International Journal of Fuzzy Mathematics and Systems.
ISSN 2248-9940 Volume 3, Number 4 (2013), pp. 275-283

http://www.ripublication.com

On Fuzzy σ-Baire Spaces

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Abstract

In this paper the concepts of fuzzyσ-Baire spaces are introduced and characterizations of fuzzy σ-Baire spaces are studied. Several examples are given to illustrate the concepts introduced in this paper.

KEYWORDS: Fuzzy F_σ-set, fuzzyG_δ-set, fuzzy nowhere dense set, Fuzzyσ-nowhere denses et, Fuzzy σ-first category, Fuzzy σ-second category and Fuzzyσ-Baire spaces,

2000 AMS Classification: 54A40, 03E72.

INTRODUCTION.

The fuzzy concept has invaded almostall branchesofMathematicsever since the introduction offuzzy setby L. A. ZADEH [12] . Thetheoryof fuzzy topologicalspaces was introducedand developed by C. L. CHANG [3] . Since thenmuchattention has been paid togeneralize thebasicconcepts of GeneralTopologyinfuzzy settingand thus a modern theoryof fuzzy topology hasbeen developed. Inrecentyears, fuzzy topology has been foundto be very useful insolvingmanypractical problems. In [5] El. Naschieshowed that thenotion offuzzy topology mightbe relevant to Quantum Particle Physics in connection with String Theory. It has been shown that the fuzzyKähler manifoldswhich are based on a topology, play the important rule in $\in^\infty$theory. In this paper we introduce the concepts of fuzzy σ-Baire spaces. Alsowe discussseveralcharacterizationsof fuzzyσ-Bairespaces.

Several examples are given to illustrate theconcepts introduced inthis paper.

2. PRELIMINARIES

Byafuzzy topologicalspace we shall meana non –emptyset Xtogetherwitha fuzzy topologyT (in the sense of Chang) and denote it by (X, T).

DEFINTION 2. 1: Let λandμ beany twofuzzy sets in (X, T). Then wedefine$\lambda \vee \mu$: X $\to$ [0, 1] as follows: $(\lambda \vee \mu)$ (x) = Max { λ (x), μ (x) }. Alsowe define $\lambda \wedge \mu$: X $\to$ [0, 1] as follows: $(\lambda\wedge\mu)$ (x) =Min { λ(x), μ (x)}.

DEFINTION 2. 2`` [1] . : Let (X, T) be any fuzzy topological space and λbe any fuzzy setin (X, T). We define Cl(λ)=$\wedge${ μ / $\lambda \le \mu$, 1–$\mu\in$ T}and int(λ)=$\vee${ μ / $\mu \le \lambda$, $\mu\in$ T}.

For anyfuzzy setina fuzzy topological space(X, T), itiseasy to see that 1— cl (λ)=int (1 — λ)and 1 – int(λ)=cl (1 — λ)

DEFINTION 2. 3: Let (X, T) and (Y, S) be any two fuzzy topological spaces. Letfbe a function from the fuzzy topological space (X, T) tothe fuzzy topological space (Y, S).

Let λbea fuzzy set in (Y, S). The inverse imageof λ underf written as$f^{-1}(\lambda)$ is the fuzzy set in(X, T) defined by $f^{-1}(\lambda)$ (x)=λ (f (x)) for all x $\in$X. Also the image of λin (X, T) under f written as f (λ) is the fuzzy set in(Y, S)defined by

$$f(\lambda)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \lambda(x) & if\ f^{-1}(y) \text{ is non} - empty; \\ 0 & otherwise. \end{cases} \quad \text{for each } y\in Y.$$

Lemma2. 1 [3] : Letf: (X, T) $\to$(Y, S) be amapping. Forfuzzy setsλ and$\mu$of (X, T) and (Y, S) respectively, the following statementshold:

1. $ff^{-1}(\mu) \le \mu$;
2. $f^{-1}f(\lambda) \ge \lambda$;
3. $f(1 - \lambda) \ge 1 - f(\lambda)$;
4. $f^{-1}(1 - \mu) = 1 - f^{-1}(\mu)$;
5. Iffis one-to-one, then $f^{-1} f(\lambda)=\lambda$;
6. If f is onto, then $f f^{-1}(\mu)= \mu$;
7. If f isone-to-one andonto, then $f(1 - \lambda)=1 - f(\lambda)$.

DEFINTION 2. 4 [8] : A fuzzyset λ in a fuzzytopological space (X, T) is calledfuzzy dense if there existsno fuzzy closedsetμ in (X, T) such that$\lambda < \mu < 1$.

DEFINITION 2. 5 [3] : A fuzzy set λ in a fuzzy topologicalspace (X, T) is called a fuzzy F_σ-set in(X, T) if $\boldsymbol{\lambda} = \bigvee_{i=1}^{\infty} \boldsymbol{\lambda_i}$, where$1 - \lambda_i \in$ T for i$\in$I.

DEFINITION 2. 6 [3] : A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzyG_δ-set in (X, T) if$\boldsymbol{\lambda} = \bigwedge_{i=1}^{\infty} \boldsymbol{\lambda_i}$, where$\lambda_i \in$ T for i$\in$I.

DEFINITION 2. 7 [1] : Afuzzyset λinafuzzytopologicalspace(X, T) is called a fuzzy semi-open if $\lambda \leq$ clint(λ). The complementof λ in (X, T)iscalledafuzzy semi-closed set in (X, T).

DEFINITION 2. 8 [10] : A fuzzytopologicalspace (X, T)is called an fuzzy open hereditarilyirresolvable space ifint cl (λ)≠0, then int(λ)≠ 0 for any non-zero fuzzy set in (X, T) .

DEFINITION 2. 9. [8] : A fuzzy topological space (X, T) is called fuzzy first category if the fuzzy set $\mathbf{1_X}$ is a fuzzy first category set in (X, T). That is, $1_X = \bigvee_{i=1}^{\infty}(\lambda_i)$where λ_i 's are fuzzynowhere dense sets in (X, T). Otherwise, (X, T) will be called a fuzzy second category space.

3. FUZZY σ-NOWHERE DENSESETS

Motivated by the classical concept introduced in [5] we shall now define:

DEFINITION 3. 1: Let(X, T) beafuzzy topological space. A fuzzy set λin (X, T) is called a fuzzyσ-nowhere dense setifλ is a fuzzy F_σ-set in (X, T) suchthat int (λ) = 0.

EXAMPLE 3. 1: Let X ={a, b, c}. The fuzzy sets λ, μ and υare definedon X as follows:

- λ: X → [0, 1] isdefined asλ (a) =0. 3; λ(b)=0. 7; λ (c)=0. 4.
- μ: X → [0, 1] isdefined as μ (a)=0. 5; μ (b) = 0. 4;μ (c) =0. 8.
- υ: X → [0, 1] isdefinedasυ(a) =0. 2; υ(b) =0. 4; υ (c) =0. 6.

Then, T = {0, λ, μ, υ, λ ∨ μ, λ ∨υ, λ ∧ μ, λ ∧υ, υ∨ (λ ∧ μ), 1} is clearly a fuzzy topology on X. Now consider the fuzzy setδ= [1 − (λ ∨ μ)] ∨ [1 − (λ ∨υ)] in (X, T). Then δ is a fuzzy F_σ-set in (X, T)and int (λ)=0 and henceδis a fuzzy σ-nowhere dense set in (X, T). The fuzzy set β = (1 −λ)∨(1 −μ) ∨(1 −υ)is a fuzzy F_σ-set in (X, T) and int (λ)≠ 0 and hence β isnot a fuzzy σ-nowhere dense set in (X, T).

REMARKS 3. 1: If λ and μare fuzzyσ-nowhere dense sets, then λ ∨ μ need not be a fuzzyσ-nowhere dense set in (X, T). For, consider the following example:

EXAMPLE 3. 2: Let X = {a, b, c}. The fuzzy sets λ, μ andυare definedon X as follows:

- λ: X → [0, 1] defined as λ (a) = 0. 6 ; λ(b)= 0. 7 ; λ (c) = 0. 5.
- μ: X → [0, 1] defined as μ (a) = 0. 5 ; μ (b) = 0. 4 ; μ (c) =0. 8.
- υ: X → [0, 1] defined as υ (a) = 0. 7 ; υ (b) = 0. 6 ; υ (c) =0. 5 .

Now α = [(1 – μ)∨{1 – (λ ∨ μ)}∨ {1 – (μ∨υ) }] and β = [(1 –λ)∨(1 –υ)] are fuzzy F_σ-sets in (X, T). Also int (α) = 0 and int (β) = 0. Therefore α and β are fuzzy σ-nowhere dense sets in (X, T). But α∨ βisnot afuzzyσ – nowhere dense set in (X, T), since $\alpha\vee\beta$ isa fuzzy F_σ-set in (X, T)and int $(\alpha\vee\beta) = \mu\wedge\upsilon \neq 0$.

PROPOSITION 3. 1: In a fuzzy topological space (X, T)a fuzzy set λ isfuzzy σ-nowhere dense in (X, T) if and only if $1-\lambda$ is a fuzzy dense and fuzzy G_δ-set in (X, T).

PROOF: Let λ bea fuzzy σ-nowhere dense set in (X, T). Then $\boldsymbol{\lambda} = \vee_{i=1}^{\infty}(\boldsymbol{\lambda}_i)$where $1-\lambda_i \in T$, for $i\in I$ and int $(\lambda) = 0$. Then $1 - \text{int}(\lambda) = 1 - 0 = 1$ implies thatcl $(1-\lambda) = 1$.

Also$(1-\lambda) = 1 - \vee_{i=1}^{\infty}(\boldsymbol{\lambda}_i) = \wedge_{i=1}^{\infty}(1-\lambda_i)$where$1-\lambda_i \in T$, for $i\in I$. Hence we have $1-\lambda$ is a fuzzy dense and fuzzy G_δ-set in (X, T).

Conversely, let λ be a fuzzy dense and fuzzy G_δ-set in (X, T). Then $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$where$\lambda_i \in T$, for $i\in I$. Now $(1-\lambda) = 1 - \wedge_{i=1}^{\infty}(\lambda_i) = \vee_{i=1}^{\infty}(1-\boldsymbol{\lambda}_i)$.

Hence $1-\lambda$ is a F_σ-set in (X, T) and int $(1-\lambda) = 1 - \text{cl}(\lambda) = 1 - 1 = 0$. [since λ is a fuzzy dense] . Therefore $1-\lambda$ is a fuzzy σ-nowhere dense set in (X, T).

PROPOSITION 3. 2: If λ is a fuzzy dense set in (X, T) such that $\mu \leq (1-\lambda)$, where μ is a fuzzy F_σ-set in (X, T), then μ is a fuzzyσ-nowhere dense set in (X, T).

PROOF: Let λ be a fuzzy dense set in (X, T) such that $\mu \leq (1-\lambda)$. Now $\mu \leq (1-\lambda)$ implies that $\text{int}(\mu) \leq \text{int}(1-\lambda) = 1 - \text{cl}(\lambda) = 1 - 1 = 0$ and hence $\text{int}(\mu) = 0$.

Therefore μ is a fuzzy σ-nowhere dense set in (X, T).

PROPOSITION 3. 3: If λ is a fuzzy F_σ-set and fuzzy nowhere dense set in (X, T), then$\lambda$is a fuzzyσ-nowhere dense set in (X, T).

PROOF: Now $\lambda \leq \text{cl}(\lambda)$ for any fuzzy set in (X, T). Then, int $(\lambda) \leq$ intcl (λ). Since λ is a fuzzy nowhere dense set in (X, T), intcl $(\lambda) = 0$ and hence $\text{int}(\lambda) = 0$ andλ is a fuzzy F_σ-set implies that $\lambda$is a fuzzy σ-nowhere dense set in (X, T).

REMARKS 3. 2: If λ is a fuzzy F_σ-set and fuzzy σ-nowhere dense set in (X, T), thenλ need not be a fuzzynowhere dense set in (X, T). For, consider the following example:

EXAMPLE 3. 3: Let X = {a, b, c}. The fuzzy sets λ, μ and$\upsilon$are definedon X as follows:

- $\lambda: X \to [0, 1]$ defined as $\lambda(a) = 0.8$; $\lambda(b) = 0.6$; $\lambda(c) = 0.7$.
- $\mu: X \to [0, 1]$ defined as $\mu(a) = 0.6$; $\mu(b) = 0.9$; $\mu(c) = 0.8$.
- $\upsilon: X \to [0, 1]$ defined as $\upsilon(a) = 0.7$; $\upsilon(b) = 0.5$; $\upsilon(c) = 0.9$.

Then $T = \{0, \lambda, \mu, \upsilon, \lambda\vee\mu, \lambda\vee\upsilon, \mu\vee\upsilon, \lambda\wedge\mu, \lambda\wedge\upsilon, \mu\wedge\upsilon, \lambda\vee(\mu\wedge\upsilon), \mu\vee(\lambda\wedge\upsilon), \upsilon\vee(\lambda\wedge\mu), 1\}$ is a fuzzy topology on X. Now the fuzzy set $\eta = (1-\mu)\vee(1-[\lambda\wedge\upsilon])\vee(1-\upsilon)$ and int $(\eta) = 0$ and hence η is a fuzzy F_σ-set and fuzzy σ-nowhere dense set in (X, T) but η is not a fuzzynowhere dense set in (X, T) since int $\text{cl}(\eta) \neq 0$.

PROPOSITION 3. 4: If(X, T) is a fuzzy open hereditarily irresolvable space, any fuzzy σ-nowhere dense set in (X, T) is a fuzzy nowhere dense set in (X, T).

PROOF: Let λ be a fuzzy σ-nowhere dense set in an fuzzy open hereditarily irresolvable space (X, T). Thenλ is a fuzzy F_σ-set in (X, T) suchthat int (λ) =0. Since(X, T) is anfuzzy open hereditarily irresolvable space, int(λ)= 0 implies that int cl(λ) =0. Henceλ is a fuzzy nowhere dense set in (X, T).

DEFINITION 3. 2: Let(X, T) be a fuzzy topological space. A fuzzy set λ in (X, T) is called fuzzy σ-first category if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ where $\lambda_i's\ are$ fuzzyσ-nowheredense sets in (X, T). Any otherfuzzy set in (X, T) is said to be fuzzy σ-second categoryin (X, T) .

DEFINITION 3. 3: Let λ be a fuzzy σ-first category set in (X, T). Then 1 – λ is called a fuzzy σ-residual set in (X, T).

DEFINITION 3. 4: A fuzzy topological space (X, T) is called fuzzy σ-first category if the fuzzy set $\mathbf{1_X}$ is a fuzzy σ-first category set in (X, T). That is, $1_X = \vee_{i=1}^{\infty}(\lambda_i)$where λ_i 's are fuzzyσ-nowhere dense sets in (X, T). Otherwise, (X, T) will be called a fuzzy σ-second category space.

4. FUZZY σ-BAIRE SPACE

Motivated by the classical concept introduced in [6] we shall now define:

DEFINITION 4. 1: Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy σ-Baire Spaceif int $(\vee_{i=1}^{\infty}\ (\lambda_i)) = 0$ where $\lambda_i's\ are$ fuzzy σ-nowheredensesets in (X, T).

EXAMPLE 4. 1: Let X = {a, b, c}. The fuzzy sets λ, μ andυ are defined on X as follows:

- λ: X → [0, 1] is defined as λ (a)= 0. 8; λ(b) = 0. 6 ; λ (c) = 0. 7.
- μ: X → [0, 1] isdefined as μ (a) = 0. 6 ; μ (b) =0. 9; μ (c) = 0. 8.
- υ: X → [0, 1] is defined as υ (a) = 0. 7 ; υ (b) = 0. 5 ; υ (c) =0. 9.

Then T = {0, λ, μ, υ, λ ∨ μ, λ ∨υ, μ ∨υ, λ ∧ μ, λ ∧υ, μ ∧υ, λ∧(μ ∨υ), λ∨ (μ ∧υ), μ∧ (λ ∨υ), μ∨(λ ∧υ), υ∧ (λ ∨ μ), υ∨ (λ ∧ μ), 1} is a fuzzy topology on X. Now

- α= (1 –λ)∨(1 – [λ∨ (μ ∧υ)]) ∨ [1 – (λ ∨ μ∨υ)] and int (α) = 0.
- β = (1 –μ) ∨(1 – [λ∨υ]) ∨ [1 – (μ∨(λ ∧υ))] and int (β) = 0.
- = (1 –υ) ∨(1 – [λ∨μ]) ∨ [1 –(λ ∧μ)] ∨(1 – [μ ∨υ]) and int (δ) = 0
- η= (1 – [λ ∧υ]) ∨(1 – [μ ∧υ]) ∨ [1 –(λ ∧ [μ ∨υ] , ∨(1 – [μ∧(λ ∨υ)]) and int (η) = 0.

Then α, β, δ and η are fuzzy σ-nowhere dense sets in (X, T) and also int $(\alpha\vee\beta\vee\eta\vee\eta) = 0$ and therefore (X, T) is a fuzzy σ-Baire Space.

PROPOSITION4. 1: Let (X, T) be a fuzzy topological space. Then the following are equivalent:

1. (X, T) is a fuzzy σ-Baire space.
2. Int $(\lambda) = 0$ for every fuzzy σ-first category set λ in (X, T).
3. cl $(\mu) = 1$ for every fuzzy σ-residual set μ in (X, T).

PROOF: (1) $\Rightarrow$(2). Let λ be a fuzzy σ-first category set in (X, T). Then $\lambda = (\vee_{i=1}^{\infty}(\lambda_i))$ where λ_i's are fuzzy σ-nowhere dense sets in (X, T). Then, we have int $(\lambda)=\text{int}(\vee_{i=1}^{\infty}(\lambda_i))$.

Since (X, T) is a fuzzy σ-Baire space, $\text{int}(\vee_{i=1}^{\infty}(\lambda_i)) = 0$. Hence $\text{int}(\lambda)= 0$ for any fuzzy σ-first category set λ in (X, T).

(2) $\Rightarrow$ (3). Let μ be a fuzzy σ-residual set in (X, T). Then $(1 - \mu)$ is a fuzzy σ – first category set in (X, T). By hypothesis, int $(1 - \mu) = 0$. Then $1 - \text{cl}(\lambda) = 0$. Hence cl $(\lambda) = 1$for any fuzzy σ-residual set μ in (X, T).

(3) $\Rightarrow$(1). Let λ be a fuzzy σ-first category set in (X, T). Then $\lambda = (\vee_{i=1}^{\infty}(\lambda_i))$ where λ_i's are fuzzy σ-nowhere dense sets in(X, T). Now λ is afuzzy first σ-category set in (X, T) implies that $(1 - \lambda)$ is a fuzzyσ-residual set in(X, T). By hypothesis, we have cl $(1 - \lambda) = 1$. Then $1 - \text{int}(\lambda) = 1$. Henceint $(\lambda) = 0$. Thatis, $\text{int}(\vee_{i=1}^{\infty}(\lambda_i)) = 0$ where λ_i's are fuzzy σ – nowheredense sets in (X, T). Hence (X, T) is a fuzzy σ-Bairespace.

PROPOSITION4. 2: If $cl(\wedge_{i=1}^{\infty}(\lambda i)) = 1$ where λ_i's arefuzzy dense G_δ-sets in (X, T), then (X, T) is a fuzzy σ-Baire space.

PROOF: Now $cl(\wedge_{i=1}^{\infty}(\lambda_i)) = 1$ implies that $1 - cl(\wedge_{i=1}^{\infty}(\lambda_i)) = 0$. Then we have int $(1 - (\wedge_{i=1}^{\infty}(\lambda_i)) = 0$, which implies that int $(\vee_{i=1}^{\infty} (1 - \lambda_i)= 0$. Let$\mu_i= 1 -\lambda_i$.

Then $\text{int}(\vee_{i=1}^{\infty}(\mu_i))= 0$. Since $\lambda_i$is a fuzzy dense G_δ-setin(X, T), by proposition 3. 1, $1 -\lambda_i$ isa fuzzy σ-nowhere dense set in(X, T). Henceint$(\vee_{i=1}^{\infty}(\mu_i))= 0$, where $\mu_i's$ are fuzzy σ-nowhere dense setsin(X, T). Therefore(X, T) is a fuzzy σ-Bairespace.

PROPOSITION 4. 3: If the fuzzy topological space (X, T) is a fuzzyσ-Baire space, then (X, T) is a fuzzyσ-second category space.

PROOF: Let (X, T) be a fuzzyσ-Baire space. Then $\text{int}(\vee_{i=1}^{\infty}(\lambda_i)) = 0$ where λ_i's are fuzzy σ-nowhere dense sets in (X, T). Then$\vee_{i=1}^{\infty}(\lambda_i) \neq 1_x$ [Other wise, $\vee_{i=1}^{\infty}(\lambda_i) = 1_x$ implies thatint $(\vee_{i=1}^{\infty}(\lambda_i)) = \text{int } 1_x = 1_x$, which implies that 0 = 1, a contradiction] . Hence (X, T) is a fuzzyσ-second category space.

REMARKS: The converse of the above proposition need not be true. A fuzzy σ-second category space need not be a fuzzy σ-Baire space. For, consider the following example:

EXAMPLE4. 2: Let X = {a, b, c} and λ, μ, υ be the fuzzy sets defined on X as follows:

- λ: X → [0, 1] isdefined as λ (a) = 0. 5; λ(b) = 0. 6 ; λ (c)= 0. 7.
- μ: X → [0, 1] isdefined as μ (a) = 0. 8 ; μ (b) = 0. 4; μ (c) =0. 5.
- υ: X → [0, 1] isdefined as υ(a) = 0. 7 ; υ(b) = 0. 5 ; υ (c) =0. 8

Then T = { 0, λ, μ, υ, λ ∨ μ, λ∨υ, μ∨υ, λ∧μ, λ ∧υ, μ∧υ, λ ∨(μ∧υ), μ∨ (λ ∧υ), υ∧ (λ ∨ μ), λ ∨ μ∨υ, 1 }is a fuzzy topology on X. Now the fuzzy set α = { (1 – μ) ∨ (1-υ)∨ (1 – (λ∨ μ))∨(1 – (λ∨υ))∨ (1 – (μ∨υ))∨(1 – (λ∨μ∨υ)}, is a fuzzy F_{σ}-set in (X, T) and int (α) = 0 and hence is a fuzzy σ-nowhere dense set in (X, T).

Also β={ (1 –λ) ∨(1 – [λ ∧υ]) ∨(1 – (λ∨ [μ∧υ]))∨((1 – [μ∨ (λ ∧υ)])∨(1 – [υ∧ (λ∨μ)])} Is a fuzzy F_{σ}-set in (X, T) and int (β) = 0 and hence is a fuzzy σ-nowhere dense set in (X, T). Now (α∨β) ≠ 1_X . Therefore (X, T) is a fuzzyσ-second category space. But int (α∨β) =λ∧μ≠0 and therefore (X, T) is not a fuzzyσ-Baire space.

REMARKS: A fuzzy σ-Baire spaceneed not be a fuzzyBairespace. For, consider the following example:

EXAMPLE 4. 3: Let X = {a, b, c}. The fuzzy sets λ, μ andυaredefinedon X as follows:

- λ: X → [0, 1] is defined as λ(a) = 1; λ(b)= 0. 2 ; λ (c) = 0. 7.
- μ: X → [0, 1] is defined as μ(a) = 0. 3 ; μ (b) = 1; μ (c) =0. 2.
- υ: X → [0, 1] is defined as υ (a) = 0. 7 ; υ(b) = 0. 4 ; υ(c)=1.

Then T = { 0, λ, μ, υ, λ ∨ μ, λ∨υ, μ∨υ, λ∧μ, λ ∧υ, μ∧υ, μ∨ (λ ∧υ), υ∨ (λ∧μ), υ∧ (λ ∨ μ), λ ∨ μ∨υ, 1 }is a fuzzy topology on X. Now 1 – λ, 1 – μ, 1 – υ, 1 – (λ ∨ μ), 1 – (λ∨υ), 1– (μ∨υ), 1– (λ ∧υ), 1– [μ∨ (λ ∧υ)] , 1– [υ∨ (λ∧μ)] , 1– [υ∧ (λ ∨ μ)] arefuzzy nowhere dense sets in (X, T). 1 – (λ∧μ) = {(1 – λ) ∨ (1 – μ) ∨ (1-υ)∨ (1 – (λ∨ μ))∨(1 – (λ∨υ))∨ (1 – (μ∨υ))∨(1– [μ∨ (λ ∧υ)])∨ (1– [υ∨ (λ∧μ)])∨ (1– [υ∧(λ ∨ μ)])} . Therefore, 1 – (λ∧μ) is a fuzzy first categoryset in(X, T). But int (1 – (λ∧μ)) = λ∧μ ≠0. Hence (X, T) is not a fuzzy Baire Space. Now α= (1 –υ) ∨(1 – [λ∨υ]) ∨ [1 – (μ∨υ)] and int (α) = 0. β = (1 –λ) ∨(1 – [λ∨μ]) ∨ [1 – (λ ∧υ)] ∨(1– [μ∨ (λ ∧υ)] ∨ [1– [υ∨ (λ∧μ)] and int (β) = 0. Hence α and β arefuzzy σ-nowhere dense sets in (X, T). Now the fuzzy (α∨ β)is a fuzzyσ-first categoryset in (X, T)and int ((α∨ β)=0. Hence (X, T) is a fuzzy σ-Baire space.

PROPOSITION 4. 4: If the fuzzy topologicalspace (X, T) is a fuzzyσ-Baire and fuzzy open hereditarilyirresolvabespace, then (X, T) is a fuzzyBaire Space.

PROOF: Let (X, T)be a fuzzyσ-BaireSpace and fuzzyopen hereditarily irresolvabe space. Then, $\mathrm{int}(\vee_{i=1}^{\infty}(\lambda_i)) = 0$where λ_i's are fuzzy σ-nowhere dense sets in (X, T).

By proposition 3. 4, λ_i' s are fuzzynowhere dense sets in (X, T). Hence, $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where λ_i' s are fuzzy nowhere dense sets in (X, T). Therefore (X, T) is a fuzzyBaire Space.

PROPOSITION 4. 5: Ifthe fuzzy topologicalspace (X, T) is a fuzzy BaireSpace and if the fuzzy nowhere dense sets in (X, T) are fuzzy F_σ-sets in (X, T), then (X, T) is a fuzzyσ-Baire Space.

PROOF: Let (X, T)be a fuzzyBaireSpacesuch that every fuzzy nowhere dense set $\lambda_i$is a fuzzy F_σ-set in (X, T). Then, $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$ where λ_i's are fuzzy nowhere dense sets in (X, T). By proposition 3. 3, $\lambda_i$is a fuzzyσ-nowhere dense set in (X, T). Henceint$(\vee_{i=1}^{\infty}(\lambda_i)) = 0$ where λ_i's are fuzzy σ-nowhere dense set in (X, T). Therefore (X, T) is a fuzzyσ-Baire Space.

PROPOSITION 4. 6: Let (X, T) be a fuzzy topologicalspace. If$\wedge_{i=1}^{\infty}(\lambda i) \neq 0$, where λ_i's are fuzzy dense and fuzzy G_δ-sets in (X, T), then (X, T) is a fuzzy σ-second category space.

PROOF: Now$\wedge_{i=1}^{\infty}(\lambda_i) \neq 0$ implies that $1 - (\wedge_{i=1}^{\infty}(\lambda_i)) \neq 1 - 0 = 0$. Then we have $(\vee_{i=1}^{\infty}(1 - \lambda_i)) \neq 1$. Since $\lambda_i$is a fuzzy dense and fuzzy G_δ-setin(X, T), by proposition 3. 1, $1 - \lambda_i$ is a fuzzy σ-nowhere dense set in(X, T). Hence $\vee_{i=1}^{\infty} (1 - \lambda_i) \neq 1$, where $(1 - \lambda_i)$'s are fuzzy σ –nowheredense sets in(X, T). Hence (X, T) is not a fuzzy σ –first category space. Therefore (X, T) is a fuzzy σ-second category space.

5. FUNCTIONSAND FUZZYσ-BAIRE SPACES

DEFINITION 5. 1 [1] : A functionf: (X, T) → (Y, S) from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S), is said to fuzzy open if the image of every fuzzy open set in (X, T), is fuzzy open in (Y, S).

Definition 5. 2 [9] : A function f: (X, T) → (Y, S) from a fuzzy topologicalspace (X, T) into another fuzzy topological space (Y, S), is called fuzzy contra-continuous if $f^{-1}(\lambda)$ is fuzzy closed (open) in (X, T), for each fuzzy open (closed) set λ in (Y, S).

Proposition 5. 1: If f: (X, T) → (Y, S) is an fuzzy contra –continuous and fuzzy open function froma topological space(X, T)onto a fuzzy open hereditarily irresolvable space(Y, S), then (Y, S) is a fuzzy σ-Baire space.

PROOF: Let λ be a fuzzy σ-first category set in (Y, S). Then $\lambda = (\vee_{i=1}^{\infty}(\lambda_i))$ where λ_i's are fuzzy σ-nowhere dense sets in (Y, S). Suppose that int $(\lambda) \neq 0$. Then there exists an fuzzy open set $\mu \neq 0$ in (Y, S) such that $\mu \leq \lambda$. Then $f^{-1}(\mu) \leq f^{-1}(\lambda) = f^{-1}((\vee_{i=1}^{\infty}(\lambda_i)) = \vee_{i=1}^{\infty} f^{-1}(\lambda_i)$. Hence$f^{-1}(\mu) \leq \vee_{i=1}^{\infty} f^{-1}(cl(\lambda_i))$ Since f is afuzzy contra – continuousfunction and cl (λ_i) is a fuzzy closed set in (Y, S), $f^{-1}(cl(\lambda_i))$ is fuzzy open in(X, T). Hence we have, $f^{-1}(\mu) \leq \vee_{i=1}^{\infty} f^{-1}(cl(\lambda_i)) = \vee_{i=1}^{\infty} int(f^{-1}(cl(\lambda_i))$

Since f is fuzzy open and onto, int $(f^{-1}(\lambda_i)) \leq (f^{-1}$ int $(\lambda_i))$ and therefore we have $f^{-1}(\mu) \leq \vee_{i=1}^{\infty} f^{-1}(\text{int cl}(\lambda_I))$. Since (Y, S) is a fuzzy open hereditarily irresolvable space, by proposition 3. 4, the fuzzyσ-nowhere dense setsλ$_i$'s are fuzzy nowhere dense sets in (Y, S) . Hence we have int cl $(\lambda_i)=0$. Then $f^{-1}(\mu) \leq \vee_{i=1}^{\infty} f^{-1}(0) = 0$.

That is, $f^{-1}(\mu) \leq 0$ and hence $f^{-1}(\mu) = 0$ which implies that $\mu = 0$, a contradiction to $\mu \neq 0$. Hence we must have int$(\lambda) = 0$ where λ is a fuzzy σ-first categorysetin (Y, S). Hence by proposition 4. 1, (Y, S) is a fuzzy σ-Baire space.

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