International Journal of Fuzzy Mathematics and Systems.
Volume 1, Number 1 (2011), pp. 81-92

http://www.ripublication.com

Some Fixed Point Theorems for Occasionally Weakly Compatible Mappings in Fuzzy 2-Metric Spaces.

Priyanka Nigam and Neeraj Malviya

NRI Institute of Information Science and Technology, Bhopal, India
E-mail: priyanka_nigam01@yahoo.co.in, maths.neeraj@gmail.com

Abstract

The Objective of this paper is to obtain some common fixed point theorems for occasionally weakly compatible mappings in fuzzy 2-metric spaces.

Keywords: Occasionally weakly compatible mappings, fuzzy 2- metric space.

Introduction

Fuzzy set was defined by Zadeh [30]. Kramosil and Michalek [16] introduced fuzzy metric space, George and Veeramani [8] modified the notion of fuzzy metric spaces with the help of continuous t-norms. Many researchers have obtained common fixed point theorems for mappings satisfying different types of commutativity conditions. Vasuki [27] proved fixed point theorems for R-weakly commutating mappings. Pant [20, 21, 22] introduced the new concept reciprocally continuous mappings and established some common fixed point theorems. Balasubramaniam et al.[6], have shown that Rhoades [24] open problem on the existence of contractive definition which generates a fixed point but does not force the mappings to be continuous at the fixed point, posses an affirmative answer. Pant and Jha [22] obtained some anologus results proved by Balasubramanium et al. Recent literature on fixed point in fuzzy metric space can be viewed in [1, 2, 3, 4, 11, 18, 26].

The concept of 2-metric space was initiated by Gahler [28] whose abstract properties were suggested by the area function in Euclidian space. In a paper Sanjay kumar [29] discussed the concept of fuzzy 2-metric space akin to 2-metric spaces introduced by Gahler [28].

This paper presents some common fixed point theorems for more general commutative condition i.e. occasionally weakly compatible mappings in complete fuzzy 2-metric space

Preliminary Notes

Definition 2.1 [30] A fuzzy set A in X is a function with domain X and values in [0,1].

Definition 2.2 [25] A binary operation $*:[0,1]\times[0,1]\to[0,1]$ is a continuous t-norms if $*$ is satisfying conditions:

$*$ is an commutative and associative;

$*$ is continuous;

$a*1=a$ for all $a\in[0,1]$;

$a*b\le c*d$ whenever $a\le c$ and $b\le d$ and a, b, c, d $\in[0,1]$.

Definitions 2.3 [8] A 3-tuple (X,M,$*$) is said to be a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2\times(0,\infty)$ satisfying the following conditions, for all x, y, z $\in$ X, s, t >0,

$M(x,y,t)>0$;

$M(x,y,t)=1$ if and only if $x=y$;

$M(x,y,t)=M(y,x,t)$;

$M(x,y,t)*M(y,z,s)\le M(x,z,t+s)$;

$M(x,y,\cdot):(0,\infty)\to(0,1]$ is continuous.

Then M is called a fuzzy metric on X. Then $M(x,y,t)$ denotes the degree of nearness between x and y with respect to t.

Example 2.4 (Induced fuzzy metric [8]) Let (X,d) be a metric space. Denote $a*b=ab$ for all $a,b\in[0,1]$ and let M_d be fuzzy sets on $X^2\times(0,\infty)$ defined as follows:

$$M_d=\frac{t}{t+d(x,y)}.$$

Then $(X,M_d,*)$ is a fuzzy metric space.

Definition 2.5 A binary operation $*:[0,1]\times[0,1]\times[0,1]\to[0,1]$ is called a continuous t-norm if $([0,1],\cdot)$ is an abelian topological monoid with unit 1 such that $a*b*c\le d*e*f$ whenever $a\le d, b\le e$ and $e\le f$ for all $a,b,c,d,e,f\in[0,1]$.

In a paper Sanjay kumar [29] discussed the concept of fuzzy 2-metric space akin to 2-metric spaces introduced by Gahler [28]. We borrow some definitions, examples and remark given in [29].

Definition 2.6 [29] A triplet $(X,M,*)$ is a fuzzy 2-metric space if X is an arbitrary

set, $*$ is a continuous t-norm, and M is a fuzzy set in $X^3 \times [0,\infty)$ satisfying the following conditions:

$M(x,y,a,0)=0$

$M(x,y,a,t)=1$ for all $t>0$ if and only if at least two of them are equal.

$M(x,y,a,t)=M(y,a,x,t)=M(a,y,x,t)$ (symmetric)

$M(x,y,a,r+s+t) \geq M(x,y,z,r) * M(x,z,a,s) * M(z,y,a,t)$ for all $x,y,z \in X$ and $r,s,t>0$.

$M(x,y,a,.):[0,\infty) \to [0,1]$ is left continuous for all $x,y,z,a \in X$ and $r,s,t>0$.

$\lim_{n\to\infty} M(x,y,a,t)=1$ for all $x,y,a \in X, t>0$.

Example 2.7 [29] Let X be the set {1, 2, 3, 4} with 2-metric d defined by

$$d(x,y,z)=\begin{cases} 0, if \text{ x=y,y=z,z=x and \{x,y,z\}=\{1,2,3\}} \\ \frac{1}{2}, \text{ otherwise.} \end{cases}$$

For each $t \in [0,\infty)$, define $a*b*c=abc$ and

$$M(x,y,z,t)=\begin{cases} 0, \text{ if t=0.} \\ \frac{t}{t+d(x,y,z)}, \text{ if t>0, where x, y, z } \in \text{X.} \end{cases}$$

Then $(X,M,*)$ is a fuzzy 2-metric space.

Definition 2.8 [29] (a) A sequence $\{x_n\}$ in $(X,M,*)$ is convergent to $x \in X$ if $\lim_{n\to\infty} M(x_n,x,a,t)=1$ for each $t>0$.

(b) A fuzzy 2-metric space, $(X,M,*)$ is called Cauchy if $\lim_{n,m\to\infty} M(x_n,x_m,a,t)=1$ for each $t>0$.

(c) A fuzzy 2-metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.9 A pair of self-mappings (f, g) of a fuzzy 2- metric space (X,M,$*$) is said to be weakly commuting if $M(fgx,gfx,a,t) > M(fx,gx,a,t)$ for all $x \in X$ and $t>0$.

R-weakly commuting if there exist some R>0 such that $M(fgx,gfx,a,t) \geq M(fx,gx,a,\frac{t}{R})$.

Definition 2.10 Two self mappings f and g of a fuzzy 2- metric space (X,M,$*$) are

called compatible if $\lim_{n\to\infty} M(fgx_n, gfx_n, a, t) = 1$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$ for some x in X.

Definition 2.11 Two self maps f and g of a fuzzy 2-metric space (X,M,$*$) are called reciprocally continuous on X if $\lim_{n\to\infty} fgx_n = fx$ and $\lim_{n\to\infty} gfx_n = gx$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$ for some x in X.

Lemma 2.12 Let (X, M,$*$) be a fuzzy 2-metric space. If there exists $q \in (0,1)$ such that M(x, y ,a , qt) $\geq$ M(x ,y ,a ,t) for all x, y $\in$ X and t>0, then x = y.

Definition 2.13 [1] Let X be a set, f, g selfmaps of X. A point x in X is called a coincidence point of f and g iff fx = gx. We shall call w = fx = gx a point of coincidence of f and g.

Definition 2.14 [13] A pair of maps S and T is called weakly compatible pair if they commute at coincidence points.

Definition 2.15 [1]Two self maps f and g of a set X are occasionally weakly compatible (owc) iff there is a point x in X which is a coincidence point of f and g at which f and g commute.

A. Al-Thagafi and Naseer Shahzad [5] shown that occasionally weakly is weakly compatible but converse is not true.

Definition 2.16 [5] Let R be the usual metric space. Define S, T: R$\to$R by Sx = 2x and Tx = x^2 for all $x \in R$. Then Sx = Tx for x = 0,2 but ST0 =TS0, and ST2 $\neq$ TS2. S and T are occasionally weakly compatible self maps but not weakly compatible.

Lemma 2.17 [14] Let X be a set, f, g owc self maps of X. If f and g have a unique point of coincidence, w = fx = gx, then w is the unique common fixed point of f and g.

Main Results

Theorem 3.1 Let (X, M, $*$) be a complete fuzzy 2-metric space and Let A, B, S and T be self mappings of X. Let the pairs {A, S} and {B, T} be owc. If there exist $q \in (0,1)$ such that

$$M(Ax, By, a, qt) \geq \alpha_1 M(Sx, Ty, a, t) + \alpha_2 M(Ax, Ty, a, t) + \alpha_3 M(By, Sx, a, t) \qquad (3.1)$$

For all x, y $\in$ X, where $\alpha_1, \alpha_2, \alpha_3 > 0$, $\alpha_1 + \alpha_2 + \alpha_3 > 1$ then there exist a unique point $w \in X$ such that $Aw = Sw = w$ and a unique point $z \in X$ such that $Bz = Tz = z$. Moreover, $z = w$, so that there is a unique common fixed point of A, B, S and T.

Proof: Let the pairs {A, S} and {B, T} be owc, so there are points $x, y \in X$ such that

$Ax = Sx$ and $By = Ty$. We claim that, $Ax = By$. If not, by inequality (3.1)

$$M(Ax, By, a, qt) \geq \alpha_1 M(Sx, Ty, a, t) + \alpha_2 M(Ax, Ty, a, t) + \alpha_3 M(By, Sx, a, t)$$
$$= \alpha_1 M(Ax, By, a, t) + \alpha_2 M(Ax, By, a, t) + \alpha_3 M(By, Ax, a, t)$$
$$= (\alpha_1 + \alpha_2 + \alpha_3) M(Ax, By, a, t)$$

a contradiction, since $(\alpha_1 + \alpha_2 + \alpha_3) > 1$. And by Lemma (2.12) Ax=By, i.e. $Ax = Sx = By = Ty$. Suppose that there is another point z such that $Az = Sz$ then by (3.1) we have $Az = Sz = By = Ty$. So, $Ax = Az$ and $w = Ax = Sx$ is the unique point of coincidence of A and S. By Lemma (2.17) w is the only common fixed point of A and S, i.e. $w = Aw = Sw$. Similarly there is a unique point $z \in X$ such that $z = Bz = Tz$.

Assume that $w \neq z$. We have

$$M(w, z, a, qt) = M(Aw, Bz, a, qt)$$
$$\geq \alpha_1 M(Sw, Tz, a, t) + \alpha_2 M(Aw, Tz, a, t) + \alpha_3 M(Bz, Sw, a, t)$$
$$= \alpha_1 M(w, z, a, t) + \alpha_2 M(w, z, a, t) + \alpha_3 M(z, w, a, t)$$
$$= (\alpha_1 + \alpha_2 + \alpha_3) M(w, z, a, t)$$

a contradiction, since $(\alpha_1 + \alpha_2 + \alpha_3) > 1$. And by Lemma (2.12) z=w. Also by Lemma (2.17), z is the common fixed point of A, B, S and T. The uniqueness of the fixed point holds from (3.1).

Theorem 3.2 Let (X, M, $*$) be a complete fuzzy 2-metric space and Let A, B, S and T be self mappings of X. Let the pairs {A, S} and {B, T} be owc. If there exist $q \in (0,1)$ such that

$$M(Ax, By, a, qt) \geq \alpha \min\{M(Sx, Ty, a, t), M(Sx, Ax, a, t)\}$$
$$+ \beta \min\{M(By, Ty, a, t), M(Ax, Ty, a, t)\}$$
$$+ \gamma M(By, Sx, a, t) \tag{3.2}$$

For all x, y $\in$ X, where, $\alpha, \beta, \gamma > 0$, $(\alpha + \beta + \gamma) > 1$ then there exist a unique point $w \in X$ such that $Aw = Sw = w$ and a unique point $z \in X$ such that $Bz = Tz = z$. Moreover, $z = w$, so that there is a unique common fixed point of A, B, S and T.

Proof: Let the pairs {A, S} and {B, T} be owc, so there are points $x, y \in X$ such that $Ax = Sx$ and $By = Ty$. We claim that, $Ax = By$. If not, by inequality (3.2)

$$M(Ax, By, a, qt) \geq \alpha \min\{M(Sx, Ty, a, t), M(Sx, Ax, a, t)\}$$
$$+ \beta \min\{M(By, Ty, a, t), M(Ax, Ty, a, t)\}$$
$$+ \gamma M(By, Sx, a, t)$$

$$= \alpha \min\{M(Ax, By, a, t), M(Ax, Ax, a, t)\}$$
$$+ \beta \min\{M(By, By, a, t), M(Ax, By, a, t)\}$$
$$+ \gamma M(By, Ax, a, t)$$
$$= \alpha \min\{M(Ax, By, a, t), 1\} + \beta \min\{1, M(Ax, By, a, t)\}$$
$$+ \gamma M(By, Ax, a, t)$$
$$= \alpha M(Ax, By, a, t) + \beta M(Ax, By, a, t) + \gamma M(Ax, By, a, t)$$
$$= (\alpha + \beta + \gamma) M(Ax, By, a, t)$$

a contradiction, since $(\alpha + \beta + \gamma) > 1$. And by Lemma (2.12) Ax=By, i.e. $Ax = Sx = By = Ty$. Suppose that there is another point z such that $Az = Sz$ then by (3.2) we have $Az = Sz = By = Ty$. So, $Ax = Az$ and $w = Ax = Sx$ is the unique point of coincidence of A and S. By Lemma (2.17) w is the only common fixed point of A and S, i.e. $w = Aw = Sw$. Similarly there is a unique point $z \in X$ such that $z = Bz = Tz$.

Assume that $w \neq z$. We have

$$M(w, z, a, qt) = M(Aw, Bz, a, qt)$$
$$\geq \alpha \min\{M(Sw, Tz, a, t), M(Sw, Aw, a, t)\}$$
$$+ \beta \min\{M(Bz, Tz, a, t), M(Aw, Tz, a, t)\}$$
$$+ \gamma M(Bz, Sw, a, t)$$
$$= \alpha \min\{M(w, z, a, t), 1\} + \beta \min\{1, M(w, z, a, t)\}$$
$$+ \gamma M(z, w, a, t)$$
$$= (\alpha + \beta + \gamma) M(w, z, a, t)$$

a contradiction, since $(\alpha + \beta + \gamma) > 1$. And by Lemma (2.12) $z = w$. Also by Lemma (2.17) $z = w$ is the common fixed point of A, B, S and T. Therefore the uniqueness of the fixed point holds from (3.2).

Theorem 3.3 Let (X, M, $*$) be a complete fuzzy 2-metric space and Let A, B, S and T be self mappings of X. Let the pairs {A, S} and {B, T} be owc. If there exist $q \in (0,1)$ such that

$$M(Ax, By, a, qt) \geq \min\{M(Sx, Ty, a, t), M(Ax, Sx, a, t), M(Ty, Ax, a, t), M(By, Ty, a, t)), M(By, Sx, a, t)\} \quad (3.3)$$

For all x, y $\in$ X, and $t > 0$ then there exist a unique point $w \in X$ such that $Aw = Sw = w$ and a unique point $z \in X$ such that $Bz = Tz = z$. Moreover, $z = w$, so that there is a unique common fixed point of A, B, S and T.

Proof: Let the pairs {A, S} and {B, T} be owc, so there are points $x, y \in X$ such that $Ax = Sx$ and $By = Ty$. We claim that, $Ax = By$. If not, by inequality (3.3)

$$M(Ax, By, a, qt) \geq \min\{M(Sx, Ty, a, t), M(Ax, Sx, a, t), M(Ty, Ax, a, t), M(By, Ty, a, t),$$
$$M(By, Sx, a, t)\}$$
$$= \min\{M(Ax, By, a, t), M(Ax, Ax, a, t), M(By, Ax, a, t), M(By, By, a, t)$$
$$, M(By, Ax, a, t)\}$$
$$= \min\{M(Ax, By, a, t), 1, M(By, Ax, a, t), 1, M(By, Ax, a, t)\}$$
$$= M(Ax, By, a, t)$$

Therefore $Ax = By$, i.e. $Ax = Sx = By = Ty$. Suppose that there is another point z such that $Az = Sz$ then by (3.3) we have $Az = Sz = By = Ty$. So, $Ax = Az$ and $w = Ax = Sx$ is the unique point of coincidence of A and S. By Lemma (2.17) w is the only common fixed point of A and S, i.e. $w = Aw = Sw$. Similarly there is a unique point $z \in X$ such that $z = Bz = Tz$.

Assume that $w \neq z$. We have

$$M(w, z, a, qt) = M(Aw, Bz, a, qt)$$
$$\geq \min\{M(Sw, Tz, a, t), M(Aw, Sw, a, t), M(Tz, Aw, a, t), M(Bz, Tz, a, t),$$
$$M(Bz, Sw, a, t)\}$$
$$= \min\{M(w, z, a, t), M(w, w, a, t), M(z, w, a, t), M(z, z, a, t), M(z, w, a, t)\}$$
$$= M(w, z, a, t)$$

Therefore we have $z = w$ by Lemma (2.17) and z is the common fixed point of A, B, S and T. The uniqueness of fixed point holds from (3.3).

Theorem 3.4 Let (X, M, $*$) be a complete fuzzy 2-metric space and Let A, B, S and T be self mappings of X. Let the pairs {A, S} and {B, T} be owc. If there exist $q \in (0,1)$ such that

$$M(Ax, By, a, qt) \geq \phi\{M(Sx, Ty, a, t), M(Sx, Ax, a, t), M(By, Ty, a, t), M(Ax, Ty, a, t)$$
$$, M(By, Sx, a, t)\} \tag{3.4}$$

For all x, y $\in$ X, and $\phi : [0,1]^5 \to [0,1]$ such that $\phi(t,1,1,t,t) > t$ for all $0 < t < 1$ then there exist a unique common fixed point of A, B, S and T.

Proof: Let the pairs {A, S} and {B, T} be owc, there are points $x, y \in X$ such that $Ax = Sx$ and $By = Ty$. We claim that, $Ax = By$. If not, by inequality (3.4)

$$M(Ax,By,a,qt) \geq \phi\{M(Sx,Ty,a,t), M(Sx,Ax,a,t), M(By,Ty,a,t), M(Ax,Ty,a,t)$$
$$, M(By,Sx,a,t)\}$$
$$= \phi\{M(Ax,By,a,t), M(Ax,Ax,a,t), M(By,By,a,t), M(Ax,By,a,t)$$
$$, M(By,Ax,a,t)\}$$
$$= \phi\{M(Ax,By,a,t),1,1, M(Ax,By,a,t), M(By,Ax,a,t)\}$$
$$> M(Ax,By,a,t)$$

a contradiction, therefore by Lemma (2.12) Ax=By, i.e. $Ax = Sx = By = Ty.$ Suppose that there is another point z such that $Az = Sz$ then by (3.4) we have $Az = Sz = By = Ty.$ So, $Ax = Az$ and $w = Ax = Sx$ is the unique point of coincidence of A and S. By Lemma (2.17) w is the only common fixed point of A and S, i.e. $w = Aw = Sw.$ Similarly there is a unique point $z \in X$ such that $z = Bz = Tz.$

Thus z is a common fixed point of A, B, S and T. The uniqueness of the fixed point holds from (3.4).

Theorem 3.5 Let (X, M, $*$) be a complete fuzzy 2-metric space and Let A, B, S and T be self mappings of X. Let the pairs {A, S} and {B, T} be owc. If there exist $q \in (0,1)$ such that

$$M(Ax,By,a,qt) \geq \phi\{M(Sx,Ty,a,t), M(Ax,Ty,a,t), M(By,Sx,a,t)\} \tag{3.5}$$

For all x, y $\in$ X, and $\phi : [0,1]^3 \to [0,1]$ such that $\phi(t,t,t) > t$ for all $0 < t < 1$ then there exist a unique common fixed point of A, B, S and T.

Proof: The proof follows from Theorem 3.4

Theorem 3.6 Let (X, M, $*$) be a complete fuzzy 2-metric space and Let A, B, S and T be self mappings of X. Let the pairs {A, S} and {B, T} be owc. If there exist $q \in (0,1)$ such that

$$M(Ax,By,a,qt) \geq \phi\{\min\{M(Sx,Ty,a,t), M(Sx,Ax,a,t), M(By,Ty,a,t),$$
$$M(Ax,Ty,a,t), M(By,Sx,a,t)\} \tag{3.6}$$

For all x, y $\in$ X, and $\phi : [0,1] \to [0,1]$ such that $\phi(t) > t$ for all $0 < t < 1$ then there exist a unique common fixed point of A, B, S and T.

Proof: The Proof follows from Theorem 3.4.

Theorem 3.7 Let (X, M, $*$) be a complete fuzzy 2-metric space and Let A, B, S and T be self mappings of X. Let the pairs {A, S} and {B, T} be owc. If there exist a point $q \in (0,1)$ for all $x, y \in X$ and $t > 0$ such that

$$M(Ax, By, a, qt) \geq M(Sx, Ty, a, t) \tag{3.7}$$

then there exist a unique common fixed point of A, B, S and T.

Proof: Let the pairs {A, S} and {B, T} be owc, there are points $x, y \in X$ such that $Ax = Sx$ and $By = Ty$. We claim that, $Ax = By$. If not, by inequality (3.7)

$$M(Ax, By, a, qt) \geq M(Sx, Ty, a, t)$$
$$= M(Ax, By, a, t)$$

a contradiction, therefore by Lemma (2.12) Ax=By, i.e. $Ax = Sx = By = Ty$. Suppose that there is another point z such that $Az = Sz$ then by (3.7) we have $Az = Sz = By = Ty$. So, $Ax = Az$ and $w = Ax = Sx$ is the unique point of coincidence of A and S. By Lemma (2.17) w is the only common fixed point of A and S, i.e. $w = Aw = Sw$. Similarly there is a unique point $z \in X$ such that $z = Bz = Tz$.

Assume that $w \neq z$. We have

$$M(w, z, a, qt) = M(Aw, Bz, a, t)$$
$$\geq M(Sw, Tz, a, t)$$
$$= M(w, z, a, t)$$

Therefore we have $z = w$ by Lemma 2.17 and z is the common fixed point of A, B, S and T. The uniqueness of the fixed point holds from (3.7).

Theorem 3.8 Let (X, M, $*$) be a complete fuzzy 2-metric space and Let A and S be self mappings of X. Let A and S are owc. If there exist a point $q \in (0,1)$ for all $x, y \in X$ and $t > 0$ such that

$$M(Sx, Sy, a, qt) \geq \alpha_1 M(Ax, Ay, a, t) + \alpha_2 M(Sx, Ay, a, t) + \alpha_3 M(Sy, Ax, a, t) \tag{3.8}$$

for all $x, y \in X$, where $\alpha_1, \alpha_2, \alpha_3 > 0, (\alpha_1 + \alpha_2 + \alpha_3) > 1$, then there exist a unique common fixed point of A and S.

Proof: Let the pair {A, S} be owc, so there is a point $x \in X$ such that $Ax = Sx$. We claim that $Sx = Sy$. By inequality (3.8)

$$M(Sx, Sy, a, qt) \geq \alpha_1 M(Ax, Ay, a, t) + \alpha_2 M(Sx, Ay, a, t) + \alpha_3 M(Sy, Ax, a, t)$$
$$= \alpha_1 M(Sx, Sy, a, t) + \alpha_2 M(Sx, Sy, a, t) + \alpha_3 M(Sy, Sx, a, t)$$
$$= (\alpha_1 + \alpha_2 + \alpha_3) M(Sx, Sy, a, t)$$

a contraction, since $(\alpha_1 + \alpha_2 + \alpha_3) > 1$. Therefore $Sx = Sy$. Therefore $Ax = Ay$ and

Ax is unique. From Lemma 2.17, A and S have a unique fixed point.

Theorem 3.9 Let (X, M, $*$) be a complete fuzzy 2-metric space and Let A and S be self mappings of X. Let A and S are owc. If there exist a point $q \in (0,1)$ for all $x, y \in X$ and $t > 0$ such that

$$M(Sx, Sy, a, qt) \geq \alpha M(Ax, Ay, a, t) + \beta \min\{M(Ax, Ay, a, t), M(Sx, Ax, a, t), , M(Sy, Ay, a, t)\} \quad (3.9)$$

for all $x, y \in X$, where $\alpha, \beta > 0$, $(\alpha + \beta) > 1$. Then A and S have a unique common fixed point.

Proof: Let the pair {A, S} be owc, so there is a point $x \in X$ such that $Ax = Sx$. We claim that $Sx = Sy$. By inequality (3.9)

$$M(Sx, Sy, a, qt) \geq \alpha M(Ax, Ay, a, t) + \beta \min\{M(Ax, Ay, a, t), M(Sx, Ax, a, t), M(Sy, Ay, a, t)\}$$

$$= \alpha M(Sx, Sy, a, t) + \beta \min\{M(Sx, Sy, a, t), M(Sx, Sx, a, t), M(Sy, Sy, a, t)\}$$

$$= \alpha M(Sx, Sy, a, t) + \beta \min\{M(Sx, Sy, a, t), 1, 1\}$$

$$= (\alpha + \beta) M(Sx, Sy, a, t)$$

a contraction, since $(\alpha + \beta) > 1$. Therefore $Sx = Sy$. Therefore $Ax = Ay$ and Ax is unique. From Lemma 2.17, A and S have a unique fixed point.

Conclusion

In this paper we prove some fixed point theorems for a pair of occasionally weakly compatible mappings in fuzzy 2-metric space.

Acknowledgement

The authors are thankful to Prof. S. S. Pagey [Institute for Excellence in Higher Education, Bhopal (M. P)] for constant encouragement and helpful discussion in the presentation of this paper.

References

[1] C. T. Aage, J. N. Salunke,"On Fixed Point Theorems in Fuzzy Metric Spaces", International Journal of Pure and Applied Mathematics,2(3) , 2010, pp 123-131.

[2] C. T. Aage, J. N. Salunke, "Common Fixed Point Theorems in Fuzzy Metric Spaces", International Journal of Pure and Applied Mathematics, 56(2), 2009, pp 155-164.

[3] C. T. Aage, J. N. Salunke, "Some Fixed Point Theorems in Fuzzy Metric Spaces", International Journal of Pure and Applied Mathematics, 56(3), 2009, pp 311-320.
[4] C.T. Aage, J. N. Salunke,"On Fixed Point Theorems in Fuzzy Metric Spaces Using A Control Function", International Journal of Nonlinear Analysis and Applications, 6(2), 2010, pp 50-57.
[5] A. Al-Thagafi and Naseer Shahzad," Generalized I-Nonexpansive Selfmaps and Invariant Approximations, Acta Mathematica Sinica, English Series May, 2008, Vol. 24, No. 5, pp. 867876.
[6] P. Balasubramaniam, S. Muralisankar, R.P. Pant ,"Common fixed points of four mappings in a fuzzy metric space", J. Fuzzy Math. 10(2) (2002), 379-384.
[7] Y. J. Cho, H. K. Pathak, S. M. Kang, J. S. Jung," Common Fixed points of compatibility maps of type (A) on fuzzy Metric Spaces", Fuzzy Sets and Systems 93 (1998), 99-111.
[8] A.George, P. Veeramani," On some results in Fuzzy metric spaces", Fuzzy Sets and Systems, 64(1994), 395-399.
[9] M. Grabiec," Fixed points in fuzzy metric spaces", Fuzzy Sets and Systems, 27(1988), 385-389.
[10] O.Hadzic," Common Fixed Points Theorems for families of mapping in complete metric space", Math, Japan. 29 (1984), 127-134.
[11] Mohd. Imdad and Javid Ali," Some Common fixed point theorems in fuzzy metric spaces", Mathematical Communications 11(2006), 153-163.
[12] G. Jungck," Compatible mappings and common fixed points(2)", International. J. Math. Sci. (1988), 285-288.
[13] G. Jungck and B. E. Rhodes," Fixed Point for Set Valued functions without Continuity", Indian J. Pure Appl. Math., 29(3), (1998), pp.771-779.
[14] G.Jungk and B. E. Rhoades," Fixed Point Theorems for Occasionally Weakly Compatible Mappings", Fixed Point Theory, Vol 7, No. 2, 2006, 287-296.
[15] G. Jungck and B.E. Rhoades," Fixed Point Theorems for Occasionally Weakly compatible Mappings", Erratum, Fixed Point Theory, Vol 9, No. 1, 2008, 383-384.
[16] O. Kramosil and J. Michalek," Fuzzy metric and Statistical metric spaces", Kybernetika , 11(1975), 326-334.
[17] S. Kutukcu,"A fixed point theorem for contractive type mappings in Menger spaces", Am. J. App. Sci. 4(6) (2007), 371-373.
[18] Servet Kutukcu, Sushil Sharma and Hanifi Tokgoz," A Fixed Point Theorem in Fuzzy Metric Spaces", Int. Journal of Math. Analysis, Vol. 1, 2007, no. 18, 861-872.
[19] S. N. Mishra," Common fixed points of compatible mappings in Pm-spaces", Math. Japan. 36 (1991), 283-289.
[20] R. P. Pant," Common fixed points of four mappings", Bull, Cal. Math. Soc. 90 (1998), 281-286.
[21] R. P. Pant," Common fixed point theorems for contractive maps", J. Math. Anal. Appl. 226 (1998), 251-258.

[22] R. P. Pant, K. Jha," A remark on common fixed points of four mappings in a fuzzy metric space", J. Fuzzy Math. 12(2) (2004), 433-437.
[23] H. K. Pathak and Prachi Singh," Common Fixed Point Theorem for Weakly Compatible Mapping", International Mathematical Forum, 2, 2007, no. 57, 2831-2839.
[24] B. E. Rhoades," Contractive definitions and continuity", Contemporary Math. 72 (1988), 233-245.
[25] B. Schweizer and A. Sklar," Statistical metric spaces", Pacific J. Math. 10 (1960), 313-334.
[26] Seong Hoon Cho," On common fixed point in fuzzy metric space", Int. Math. Forum, 1, 2006, 10, 471-479.
[27] R. Vasuki, Common fixed points for R-weakly commuting maps in fuzzy metric spaces, Indian J. Pure Appl. Math. 30 (1999), 419-423.
[28] Gahler .S," 2-metriche reume und ihre tapologische structure", Math. Nauhr., 26 (1963/64) 115-148.
[29] Sanjay kumar, "Common fixed point theorem in fuzzy 2-metric spaces, Uni. Din. Bacau. Studii Si Cercetiri Sciintifice, Seria: Mathematica Nr 18(2008) 111-116.
[30] L. A. Zadeh, Fuzzy sets, Inform and Control 8 (1965), 338-353.