e^{ikx_2} \tag{27$b$}$$

where $C_1,\, D_1$ are constants. The magnetic field $H_1$ from Eq. (10) is then given by

$$H_1 = \frac{\gamma_1^2}{\gamma_0^2}\left(C_1\, e^{i\,\lambda_1 x_3} + D_1\, e^{-i\lambda_1 x_3}\right) e^{ikx_2}\, e^{i\,\omega\phi} \tag{28}$$

The boundary conditions at the interfaces $x_3 = 0$ and $x_3 = -h$ are that the tangential components of $\mathbf{E}$ and $\mathbf{H}$ are continuous across the two interfaces. This means the continuity of $E_1$ and $H_2$ in Case$(a)$ and the continuity of $E_2$ and $H_1$ in Case $(b)$ at $x_3 = 0$ and $x_3 = -h$.

## 4. PLANE WAVE TRANSMISSION IN THE TWO CASES

$Case(a)$: As the incident wave strikes the film at an angle $\theta$, a part of it is reflected back in the free space $x_3 > 0$. The electromagnetic field in this domain is therfore represented by Eqs. (15) and $(16a,\, b)$ with $B =: E_0$ and $A =: A_0$ = amplitude of the reflected wave. Also, in terms of $\theta$, $k = \gamma_0 \sin\theta,\ \lambda_0 = \gamma_0 \cos\theta$. In the multiferroic layer $-h < x_3 < 0$, the representations of $E_1$, $H_1$ and $H_2$ are given by Eqs. (20) and $(21a,\, b)$. In the transmission domain $x_3 < -h$, there is only a transmitted wave travelling in the negative $x_3$ - axis direction. Hence, its representation following Eqs. (15) and $(16a,\, b)$ is given by the equations

$$E_1 = B_2\, e^{-i\lambda_0 x_3}\, e^{ikx_2} \tag{29a}$$

$$H_2 = -\frac{\lambda_0}{\omega\mu_0}\, B_2\, e^{-i\lambda_0 x_3}\, e^{ikx_2} \tag{29b}$$

$$H_3 = -\frac{k}{\omega\mu_0}\, B_2\, e^{-i\lambda_0 x_3}\, e^{ikx_2} \tag{29c}$$

The boundary conditions on the two interfaces $x_3 = 0$ and $x_3 = -h$ are the continuity of $E_1$ and $H_2$. These conditions yield a system of four equations viz.

$$A_0 + E_0 = A_1 + B_1 \tag{30a}$$

$$\lambda_0\, (A_0 - E_0) = \frac{\lambda_1}{\kappa_m}\, (A_1 - B_1) \tag{30b}$$

$$A_1\, e^{-il_1 h} + B_1\, e^{i\lambda_1 h} = B_2\, e^{i\lambda_0 h} \tag{30c}$$

$$\frac{\lambda_1}{\kappa_m}\, (A_1\, e^{-i\lambda_1 h} - B_1\, e^{i\lambda_{1|h}}) = -\lambda_0\, B_2\, e^{i\lambda_0 h} \tag{30d}$$

The solution of the Eqs. $(30a - d)$ yields the value of $B_2$ in terms of $E_0$. Hence from Eqs. $(21a,\, b)$ the transmitted $\mathbf{H}$ field is given by

$$H_2 = \frac{4E_0\, \lambda_1}{\omega\mu_0\kappa_m\Delta_a}\, e^{-i\,(\lambda_0+\lambda_1)\,h}\, e^{i\,\omega\phi}\, e^{-i\lambda_0 x_3}\, e^{ikx_2} \tag{31a}$$

$$H_3 = \frac{4E_0\, k\lambda_1}{\omega\mu_0\kappa_m\Delta_a}\, e^{-i\,(\lambda_0+\lambda_1)\,h}\, e^{i\,\omega\phi}\, e^{-i\lambda_0 x_3}\, e^{ikx_2} \tag{31b}$$

where $k/\lambda_0 = \gamma_0\,\sin\theta/\gamma_0\,\cos\theta = \tan\theta$, and

$$\Delta_a = \Big(1 - \frac{\lambda_1}{\lambda_0\kappa_m}\,e^{i\,\omega\phi}\Big)^2 - \Big(1 + \frac{\lambda_1}{\lambda_0\kappa_m}\,e^{i\,\omega\phi}\Big)^2\,e^{-2i\lambda_1 h} \tag{32}$$

The magnitude of the resultant transmitted magnetic intensity **H** is thus

$$|H| = \frac{4E_0\,\lambda_1\,\sec\theta}{\omega\mu_0\kappa_m|\Delta_a|} \tag{33}$$

$Case(b)$: In this case $H_1$, $E_2$, $E_3$ in the domain $x_3 > 0$ are given by Eqs. (25) and $(26a,\,b)$ with $C =: C_0$ = amplitude of the reflected **H** field, and $D =: D_0$ so that $\lambda_0 D_0/(\omega\epsilon_0)$ and $kD_0/(\omega\epsilon_0)$ are the components of the given incident $\mathbf{E}_0$ field. Hence, $E_0^2 = (\lambda_0^2 + k^2)\,D_0^2/(\omega^2\epsilon_0^2)$ or, $D_0 = \epsilon_0 c\,E_0$ since $k = \gamma_0\,\sin\theta$, and $\lambda_0 = \gamma_0\,\cos\theta$. Eqs. (28) and $(27a,\,b)$ respectively. In the domain $x_3 < -h$, since the transmitted wave travels opposite to the $X_3$ - axis,

$$H_1 = D_2\,e^{-i\lambda_0 x_3}\,e^{ikx_2} \tag{34}$$

$$E_2 = \frac{\lambda_0}{\omega\,\epsilon_0} D_2\,e^{-i\lambda_0 x_3}\,e^{ikx_2} \tag{35$a$}$$

$$E_3 = \frac{k}{\omega\,\epsilon_0} D_2\,e^{-i\lambda_0 x_3}\,e^{ikx_2} \tag{35$b$}$$

The boundary conditions on $x_3 = 0$ and $x_3 = -h$ are the continuity of $H_1$ and $E_2$. These conditions yield the system of equations

$$C_0 + D_0 = \frac{\gamma_1^2}{\kappa_m\gamma_0^2}\,(C_1 + D_1)\,e^{i\,\omega\phi} \tag{36$a$}$$

$$\lambda_0\,(C_0 - D_0) = \lambda_0\,(C_1 - D_1)\,e^{i\,\omega\phi} \tag{36$b$}$$

$$\frac{\gamma_1^2}{\kappa_m\gamma_0^2}\,(C_1\,e^{-i\lambda_0 h} + D_1\,e^{i\lambda_1 h})\,e^{i\,\omega\phi} = D_2\,e^{i\lambda_0 h} \tag{36$c$}$$

$$-\lambda_1\,(C_1\,e^{-i\lambda_0 h} - D_1\,e^{i\lambda_1 h})\,e^{i\,\omega\phi} = \lambda_0\,D_2\,e^{i\lambda_0 h} \tag{36$d$}$$

The solution of Eqs. $(36a - d)$ yields the value of $D_2$ in terms of $D_0 = \epsilon_0 c\,E_0$. Hence, according to Eq. (34) the transmitted **H** field is given by the equation

$$H_1 = -\frac{4\epsilon_0 c\,E_0}{\kappa_m\Delta_b}\,\frac{\lambda_1\gamma_1^2}{\lambda_0\gamma_0^2}\,e^{-i(l_0+l_1)h}\,e^{i\,\omega\phi}\,e^{-i\lambda_0 x_3}\,e^{ikx_2} \tag{37}$$

or,

$$|H_1| = \frac{4\epsilon_0 c\,E_0}{\kappa_m|\Delta_b|}\,\frac{\lambda_1\gamma_1^2}{\lambda_0\gamma_0^2} \tag{38}$$

where

$$\Delta_b = \Big(1 - \frac{\lambda_0\gamma_1^2}{\kappa_m\lambda_1\gamma_0^2}\,e^{i\,\omega\phi}\Big)^2 - \Big(1 + \frac{\lambda_0\gamma_1^2}{\kappa_m\lambda_1\gamma_0^2}\,e^{i\,\omega\phi}\Big)^2\,e^{-2i\lambda_1 h} \tag{39}$$

## 5. EFFECT OF ELASTIC DEFORMATION

Eqs.(9) and (5) show that elastic deformation may affect the electric field $\mathbf{E}$ and the polarization $\mathbf{P}$. However, it was asserted that in the case of plane wave propagation in the multiferroic film, there is no vibration in the transverse direction, provided that, mechanically the film behaves like an unstretchable membrane. For proving this assertion, it is noted that the Kelvinic body force per unit volume on the film is (Bose [2], p. 138)

$$\mathbf{b}^{EH} = (\mathbf{P}\cdot\nabla)\,\mathbf{E} + \mu_0\,(\mathbf{M}\cdot\nabla)\,\mathbf{M} \tag{40}$$

and so its normal component is

$$\mathbf{b}^{EH}\cdot\mathbf{e}_3 = (\mathbf{P}\cdot\nabla)\,E_3 + \mu_0\,(\mathbf{M}\cdot\nabla)\,H_3 \tag{41}$$

It is sufficient to prove that the right hand side of Eq. (41) vanishes for the plane wave transmission formulated in section 3, as there is no transverse body force acting on the film.

For Case $(a)$, $E_3$ - the component of the electric field in the $X_3$ - direction is zero, while from Eq. (6) $\mathbf{M} = \mu_0\,\chi_m\mathbf{H}$. Hence Eq. (41) becomes

$$\mathbf{b}\cdot\mathbf{e}_3 = \mu_0\chi_m\left(H_2\,\frac{\partial H_3}{\partial x_2} + H_3\,\frac{\partial H_3}{\partial x_3}\right)$$

$$= -\frac{i\chi_m k e^{i\omega\phi}}{\omega\kappa_m}\left[kH_2\Big(A_1\,e^{i\lambda_1 x_3}+B_1\,e^{-i\lambda_1 x_3}\Big)+\lambda_1 H_3\Big(A_1\,e^{i\lambda_1 x_3}-B_1\,e^{-i\lambda_1 x_3}\Big)\right]e^{ikx_2} = 0 \tag{42}$$

where the expressions for $H_2$ and $H_3$ given in Eqs. $(21a,\,b)$ for the mutiferroic strip are used.

For Case $(b)$, $H_3$ - the component of the magnetic field parallel to the $X_3$ - axis is zero, while the polarization $\mathbf{P}$ is parallel to $\mathbf{E}$, say $\mathbf{P} = \epsilon_0\chi'_e\,\mathbf{E}$ where $\chi'_e$ is an electric susceptibility, assmed to be scalar due to isotropy. Hence in this case, from Eq. (41)

$$\mathbf{b}^{EH}\cdot\mathbf{e}_3 = \epsilon_0\chi'_e\left(E_2\,\frac{\partial E_3}{\partial x_2} + E_3\,\frac{\partial E_3}{\partial x_3}\right) \tag{43}$$

Using the forms of $E_2$ and $E_3$ given in Eqs. $(27a,\,b)$, the right ahnd side of of Eq. (43) equals zero, as in the case of Eq. (42).

## 6. NUMERICAL STUDY

The electromagnetic parameters of multiferroic materials differ widely. Therefore for the present study, typical data are assumed for quantitative assessment of the transmitted magnetism. Let the permittivity be $\chi_e = 100$, and the real part of of the permeability $Re(\chi_m) = 5 \times 10^{-5}$ (both nondimensional), giving $\kappa_m = 1 + 5 \times 10^{-5}$. The time lag in magnetization is assumed to be of cnstant value $0.1\,s$. Estimates of the polarization parameters $\alpha_{11}$ and $\alpha_{111}$ appearing in Eqs. (5) and (18) are estimated by assuming that $|\mathbf{P}|$ attains about $5 \times 10^{-3}\ \mu C\,cm^{-2}$ at applied $E_1 = 1\,kV\,cm^{-1}$. Ignoring the fourth power term in Eq. (18), $\alpha_{11}$ is thus estimated to be given by the equation

$$(\epsilon_0 \chi_e)^3\,\alpha_{11} = 2.40290\ \ cm^2\,kV^{-2} \tag{44}$$

wher $V$ represents the Volt. Next, since $\alpha_{111} <<< \epsilon_0 \chi_e\,\alpha_{11}^2$ as in some ferroelectric materials, it is assumed that

$$\alpha_{111} = 0.1 \times \epsilon_0 \chi_e\,\alpha_{11}^2\ \ \mu C^{-5}\,cm^9\,kV \tag{45}$$

or,

$$(\epsilon_0 \chi_e)^5\,\alpha_{111} = 0.1 \times 2.40290^2 = 0.57739\ \ cm^4\,kV^{-4} \qquad (45$$

The thickness of the multiferroic film is taken to be $h = 300\ \ nm = 3 \times 10^{-5}\ \ cm$, as shown if figure 1.

The frequency of waves in this study is considered to be in the ultra high frequency - microwave frequency GHz range. Accordingly, the frequency $\omega$ of the waves is expressed as $\omega = 2\pi \times 10^9\,f$ radians per second, where $f$ is in GHz. Inserting $\omega$ in the form just stated in Eqs. (33) and (38), one obtains in Case $(a)$

$$|H| = \frac{0.50661\,E_0\,\lambda_1\,\sec\theta}{f\,\kappa_m\,|\Delta_a|} \times 10^8 \quad \mu C\,cm^{-1}\,s^{-1} \tag{46}$$

and in Case $(b)$

$$|H_1| = \frac{0.10610\,E_0\,\lambda_0 \gamma_1^2}{\kappa_m\,|\Delta_b|\,\lambda_1 \gamma_0^2} \times 10^8 \quad \mu C\,cm^{-1}\,s^{-1} \tag{47}$$

With the above stated values, computation of $|H|$ and $H_1|$ given by Eq. (46) is undertaken for $\theta = 0^o$ and $20^o$, and that for Eq. (47) for $\theta = 20^o$ only, as the case $(b)$ for $\theta = 0^o$ is covered by Case $(a)$. The incident electric field is taken as $E_0 = 1$, 2 and 3 $kV\,cm^{-1}$. The value of the other dependent parameters become $\gamma_0 = \omega/c = 0.20944f$ $GHz\,cm^{-1}$, $\lambda_0 = \gamma_0\,\cos\theta$ and $\lambda_1 = \sqrt{\gamma_1^2 - \gamma_0^2 \sin^2\theta}$, where $\gamma_1^2$ is given by Eq. (18) or,

$$\gamma_1^2 = \gamma_0^2\,[1 + 100\,(1 - 2.40298\,E_0^2 + 16.74440\,E_0^4)] \tag{48}$$

where $|E_1|$ is replaced by $E_0$. The results of computation are shown in figures 2, 3, and 4.

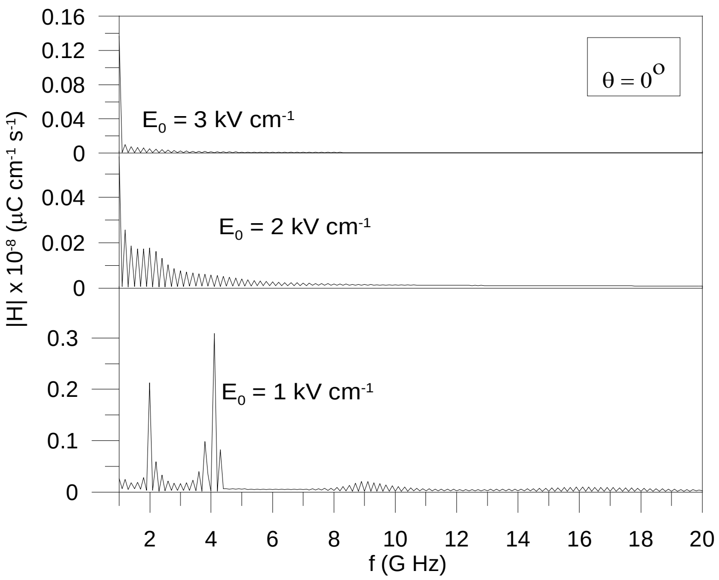


**Figure 2:** Magnitude of the transmitted magnetic field intensity $|H|$ for $\theta = 0^o$

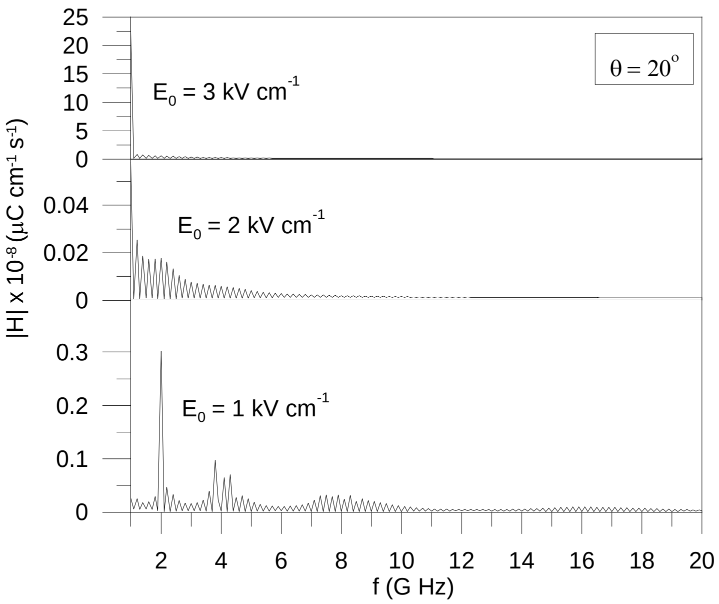


**Figure 3:** Magnitude of the transmitted magnetic field intensity $|H|$ for $\theta = 20^o$.

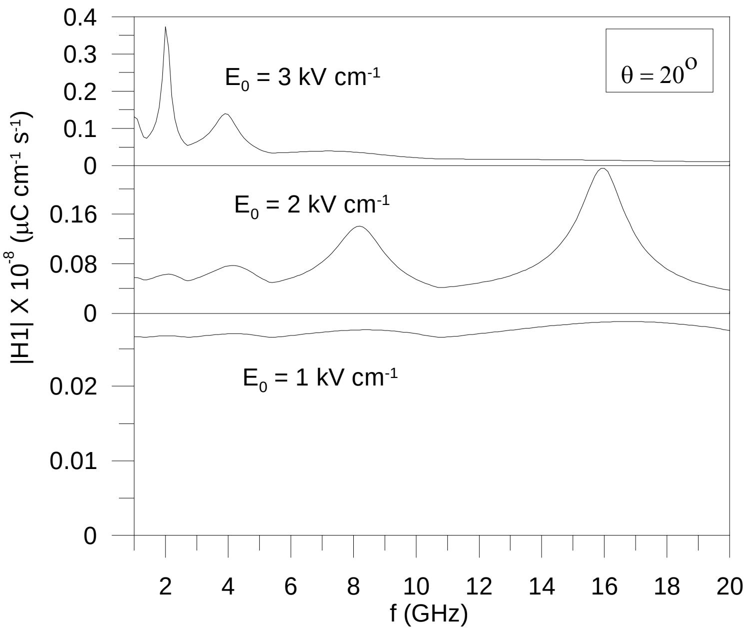


**Figure 4:** Magnitude of the transmitted magnetic field intensity $|H_1|$ for $\theta = 20^o$.

From the figures 2 and 3, it is seen that the value of $|H|$ rapidly fluctuates with increasing values of $f$, falling rapidly by an order of magnitude two, but is still very high - of the order of magnitude six. For $E_0 = 1$ there are a couple of spikes in the initial stages of increasing $f$; but for $E_0 = 2$ that becomes gradual. In the case of $E_0 = 3$ the fall is steep. In contrast, figure 4 for $|H_1|$ at $\theta = 20^o$, shows rapid fluctuations riding on patterns of wavy hills and valleys. For increasing $f$ the values of $|H_1|$ remain comparatively steady with a small amount of fall in value of $|H_1|$ for the cases $E_0 = 1$ and 2. But in the case of $E_0 = 3$ the fall in value is by an order of magnitude 1, that is, $|H_1|$ is of order seven.

## 7. CONCLUSIONS

In recent years, multiferroic materials that show both ferroelectric and ferromagnetic order are drawing much attention for applicability in order to generate significant magnetic field by direct application of electric field. Several metal oxides/pervoskites are under investigation that exhibit this property in different temperature ranges. The present paper sets up the basic governing equations for the electromagnetic field in a conglomerate of ferromagnetic particles larger than micro scale, so that a continuum theory can be set up. Bose [2] gives a development of the basic equations of such a theory, consisting of the Maxwell's equations for a moving medium, as the material also exhibits ferroelasticity. These equations connect the electric field $\mathbf{E}$, the magnetic field $\mathbf{H}$ and the velocity of motion $\dot{\mathbf{u}}$. As the medium is also dielectric, the polarization $\mathbf{P}$ and magnetization $\mathbf{M}$ also enter in to the set of equations. General models for $\mathbf{P}$ and $\mathbf{M}$ are also presented in [2]. With this background, the transmission of an electromagnetic wave across a film of such material, bound together to form a continuum, placed in free space, is studied for obtaining the expression for the magnetic field of the transmitted wave. Special forms of the constitutive equations for $\mathbf{P}$ and $\mathbf{M}$ are developed for the problem. The study is divided in to two standard cases. In the first case, the incident $\mathbf{E}_0$ field is taken parallel to the film, and in the second, the $\mathbf{E}_0$ vector is taken to be lying in a plane perpendicular to the film. Applying the boundary conditions of the continuity of the tangential components of $\mathbf{E}$ and $\mathbf{H}$ at the interfaces, the required expressions for the transmitted $\mathbf{H}$ field are developed. Adopting representative values of the parameters appearing in the formulation, the magnitude of the transmitted $\mathbf{H}$ field is computed for the cases of normal incidence $\theta = 0^o$, and inclined incidence at $\theta = 20^o$, in GHz frequency range. The response is found to be rapidly fluctuating profiles as shown in figures 2, 3 and 4, showing significantly high magnetic field that can vary from an order of magnitude six to eight. It is also shown that the transmission does not generate any transverse vibration, if the film is perfectly flexible but unstretchable.

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