International Journal of Fuzzy Mathematics and Systems.
ISSN 2248-9940 Volume 3, Number 4 (2013), pp. 259-267

http://www.ripublication.com

Intuitionistic Fuzzy Soft N-ideals

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Abstract

The main purpose of this paper is to introduce a basic version of Intuitionistic Fuzzy Soft N-ideal theory, which extends the notion of ideals by introducing some algebraic structures in soft set, Finally we investigate some basic properties of maximal Intuitionistic Fuzzy Soft N-ideals.

Keywords: Soft set – Fuzzy soft set – Soft N-ideal – Intuitionistic Fuzzy set – Normal fuzzy set – maximal Intuitionistic Fuzzy Soft N-ideals.

Section 1: Introduction

The notion of a fuzzy set was introduced L.A. Zadeh [24] and since then this has been applied to various algebraic structures. The idea of "an intuitionistic fuzzy set" was introduced by K.T. Atanassov [3][4] as a generalization of the notion of fuzzy set. The concept Γ- near ring, a generalization of both the concepts near ring and Γ- ring was introduced by Satyanarayana [19]. Later several author such as Booth [6] and Satyanarayana [19] studied the real theory of Γ- near rings. Later Jun.et.al [8][9][10][11] considered the fuzzification of left (respectively right) ideals of Γ- near rings. In 1999 Molodtsav[17] proposed an approach for modeling vagueness and uncertainty called soft set theory. Since its inception works on Soft set theory has been applied to many different fields, such as function smoothness, Riemann integration, Pearson integration, Measurement theory, Game theory and decision making. Maji et.al [15] defined some operations on soft sets. Aktas and Naimcagman [1] generalized soft sets by defining the concept of soft groups. After them, Sun.et.al

[22] gave soft modules. Atagun and Sezgin [2] defined the concepts of soft sub rings of a ring, soft sub ideals of a field and soft submodules of a module and studied their relative properties with respect to soft set operations. Atagun and Sezgin [20] defined soft N-subgroups and soft N-ideals of an N-group. Naim cagman et.al [20] introduced the concept of union substructures of an near rings and N-subgroups. A.Solairaju et.al [21] studied the idea of union fuzzy soft n-group. In this paper, we investigate basic version of properties of maximal Intuitionistic Fuzzy Soft N-ideals and properties of maximal Intuitionistic Fuzzy Soft N-ideals.

Section 2: Preliminaries

In this section we include some elementary aspects that are necessary for this paper, from now on we denote a Γ- near ring and N is ideal unless otherwise specified.

Definition 2.1: A nonempty set R with two binary operations '+' (addition) and "." (multiplication) is called a Near-ring if it satisfies the following axioms.

- (R, +) is a group.
- (R, .) is a semigroup.
- $(x+y).\ z = x.\ z + y.z$ for all $x, y, z \in R$.

Definition 2.2: A Γ- near ring is a triple (M, +, Γ) where

- (M, +) is a group.
  - Γ is a nonempty set of binary operations on M such that for $\alpha\in\Gamma$, (M, +, α) is a near ring.
- $x\alpha(y\beta z) = (x\alpha y)\beta z$ for all $x, y, z \in M$ and $\alpha\in\Gamma$.

Definition 2.3: A subset A of a Γ- near ring M is called a left (respectively right) ideal of M if

- (A, +) is a normal divisor of (M, +).
- $u\alpha(x+v) - u\alpha v \in A$ (respectively $x\alpha u \in A$) for all $x \in A$, $\alpha\in\Gamma$ and $u, v \in M$

Definition 2.4: A fuzzy set A in a Γ- near ring m is called a fuzzy left (respectively right) ideal of M if

- $A(x-y) \geq \min\{ A(x), A(y) \}$
- $A(y+x-y) \geq A(x)$ for all $x, y \in M$
- $A(u\alpha(x+v) - u\alpha v) \geq A(x)$ for all $x, u, v \in M$ and $\alpha\in\Gamma$.

Definition 2.5: Let X be an initial universal Set and E be a set of parameters. A pair (F, E) is called a soft set over X if and only if F is a mapping from E into the set of all subsets of the set that is $F:E\rightarrow P(x)$ where P(x) is the power set of X.

Definition 2.6: The relation complement of the soft set F_A over U is denoted by $F_A^{\ r}$ where $F_A^{\ r}: A\rightarrow P(U)$ is a mapping given as $F_A^{\ r}(\alpha) = U/ F_A(\alpha)$ for all $\alpha\in A$.

Definition 2.7: Let X be a non-empty fixed μ_A: X→[0, 1] and ν_A: X→[0, 1] denote the degree of membership of each element x∈X to the set S respectively and $0 \leq \mu_A (x) + \nu_A(x) \leq 1$.

Notation: For the sake of simplicity, we shall use the symbol A = < $\mu_{A,}$ ν_A> for the IFS

A = {< x, $\mu_A(x)$, ν_A (x)> / x∈X}

Definition 2.8: Let A be an IFS in a Γ - near ring M, for each pair <α, β> ∈ [0, 1] with α+β ≤ 1, the set $A_{<\alpha, \beta>}$ = { x∈X / $\mu_A(x) \geq \alpha$ and $\nu_A (x) \leq \beta$} is called a <α, β> level subset of A.

Definition 2.9: Let F_A be a soft set over Γ. If for all x, y∈A, n∈N

- F(x-y) ≥ min{F(x), F(y)}
- F(y+x-y) ≥ F(x)
- F(n(y+x)-ny) ≥ F(x)

Then F_A is called a soft N-ideal of N.

Definition 2.10: Let U be an initial Universal Set and E be a set of parameters. A pair FS(U) denotes the fuzzy power set of U and A ⊂ E. A pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by F:A→P(U).

A fuzzy soft set is a parameterized family of fuzzy subsets of U.

Definition 2.11: An intutionistic fuzzy set A on Universe X can be defined as follows A = {< x, $\mu_A(x)$, ν_A (x)> / x∈X} where $\mu_A(x)$: X→[0, 1] and $\nu_A(x)$: X→[0, 1] with the property $0 \leq \mu_A (x) + \nu_A(x) \leq 1$, the value of μ_A (x) and $\nu_A(x)$ denote the degree of membership and non-membership of x to A, respectively. $\prod_A (x) = 1 - \mu_A(x) - \nu_A (x)$ is called the intutionistic fuzzy index.

Definition 2.12: Let U be an initial Universal Set and E be a set of parameters. A pair IFS(U) denotes the intutionistic fuzzy soft set over U and A ⊂ E. A pair (F, A) is called an intutionistic fuzzy soft set over U, where F is a mapping given by F:A→IFS(U).

Definition 2.13: An intutionistic fuzzy ideal A of M is called an intutionistic fuzzy soft N-ideal (IFSNI) of M if

- $\mu_A (x-y) \geq \min\{\mu_A(x), \mu_A(y)\}$
- $\mu_A(n(y+x-y) \geq \mu_A(x)$
- $\nu_A(x - y) \leq \max\{\nu_A(x), \nu_A(y)\}$
- $\nu_A(n(y+x)-ny) \leq \nu_A(x)$, for all x, y∈A, n∈N

Example 1: Let R be the set of all integers. Then R is a ring. Take M – Γ = R. Let a, b ∈ M, α∈Γ, Suppose a α b is the product of a, b, α∈R, then M is a Γ – near ring.

Define an IFS A = < $\mu_{A,}$ ν_A> in R as follows $\mu_A(0) = 1$ and $\mu_A(\pm 1) = \mu_A(\pm 2) = \mu_A(\pm 3) = \ldots.. = t$ and $\nu_A(0) = 1$ and $\nu_A(\pm 1) = \nu_A(\pm 2) = \nu_A(\pm 3) = \ldots.. = s$ where $t \in (0, 1)$ and $s \in (0, 1)$ and $t+s \leq 1$.

By routine calculations, clearly A is an intutionistic fuzzy soft N-ideal of M.

Example 2: Consider the additive group (Z_6, +) is a multiplication given in the following table. (Z_6, +) is a near ring.

.	0	1	2	3	4	5
0	0	0	0	0	0	0
1	3	1	5	3	1	5
2	0	2	4	0	2	4
3	3	3	3	3	3	3
4	0	4	2	0	4	2
5	3	5	1	3	5	1

But consider the N – ideal * = {0, 3}, we define soft set (g, *) by G(0)={1, 3, 5} and G(3)={1, 5} Since G(4.(2+3) - 4.2) = G(4.5 – 4.2) = G(2-2) = G(0) = {1, 3, 5} $\not\subseteq$ G(3)={1, 5}

Definition 2.14: Let M be a sub near ring of R and F_M be a soft set over N. If for all x, y $\in$ M

- $F(x-y) \geq \min\{F(x), F(y)\}$
- $F(nxy) \geq \min\{F(x), F(y)\}$ then the soft set F_M is called a soft N-near ring of R.

Definition 2.15: An IFSNI A of a Γ – near ring M is said to be a normal if $\mu_A(0)=1$, $\nu_A(0)=0$.

Section 3: Properties of Intutionistic fuzzy soft N-ideals

Proposition 3.1: Given an IFSNI A of a Γ – near ring M. Let A* be the IFS in M defined by $\mu_{A^*}(x) = \mu_A(x)+1 - \mu_A(0)$, $\nu_{A^*}(x) = \nu_A(x)+1 - \nu_A(0)$ for all $x \in M$ then A* is a normal IFSNI of M.

Proof: For all $x \in M$ use $\mu_{A^*}(x) = \mu_A(x)+1 - \mu_A(0) = 1$ and $\nu_{A^*}(x) = \nu_A(x) - \nu_A(0) = 0$

We have

$$\begin{aligned}
(\text{IFSNI}_1)\ \mu_{A^*}(x-y) &= \mu_A(x-y)+1 - \mu_A(0) \\
&\geq \min\{\mu_A(x), \mu_A(y)\}+1 - \mu_A(0) \\
&= \min\{\mu_A(x)+1- \mu_A(0), \mu_A(y) + 1 - \mu_A(0)\} \\
&= \min\{\mu_{A^*}(x), \mu_{A^*}(y)\}
\end{aligned}$$

(IFSNI$_2$) $\nu_{A*}(x\text{-}y) = \nu_A(x\text{-}y) - \nu_A(0)$
$\leq \max\{\nu_A(x), \nu_A(y)\} - \nu_A(0)$
$= \max\{\nu_A(x) - \nu_A(0), \nu_A(y) - \nu_A(0)\}$
$= \max\{\nu_{A*}(x), \nu_{A*}(y)\}$

(IFSNI$_3$) $\mu_{A*}(n(y+x)\text{-}ny) = \mu_A(n(y+x)\text{-}ny)+1 - \mu_A(0)$
$\geq \mu_A(x)+1 - \mu_A(0)$
$= \mu_{A*}(x)$

(IFSNI$_4$) $\nu_{A*}(n(y+x)\text{-}ny) = \nu_A(n(y+x)\text{-}ny) - \nu_A(0)$
$\geq \nu_A(x) - \nu_A(0)$
$= \nu_{A*}(x)$

Corollary: Let A and A* be as in Property 3.1, if there exists $x \in M$ such that A*(x)=0 then
A(x)=0.

Proposition 3.2: Let A be a IFSNI A of a Γ – near ring M and let $f:[0, \mu(0)] \rightarrow [0, 1]$, $g:[0, \nu(0)] \rightarrow [0, 1]$ are increasing functions. Then the IFS A_f: $M \rightarrow [0, 1]$ defined by $\mu_{Af} = f(\mu_A(x))$, $\nu_{Af} = f(\nu_A(x))$ is IFSNI of M.

Proof: Let $x, y \in M$ we have
$\mu_{Af}(x\text{-}y) = f(\mu_A(x\text{-}y))$
$\geq f \min\{\mu_A(x), \mu_A(y)\}$
$= \min\{f(\mu_A(x)), f(\mu_A(y))\}$
$= \min\{\mu_{Af}(x), \mu_{Af}(y)\}$

$\nu_{Af}(x\text{-}y) = f(\nu_A(x\text{-}y))$
$\leq f \max\{\nu_A(x), \nu_A(y)\}$
$= \max\{f(\nu_A(x)), f(\nu_A(y))\}$
$= \max\{\nu_{Af}(x), \nu_{Af}(y)\}$

$\mu_{Af}(n(y+x)\text{-}ny) = f(\mu_A(n(y+x)\text{-}ny)$
$\geq f(\mu_A(x))$
$= \mu_{Af}(x)$

$\nu_{Af}(n(y+x)\text{-}ny) = f(\nu_A(n(y+x)\text{-}ny)$
$\leq f(\nu_A(x))$
$= \nu_{Af}(x)$

Therefore A_f is an IFSNI of M.
If $f[\mu_A(0)] = 1$, then $\mu_{Af}(0)=1$ and $f[\nu_A(x)] = 0$ and $\nu_{Af}(x) = 0$ then A_f is normal.
Assume that $f(t) = f[\mu_A(x)] \geq \mu_A(x)$ and

$f(t) = f[\nu_A(x)] \leq \nu_A(x)$ for any $x \in M$ which gives $A \subseteq A_f$.

Proposition 3.3: Let $A \in N(M)$ is a non constant maximal element of $(N(M), \subseteq)$. Then A takes only the two values (0, 1) and (1, 0).

Proof: Since A is normal, we have $\mu_A(0)=1$ and $\nu_A(0) = 0$, and $\mu_A(x) \neq 1$ and $\nu_A(x) \neq 0$ for some $x \in M$, we consider that $\mu_A(0)=1$ and $\nu_A(0) = 0$, If not then $\exists\ x_0 \in M$ such that $0<\mu_A(x_0)<1$ and $0<\nu_A(x_0)<1$.

Define an IFS δ on M, by setting

$\mu_0(x) = [\mu_A(x\text{-}y)+ \mu_A(x_0)] / 2$

and

$\nu_0(x) = [\nu_A(x\text{-}y)+ \nu_A(x_0)] / 2$ for all $x \in M$

Then clearly δ is well defined and for all $x, y \in M$

We have

$$\begin{aligned}\mu_0(x-y) &= [\mu_A(x-y)+ \mu_A(x_0)] / 2\\ &\geq ½ \{ \min \{ \mu_A(x), \mu_A(y)\} + \mu_A(x_0)\}\\ &\geq \{ \min \{ [\mu_A(x)+ \mu_A(x_0)] / 2, [\mu_A(y)+ \mu_A(x_0)] / 2 \}\\ &\geq \min \{ \mu_A(x_0), \mu_A(y_0)\}\end{aligned}$$

$$\begin{aligned}\nu_0(x-y) &= [\nu_A(x-y)+ \nu_A(x_0)] / 2\\ &\geq ½ \{ \min \{ \nu_A(x), \nu_A(y)\} + \nu_A(x_0)\}\\ &\geq \{ \min \{ [\nu_A(x)+ \nu_A(x_0)] / 2, [\nu_A(y)+ \nu_A(x_0)] / 2 \}\\ &\geq \min \{ \nu_A(x_0), \nu_A(y_0)\}\end{aligned}$$

$$\begin{aligned}\mu_0(n(y+x)-ny) &= [\mu_A(n(y+x)-ny)+ \mu_A(x_0)] / 2\\ &\geq ½ \{ \mu_A(x) + \mu_A(x_0)\}\\ &= \mu_A(x)+ \mu_A(x_0) / 2\\ &= \mu_0(x)\end{aligned}$$

$$\begin{aligned}\nu_0(n(y+x)-ny) &= [\nu_0(n(y+x)-ny)+ \mu_A(x_0)] / 2\\ &\geq ½ \{ \nu_0(x) + \nu_0(x_0)\}\\ &= \nu_0(x)\end{aligned}$$

Therefore δ is an IFSNI of M.

Proposition 3.4: If $\{A_i / i\in I\}$ is a family of IFSNI on M, then $(\wedge_{i\in I} A_i)$ IFSNI of M.

Proof: Let $\{A_i / i\in I\}$ is a family of IFSNI on M.

For all $x, y \in M$, we have

$$\begin{aligned}(\wedge_{i\in I} A_i) (x-y) &= \inf \{ A_i (x-y) / i\in I\}\\ &\geq \inf \{ \min \{ A_i (x), A_i (y)\} / i\in I\}\\ &= \min \{ (\wedge_{i\in I} A_i) (x), (\wedge_{i\in I} A_i) (y)\}\end{aligned}$$

$(\vee_{i\in I} A_i)(x-y) = \sup\{ A_i(x-y) / i\in I\}$
$\leq \sup\{ \max\{ A_i(x), A_i(y)\} / i\in I\}$
$= \max\{ (\vee_{i\in I} A_i)(x), (\vee_{i\in I} A_i)(y)\}$

$(\wedge_{i\in I} A_i)(n(y+x)-ny) = \inf\{ A_i(n(y+x)-ny) / i\in I\}$
$\geq \inf\{ A_i(x) / i\in I\}$
$= (\wedge_{i\in I} A_i)(x)$

$(\vee_{i\in I} A_i)(n(y+x)-ny) = \sup\{ A_i(n(y+x)-ny) / i\in I\}$
$\leq \sup\{ A_i(x) / i\in I\}$
$= (\vee_{i\in I} A_i)(x)$

Hence $(\wedge_{i\in I} A_i)$ IFSNI of M.

Definition 3.1: An IFSNI A of M is said to be complete if it is normal and if there exists $x \in M$ such that $A(x)=0$.

Proposition 3.5: Let A be an IFSNI of M and let ω be a fixed element of M such that a fuzzy soft set A* in M by

$A^*(x) = [A(x) - A(\omega)] / [A(1) - A(\omega)]$ for all $x \in M$ then A* is an complete IFSNI of M.

Proof: For any $x, y \in M$, we have

$\mu A^*(x-y) = [A(x-y) - A(\omega)] / [A(1) - A(\omega)]$
$\geq [\min\{A(x), A(y)\} - A(\omega)] / [A(1) - A(\omega)]$
$= \min\{[A(x) - A(\omega)] / [A(1) - A(\omega)], [A(y) - A(\omega)] / [A(1) - A(\omega)]\}$
$= \min\{A^*(x), A^*(y)\}$

Also

$\nu A^*(x-y) = [A(x-y) - A(\omega)] / [A(1) - A(\omega)]$
$\leq [\max\{A(x), A(y)\} - A(\omega)] / [A(1) - A(\omega)]$
$= \max\{[A(x) - A(\omega)] / [A(1) - A(\omega)], [A(y) - A(\omega)] / [A(1) - A(\omega)]\}$
$= \max\{A^*(x), A^*(y)\}$

$\mu A^*(n(y+x)-ny) = [A(n(y+x)-ny) - A(\omega)] / [A(1) - A(\omega)]$
$\geq [A(x) - A(\omega)] / [A(1) - A(\omega)]$
$= A^*(x)$

$\nu A^*(n(y+x)-ny) = [A(n(y+x)-ny) - A(\omega)] / [A(1) - A(\omega)]$
$\leq [A(x) - A(\omega)] / [A(1) - A(\omega)]$
$= A^*(x)$

Therefore A* is a complete IFSNI of M.

Proposition 3.6: Let F_A be a soft set over M. Then A is a soft N-sub near ring of M if

A^r is a union soft N-near ring of M.

Proof: Let A is a soft N-sub near ring of M. Then for all x, y $\in$ M

$$F_A{}^r(x-y) = N/F_A(x-y)$$
$$\leq N/\min\{F_A(x), F_A(y)\}$$
$$= \max\{N/F_A(x), N/F_A(y)\}$$
$$= \max\{F_A{}^r(x), F_A{}^r(y)\}$$
$$= F_A{}^r(x) \cup F_A{}^r(y)$$

And

$$F_A{}^r(xy) = N/F_A(xy)$$
$$\leq N/\min\{F_A(x), F_A(y)\}$$
$$= \max\{N/F_A(x), N/F_A(y)\}$$
$$= \max\{F_A{}^r(x), F_A{}^r(y)\}$$
$$= F_A{}^r(x) \cup F_A{}^r(y)$$

Thus $F_A{}^r$ is a soft N-sub near ring of M.

Conclusion

This paper summarized the basic concepts of Intuitionistic fuzzy soft-ideals and soft N-sub near rings. By using these concepts, we studied the algebraic properties of IFSNI.

Future work

To extend this work one could study the property of IF soft sets in other algebraic structures such as groups and fields.

References:

[1] Aktas H, N.Cagman, Soft sets and soft groups, Inform. Sci. 177 (2007), 2726-2735.

[2] Atagun AO, Sezgin A, Aygun E (2011) A Note on soft near rings and idealistic soft near rings. Filomat 25(1):53-68

[3] Atanassov K.T, Remarks on the intuitionistic fuzzy sets-III. Fuzzy Sets Syst 75(1995), 401-402.

[4] Atanassov K.T, Some operations on intuitionistic fuzzy sets.Fuzzy SetsSyst 114(2000), 477-484.

[5] Aygunoglu A, H. Aygun, Introduction to fuzzy soft groups, Comput. Math. Appl. 58(2009), 1279-1286.

[6] Booth G.L, A note on Γ -near-rings Stud. Sci. Math. Hung. 23(1988), 471-475

[7] Chen D, E.C.C. Tsang, D.S. Yeung, X. Wang, The parameterization reduction of soft sets and its applications, Comput.Math.Appl.49(2005), 757- 763.
[8] Feng F, Y.B. Jun, X. Zhao, Soft semi rings, Comput. Math. Appl.(56)2008, 2621-2628.
[9] Jun Y.B, C.H. Park, Applications of soft sets in ideal theory of BCK/BCI algebras, Inform. Sci. 178(2008), 2466-2475.
[10] Jun Y.B, K.J. Lee, C.H. Park, Soft set theory applied to ideals in d-algebras, Comput. Math. appl. 2009 (57), 367-378.
[11] Jun Y.B, Soft BCK/BCI-algebras, Comput. Math. Appl. 56(2008), 1408-1413.
[12] Maji P.K, A.R. Roy, R. Biswas, An application of soft sets in a decision making problem, Comput. Math. Appl. 44(2002), 1077-1083.
[13] Maji P.K, R. Biswas, A.R. Roy, Fuzzy soft sets, J. Fuzzy Math.9(3)(2001), 589-602.
[14] Maji P.K, R. Biswas, A.R. Roy, Intuitionistic fuzzy soft sets, J. FuzzyMath. 9(3)(2001), 677-692.
[15] Maji P.K, R. Biswas, A.R. Roy, Soft set theory, Comput. Math. Appl.45(2003), 555-562.
[16] Majumdar P, S.K. Samanta, Generalised fuzzy soft sets, Comput. Math. Appl. 59 (4) (2010), 1425-1432.
[17] Molodtsov D, Soft set theory-First results, Comput. Math. Appl. 37(4-5)(1999), 19-31.
[18] Nagarajan R and N.Rajagopal, S-fuzzy version of soft N-groups, International Journal of Emerging Technology and Advanced Engineering, Vol.3, No.5 (2013), PP 322-325.
[19] Satyanarayana, Contributions to near-ring theory, Doctoral Thesis, Nagarjuna Univ.1984.
[20] Sezgin, A., Atagun, A.O. and Cagman N. Union soft substructures of near-rings and N-groups, Neural Computing and Applications 21, 133–143, 2012.
[21] Solairaju A, P.Sarangapani, R.Nagarajan, Union fuzzy soft N-groups, International Research Journal of Pure Algebra, Vol.3, No.6(2013), PP 242-246.
[22] Qiu-Mei Sun, Zi-Liong Zhang and Jing Liu, Soft sets and soft modules, Lecture Notes in Comput. Sci. 5009 (2008) 403{409.
[23] Xu W, J. Ma, S.Y. Wang, G. Hao, Vague soft sets and their properties, Comput. Math. Appl. (59)2010, 787-794.
[24] Zadeh L.A, ”Fuzzy Sets”, Information and Control Vol.8, (1965).pp.338-353.
[25] Zhan J, Y.B. Jun, Soft BL-algebras based on fuzzy sets, Comput. Math.Appl. 59(2010), 2037-2046.