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Fuzzy Baire Spaces and Functions

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Abstract

In this paper some results concerning functions that preserve fuzzy Baire spaces in the context of images and preimages are obtained. Several examples are given to illustrate the concepts introduced in this paper.

KEY WORDS: Fuzzy nowhere dense set, Fuzzy first category set, Fuzzy Baire Space, Fuzzy continuous, Fuzzy semi-continuous, Somewhat Fuzzy continuous, Fuzzy open and Somewhat Fuzzy open functions.

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INTRODUCTION

The fuzzyconcept has invaded almost all branches of Mathematics ever since the Introduction offuzzy set by L. A. ZADEH [12]. The theory of fuzzy topological spaces was introduced and developed by C. L. CHANG [2]. Since then much attention has been paid to generalize the basic concepts of General Topology in fuzzy setting and thus a modern theory ofFuzzy Topology has been developed. In recent years, fuzzy topology has been found to be very useful in solving many practical problems. In[4]El. Naschie showed that the notion of fuzzy topology might be relevant to Quantum Particle Physics and Quantum Gravity in connection with String Theory and $\in^{\infty}$Theory. Tang [5] used a slightly changed version of Chang's fuzzy topological spaces to model spatial objects for GISdata basesand Structured Query Language (SQL) forGIS. In classical topology functions that preserve Baire spaces in the context of images and pre-images are studied in [3], [9], [10]and [11]. In this papersome results concerning functions that preserve fuzzy Baire spaces in the context of images and pre-images are obtained. Several examples are given to illustrate the concepts introduced in this paper.

PRELIMINARIES

By a fuzzy topological space we shall mean a non-empty set X together with a fuzzy topology T (in the sense of Chang) and denote it by (X, T) .

Let λ and μ be any two fuzzy sets in (X, T) . Then we define $\lambda \vee \mu$: X $\to$ [0, 1] and $\lambda \wedge \mu$: X $\to$ [0, 1] as follows:

- $(\lambda \vee \mu)(x) = \text{Max}\{\lambda(x), \mu(x)\}$ and $(\lambda \wedge \mu)(x) = \text{Min}\{\lambda(x), \mu(x)\}$.

Let (X, T) be any fuzzy topological space and λ be any fuzzy set in (X, T) . We define $\text{Cl}(\lambda) = \wedge\{\mu / \lambda \le \mu, 1-\mu \in T\}$ and $\text{int}(\lambda) = \vee\{\mu / \mu \le \lambda, \mu \in T\}$. For any fuzzy set λ in a fuzzy topological space (X, T) , it is easy to see that $1 - \text{cl}(\lambda) = \text{int}(1 - \lambda)$ and $1 - \text{int}(\lambda) = \text{cl}(1 - \lambda)$ [1].

Let (X, T) and (Y, S) be any two fuzzy topological spaces. Let f be a function from the fuzzy topological space (X, T) to the fuzzy topological space (Y, S) . Let λ be a fuzzy set in (Y, S) . The inverse image of λ under f written as $f^{-1}(\lambda)$ is the fuzzy set in (X, T) defined by $f^{-1}(\lambda)(x) = \lambda(f(x))$ for all $x \in X$. Also the image of λin (X, T) under f written as $f(\lambda)$ is the fuzzy set in (Y, S) defined by

$$f(\lambda)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \lambda(x) & \textit{if } f^{-1}(y) \text{ is non}-\textit{empty}; \\ 0 & \textit{otherwise}. \end{cases}$$

for each $y \in Y$.

Lemma [2]: Let f: (X, T) $\to$ (Y, S) be a mapping. For fuzzy sets λ and μ of (X, T) and (Y, S) respectively, the following statements hold.

- $f\mathbf{f}^{-1}(\mu) \le \mu$;
- $\mathbf{f}^{-1}f(\lambda) \ge \lambda$;
- $f(1 - \lambda) \ge 1 - f(\lambda)$;
- $\mathbf{f}^{-1}(1 - \mu) = 1 - \mathbf{f}^{-1}(\mu)$;
- If f is injective, then $\mathbf{f}^{-1} f(\lambda) = \lambda$;
- If f is surjective, then $f\,\mathbf{f}^{-1}(\mu) = \mu$;
- If f is bijective, then $f(1 - \lambda) = 1 - f(\lambda)$.

A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy dense set if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$ [6].

A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy nowhere dense set if there exists no non-zero fuzzy open set μ in (X, T) such that $\mu < \text{cl}(\lambda)$. That is, $\text{int cl}(\lambda) = 0$ [8].

A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy semi-open if $\lambda \le \text{cl int}(\lambda)$ [1]. The complement of λ in (X, T) is called fuzzy semi-closed [1].

A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy first categoryif $\boldsymbol{\lambda} = \bigvee_{i=1}^{\infty}(\boldsymbol{\lambda}_i)$ where λ_i's are fuzzy nowhere dense sets in (X, T) . Any other fuzzy set in (X, T) is said to be of secondcategory[6]. Let λ be a fuzzy first category set in (X, T) .

Then $1 - \lambda$ is called a fuzzy residual set in (X, T) [8].

FUZZY BAIRE SPACES and FUNCTIONS

Definition 3. 1: Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy Baire Space if int $(\vee_{i=1}^{\infty} (\boldsymbol{\lambda}_i)) = 0$ where λ_i's are fuzzy nowhere dense sets in (X, T) [8].

Definition 3. 2: A function f: (X, T) → (Y, S) from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S) is said to be fuzzy open if the image of every fuzzy open set in (X, T) is fuzzy open in (Y, S) [1].

Definition 3. 3: A function f: (X, T) → (Y, S) from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S) is called fuzzy continuous if $\boldsymbol{f}^{-1}$ (λ) is fuzzy open in (X, T) for each fuzzy open set λ in (Y, S) [2].

Definition 3. 4: A function f: (X, T) → (Y, S) from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S) is called fuzzy semi-continuous if $\boldsymbol{f}^{-1}$ (λ) is fuzzy semi-open in (X, T) for each fuzzy open set λ in (Y, S) [1].

Definition 3. 5: A function f: (X, T) → (Y, S) from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S) is called somewhat fuzzy continuous if $\lambda \in S$ and $f^{-1}(\lambda) \neq 0$ implies that there exist a fuzzy open set μ in (X, T) such that $\mu \neq 0$ and $\mu \leq f^{-1}(\lambda)$ [6] .

Definition 3. 6: A function f: (X, T) → (Y, S) from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S) is called somewhat fuzzy open if $\lambda \in T$ and $\lambda \neq 0$ implies that there exists a fuzzy open set μ in (Y, S) such that $\mu \neq 0$ and $\mu \leq f(\lambda)$ [6].

Theorem 3. 1[8]: Let (X, T) be a fuzzy topological space. Then the following are equivalent:

1. (X, T) is a fuzzy Baire space.
2. Int $(\lambda) = 0$ for every fuzzy first category setλ in (X, T) .
3. cl $(\mu) = 1$ for every fuzzy residualset μ in (X, T) .

Let f be a function from the fuzzy topological space (X, T) to the fuzzy topological space (Y, S) . Underwhat Conditions on " f " may we assert that if (X, T) is a fuzzy Baire Space, then (Y, S) is a fuzzy Baire Space? It may be noticed that the fuzzy continuousimage of a fuzzy Baire Space may fail to be afuzzyBaire Space. For, consider the following example.

Example 3. 1: Let X = { a, b }. The fuzzy sets λ, μ and υ are defined on X as follows:

- λ: X → [0, 1] is defined as λ (a) = 0. 5 ; λ (b) = 0. 7.
- μ: X → [0, 1] is defined as μ (a) = 0. 8 ; μ (b) = 0. 4.
- υ: X → [0, 1] is defined as υ (a) = 0. 2 ; υ (b) = 0. 6.

Then, T = { 0, λ, μ, υ, λ ∨ μ, μ∨υ, λ ∧ μ, μ∧υ, 1 } and S= { 0, λ, μ, λ ∨ μ, λ ∧ μ, 1} are fuzzy topologies on X. Now the fuzzynowhere densesets in (X, T) are 1 – λ and 1 – (λ ∨ μ) and [(1 – λ) ∨{1 – (λ ∨ μ) }] =1 – λ and int [(1 – λ)] =0. Hence (X, T) is a fuzzy Baire Space. The fuzzynowhere densesets in (X, S) are 1 – λ, 1 – μ and 1 – (λ ∨ μ) and [(1 – λ) ∨ (1 – μ) ∨{1 – (λ ∨ μ) }] =1 – (λ ∧ μ) and Int [1 – (λ ∧ μ)] = (λ ∧ μ) ≠0. Hence (X, S) is not a fuzzy Baire Space.

Define a function f: (X, T) → (X, S) by f (a) =a and f (b) =b . Clearly f is a fuzzy continuous functionfrom the fuzzy Baire space (X, T) to the fuzzy topological space (Y, S) which is not afuzzy Baire Space.

It may be alsonoticed that the image of a fuzzy Baire Space under a fuzzy open function may fail to be a fuzzy Baire Space. For, Consider the following example.

Example 3. 2: Let X = { a, b, c }. The fuzzy sets λ, μ and υ are defined on X as follows:

- λ: X → [0, 1] is defined as λ (a) = 0. 6 ; λ (b) = 0. 4; λ (c) = 0. 3.
- μ: X → [0, 1] is defined as μ (a) = 0. 5 ; μ (b) = 0. 7; μ (c) = 0. 2.
- υ: X → [0, 1] is defined as υ (a) = 0. 7 ; υ (b) = 0. 5; υ (c) = 0. 6.

Then, T= { 0, λ, 1}and S= { 0, λ, μ, υ, λ ∨ μ, μ∨υ, λ ∧ μ, μ∧υ, 1 } are fuzzy topologies on X. Now the fuzzynowhere densesets in (X, T) are 1 – λ, 1 –υand [(1 – λ) ∨υ] =1 – λ and int [(1 – λ)] =0. Hence (X, T) is a fuzzy Baire Space. The fuzzynowhere densesets in (X, S) are 1 – λ, 1 – μ, 1 –υ, 1 – (λ ∨ μ) and 1 – (μ∨υ) and [(1 – λ) ∨ (1 – μ) ∨ (1 –υ) ∨ {1 – (λ ∨ μ) } ∨ {1 – (μ∨υ) }] =1 – (λ∧μ) . Now int [1 – (λ∧μ)] = (μ∧υ) ≠0. Hence (X, S) is not a fuzzy Baire Space.

Define a function f: (X, T) → (X, S) by f (a) =a and f (b) =b and f (c) =c. Clearly f is afuzzyopen functionfrom the fuzzy Baire space (X, T) to the fuzzy topological space (Y, S) , whichisnot afuzzy Baire Space.

Proposition 3. 1: Ifafunction f: (X, T) → (Y, S) from a fuzzy topological space (X, T) intoanother fuzzy topological space (Y, S) isfuzzy continuous, 1-1, onto andfuzzy open function, then for any fuzzy set λ in (X, T) int cl [f (λ)] ≤ f (int (cl (λ))) .

Proof: Letλ be a fuzzy set in (X, T) . Thenλ ≤ cl (λ) in (X, T) implies that 1 — cl (λ) ≤ (1 — λ) . Then f (1 — cl (λ)) ≤ f (1 — λ) . Since f is a fuzzy open function and 1 — cl (λ) is a fuzzy open set in (X, T) , f (1 — cl (λ)) is a fuzzy open set in (Y, S) such that f (1 — cl (λ)) ≤ f (1 — λ) . But int f (1 — λ) isthe largest fuzzy open in (Y, S) such that int [f (1 —λ)] ≤ f (1 — λ) . Hence we have that f (1 — cl (λ)) ≤ int [f (1— λ)]. Since f is 1 – 1 and onto, f (1 — λ) = 1 — f (λ) . Hence (1 — f [cl (λ)]) ≤ int [1

— f (λ)] which implies that (1 — f [cl (λ)]) ≤ (1 — cl [f (λ)) . Then cl [f (λ)] ≤ f [cl λ)], which implies that int cl [f (λ)] ≤ int (f [cl (λ)]) . Therefore we have f^{-1} (int cl [f (λ)]) ≤ f^{-1} (int (f [cl (λ)])) …. . (1) . Now int [f (cl (λ))] is an fuzzy open set in (Y, S) . Since f isfuzzy continuousf^{-1} (int (f [cl (λ)])) is an fuzzy open set in (X, T) . Hence f^{-1} (int (f [cl (λ)])) = int f^{-1} (int (f [cl (λ)])) ≤ int (f^{-1} (f [cl (λ)])) ≤ int (cl (λ)) [since f is 1-1]. That is, f^{-1} (int (f [cl (λ)]) ≤ int (cl (λ)) ……. … (2) .

From (1) and (2) , we have f^{-1} (int cl [f (λ)]) ≤ int (cl (λ))) which implies that f f^{-1} (int cl [f (λ)]) ≤ f (int (cl (λ))) . Sincef is onto, we have (int cl [f (λ)]) ≤f (int (cl (λ))) .

Proposition 3. 2: Ifafunction f: (X, T) → (Y, S) from a fuzzy topological space (X, T) intoanother fuzzy topological space (Y, S) isfuzzy continuous and fuzzy open function, then for any fuzzy set λ in (X, T) f (int (cl (λ))) ≤ int (cl [f (λ)]) .

Proof: Letλbe anyfuzzy set in (X, T) . Then int (cl (λ)) is a fuzzy open set in (X, T) .

Since f is afuzzy open function, f (int (cl (λ))) is a fuzzy open set in (Y, S) .

Now f (int (cl (λ))) ≤ f ((cl (λ))) . Since f is fuzzy continuous, f ((cl (λ))) ≤ cl (f (λ)) .

Hence we havef (int (cl (λ))) ≤ cl (f (λ)) . But int (cl f (λ)) ≤ cl (f (λ)) . Therefore, we have f (int (cl (λ))) ≤ (int (cl[f (λ)]) .

Proposition 3. 3: Ifafunction f: (X, T) → (Y, S) from a fuzzy topological space (X, T) intoanother fuzzytopological space (Y, S) isfuzzy continuous, fuzzy open, 1-1 and onto function, then for any fuzzy set λ in (X, T) , f (int (cl (λ))) = int (cl [f (λ)]) .

Proof: Letλbe anyfuzzy set in (X, T) . Since f is fuzzy continuous, 1-1, onto and fuzzy open function, by proposition 3. 1, we have int cl [f (λ)] ≤ f (int (cl (λ))) . . . … (1) .

Since f is a fuzzycontinuous and a fuzzy open function, by proposition 3. 2, we have f (int (cl (λ))) ≤ int (cl [f (λ)]) . . . . . (2) . From (1) and (2) , we have, f (int (cl (λ))) = int (cl [f (λ)]) .

Proposition 3. 4: Letthefunction f: (X, T) → (Y, S) from a fuzzy topological space (X, T) intoanother fuzzytopological space (Y, S) be fuzzy continuous, fuzzy open, 1-1 and onto function. Then, for any fuzzy set λ in (X, T) , λ is a fuzzy nowhere dense set in (X, T) if and only if f (λ) is a fuzzy nowhere dense set in (Y, S) .

Proof: Letλ be a fuzzy nowhere dense set in (X, T) . Then, int cl (λ) = 0. Since f is fuzzy continuous, fuzzy open, , 1-1 and onto function, by proposition 3. 3, we have, f (int (cl (λ))) = int (cl [f (λ)]) . Then, int (cl [f (λ)]) =f (0) =0. Hence f (λ) isa fuzzy nowhere dense set in (Y, S) .

Conversely, letf (λ) be a fuzzy nowhere dense set in (Y, S) .

Then, int (cl [f (λ)]) =0. Hencef (int (cl (λ))) =int (cl [f (λ)]) implies that f (int (cl (λ))) =0

Therefore f^{-1}f (int (cl (λ))) =f^{-1} (0) =0. Since f is 1-1, int (cl (λ)) =0. Henceλ is a fuzzy nowhere dense set in (X, T) .

Proposition 3. 5: Let thefunction f: (X, T) → (Y, S) from a fuzzy topological space (X, T) intoanother fuzzytopological space (Y, S) be fuzzy continuous, fuzzy open, 1-1 and onto function. If (X, T) is a fuzzy Baire Space, then (Y, S) is a fuzzy Baire Space.

Proof: Letλ be a fuzzy first category set in (Y, S) . Then $\boldsymbol{\lambda} = \vee_{i=1}^{\infty} (\boldsymbol{\lambda}_i)$ where λ_i's arefuzzy nowhere dense sets in (Y, S) . Since f is onto, f $\mathbf{f^{-1}}$ ($(\boldsymbol{\lambda}_i)$ = $(\boldsymbol{\lambda}_i)$ and hence $\boldsymbol{\lambda} = \vee_{i=1}^{\infty}$ $\mathbf{f\, f^{-1}}$ ($(\boldsymbol{\lambda}_i)$) and therefore, f [$\mathbf{f^{-1}}$ ($(\boldsymbol{\lambda}_i)$] are fuzzynowhere dense sets in (Y, S) . Since f is fuzzy continuous, fuzzy open, 1-1 and onto function, by proposition 3 . 4, we have $\mathbf{f^{-1}}$ ($(\boldsymbol{\lambda}_i)$) is a fuzzy nowhere dense set in (X, T) . Let μ =$\vee_{i=1}^{\infty}\mathbf{f}^{-1}$ ($(\boldsymbol{\lambda}_i)$) . **Then** μ **is** a fuzzy first category set in (X, T) . Since (X, T) is a fuzzy Baire Space, int (μ) =0. Then we have int ($\vee_{i=1}^{\infty}\mathbf{f}^{-1}$ ($(\boldsymbol{\lambda}_i)$) = 0 which implies that int $(\mathbf{f^{-1}}(\vee_{i=1}^{\infty} (\boldsymbol{\lambda}_i))$ = 0. Then, we have f (int $(\mathbf{f^{-1}}(\vee_{i=1}^{\infty} (\boldsymbol{\lambda}_i))$) $= \boldsymbol{f(0)} = \mathbf{0}$. Since f is a continuous function, we have $\mathbf{f^{-1}}$ (int$(\vee_{i=1}^{\infty} (\boldsymbol{\lambda}_i))$) $\leq$ $\mathbf{int}\left(\mathbf{f^{-1}}(\vee_{i=1}^{\infty} (\boldsymbol{\lambda}_i)\right)$.

Hence $\mathbf{f\, f^{-1}}$ (int$(\vee_{i=1}^{\infty} (\boldsymbol{\lambda}_i))$) $\leq \boldsymbol{f}$ $(\mathbf{int}\left(\mathbf{f^{-1}}(\vee_{i=1}^{\infty} (\boldsymbol{\lambda}_i)\right) = \mathbf{0} \Rightarrow \mathbf{f\, f^{-1}}$ (int$(\vee_{i=1}^{\infty} (\boldsymbol{\lambda}_i))$) $= \mathbf{0}$.

Since f is onto, $\boldsymbol{int}\,(\vee_{i=1}^{\infty} (\boldsymbol{\lambda}_i)) = \mathbf{0}$. That is, int (λ) $= \mathbf{0}$. Hence by theorem 3. 1. , (Y, S) is a fuzzy Baire Space.

Proposition 3. 6: Ifthefunction f: (X, T) → (Y, S) from a fuzzy topological space (X, T) intoanother fuzzytopological space (Y, S) is fuzzy semi-continuous and somewhat fuzzy open function and if λ is a fuzzy nowhere dense set in (Y, S) . then $\mathbf{f^{-1}}$ (λ) is a fuzzy nowhere dense set in (X, T) .

Proof: Letλ be a fuzzy nowhere dense set in (Y, S) . Then, 1 — cl (λ) is a fuzzy dense setin (Y, S) [8]. Since f is somewhat fuzzy open, $\mathbf{f^{-1}}$ (1 — cl (λ)) is a fuzzy dense set in (X, T) . That is, cl [$\mathbf{f^{-1}}$ (1 — cl (λ))] = $\mathbf{1}$ ……… . $(\mathbf{1})$. Now 1 — cl (λ) is a fuzzy open setin (Y, S) . Since f is fuzzy semi-continuous $\mathbf{f^{-1}}$ (1 — cl (λ)) is a fuzzy semi-open setin (X, T) . Then $\mathbf{f^{-1}}$ (1 — cl (λ)) ≤ $\boldsymbol{cl\ int}$ $\mathbf{f^{-1}}$ (1 — cl (λ)) implies that $\mathbf{f^{-1}}$ (1 — cl (λ)) ≤ $\boldsymbol{cl\ int}$ (1 — $\mathbf{f^{-1}}$ [cl (λ)]) . Hence $\mathbf{f^{-1}}$ (1 — cl (λ)) ≤ $\boldsymbol{cl}$ (1 — **cl** [$\mathbf{f^{-1}}$ (cl (λ))]) which implies that **cl** $\mathbf{f^{-1}}$ (1 — cl (λ)) ≤ $\boldsymbol{cl\ cl}$ (1 —**cl** [$\mathbf{f^{-1}}$ (cl (λ)]) . Then, from (1) , 1 ≤ $\boldsymbol{cl}$ (1 —**cl** [$\mathbf{f^{-1}}$ (cl (λ))]. That is, $\boldsymbol{cl}$ (1 —**cl** [$\mathbf{f^{-1}}$ (cl (λ))]) = $\mathbf{1}$ which implies that 1 — $\boldsymbol{int}$ **cl** [$\mathbf{f^{-1}}$ (cl (λ))] = $\mathbf{1}$ and hence $\boldsymbol{int}$ **cl** [$\mathbf{f^{-1}}$ (cl (λ))]) = 0. Therefore we have $\boldsymbol{int}$ **cl** [$\mathbf{f^{-1}}$ (λ)] = 0. Hence $\mathbf{f^{-1}}$ (λ) is a fuzzy nowhere dense set in (X, T) .

Proposition 3. 7: Ifthefunction f: (X, T) → (Y, S) from a fuzzy topological space (X, T) intoanother fuzzytopological space (Y, S) is fuzzy semi-continuous and somewhat fuzzy open function and if λ is a fuzzy first category set in (Y, S) . then $\mathbf{f^{-1}}$ (λ) is a fuzzy first category set in (X, T) .

Proof: Letλ be a fuzzy first category set in (Y, S) . Then $\boldsymbol{\lambda} = \vee_{i=1}^{\infty} (\boldsymbol{\lambda}_i)$ where λ_i's arefuzzy nowhere dense sets in (Y, S) . Since f is a fuzzy semi-continuous and somewhat fuzzy open function, by proposition 3. 6, $\mathbf{f^{-1}}$ ($(\boldsymbol{\lambda}_\mathbf{i})$) is a fuzzynowhere dense set in (X, T) for each fuzzynowhere dense setλ_i in (Y, S) . Then we have $\mathbf{f^{-1}}\ (\boldsymbol{\lambda}) = \mathbf{f^{-1}}\ (\vee_{i=1}^{\infty} (\boldsymbol{\lambda}_\mathbf{i})) = (\vee_{i=1}^{\infty} \mathbf{f}^{-1} (\boldsymbol{\lambda}_\mathbf{i}))$. **Since** $\mathbf{f}^{-1} (\boldsymbol{\lambda}_\mathbf{i})$) **'s are**fuzzy nowhere dense sets in (X, T) , $\mathbf{f^{-1}}$ (λ) is a fuzzy first category setin (X, T) .

Proposition 3. 8: Ifthefunction f: (X, T) → (Y, S) from a fuzzy topological space (X, T) intoanother fuzzytopological space (Y, S) is fuzzy semi-continuous and somewhat fuzzy open and somewhat fuzzy continuous function and if (X, T) is a fuzzy Baire Space (Y, S) , then (Y, S) is a fuzzy Baire Space.

Proof: Letλ be a fuzzy first category set in (Y, S) . Then, by proposition 3. 7 $\mathbf{f^{-1}}$ (λ) is a fuzzy first category set in (X, T) . Since (X, T) is a fuzzy Baire Space **int** $(\mathbf{f^{-1}}$ (λ)) = 0. Wenow claim that int (λ) = 0. Suppose int (λ) ≠0. Then thereexists a non-zero fuzzy open set μ in (Y, S) such thatμ ≤ λ. Then $\mathbf{f^{-1}}$ (μ) ≤ $\mathbf{f^{-1}}$ (λ) .

Since μ is a non-zero fuzzy open set μ in (Y, S) and $\mathbf{f^{-1}}$ (μ) ≠0 and since f is a somewhat fuzzy continuous function, there exists a non-zero fuzzy open set υ in (X, T) such that υ ≤ $\mathbf{f^{-1}}$ (μ) . Hence υ ≤ $\mathbf{f^{-1}}$ (μ) ≤ $\mathbf{f^{-1}}$ (λ) . That is, **int** $(\mathbf{f^{-1}}$ (λ)) ≠0, which is a contradiction to **int** $(\mathbf{f^{-1}}$ (λ)) = 0. Hence we must have int (λ) = 0 and therefore (Y, S) is a fuzzy Baire Space.

Theorem 3. 2 [7]: Ifthefunction f: (X, T) → (Y, S) from a fuzzy topological space (X, T) intoanother fuzzytopological space (Y, S) is somewhat fuzzy continuous and 1-1 and onto and if cl (int (λ)) = 1 for any non-zero fuzzy set λ in (X, T) , then cl [int (f (λ))] = 1 in (Y, S) .

Proposition 3. 9: Ifthefunction f: (X, T) → (Y, S) from a fuzzy topological space (X, T) intoanother fuzzytopological space (Y, S) is somewhat fuzzy continuous, 1-1 and onto function and if λ is a fuzzy nowhere dense set in (X, T) for any fuzzy set λ in (X, T) , then f (λ) is a fuzzy nowhere dense set in (Y, S) .

Proof: Letλ be a fuzzy nowhere dense set in (X, T) . Then, int cl (λ) = 0. Then 1 —int cl (λ) = 1 which implies that cl int (1 — (λ)) = 1. Since f is a somewhat fuzzy continuous function, by theorem 3. 2, cl int f [(1 — (λ))] =1. Since f is 1-1 and onto, f (1 — λ) = 1 — f (λ) . Then cl int [(1 — f (λ)) =1 and hence 1 — int cl f (λ)) =1. That is, int cl (f (λ)) = 0. Hence f (λ) isa fuzzy nowhere dense set in (Y, S) .

Proposition 3. 10: Ifthefunction f: (X, T) → (Y, S) from a fuzzy topological space (X, T) intoanother fuzzytopological space (Y, S) is a somewhat fuzzy continuous, 1-1 and onto function and if λ is a fuzzy first category set in (X, T) thenf (λ) is a fuzzy first category set in (Y, S) .

Proof: Letλ be a fuzzy first category set in (X, T) . Then $\boldsymbol{\lambda} = \vee_{i=1}^{\infty} (\boldsymbol{\lambda}_\mathbf{i})$ where λ_i's arefuzzynowhere dense sets in (X, T) . Then f $(\boldsymbol{\lambda}) = \boldsymbol{f} (\vee_{i=1}^{\infty} (\boldsymbol{\lambda}_\mathbf{i}))$ Since f is 1-1 and onto, f $(\boldsymbol{\lambda}) = \vee_{i=1}^{\infty} \mathbf{f}\ (\boldsymbol{\lambda}_\mathbf{i})$ .Since f is a somewhat fuzzy continuous and λ is a fuzzy nowhere dense set in (X, T) , by proposition 3. 9, f ($(\boldsymbol{\lambda}_i)$) is a fuzzynowhere dense set in (Y, S) . Therefore f (λ) is a fuzzy first category setin (Y, S) .

Proposition 3. 11: Ifthefunction f: (X, T) → (Y, S) from a fuzzy topological space (X, T) intoanother fuzzytopological space (Y, S) is a somewhat fuzzycontinuous, somewhat fuzzy open and 1-1 and onto function and if (Y, S) is a fuzzy Baire Space (Y, S) , then (X, T) is a fuzzy Baire Space.

Proof: Letλ be a fuzzy first category set in (X, T) . Since f is a fuzzy somewhat continuous, 1-1 and onto function, by proposition 3. 10, f (λ) is a fuzzy first category set in (Y, S) . Since (Y, S) is a fuzzy Baire space, we have int [f (λ)] = 0. We nowclaim thatint (λ) = 0. Suppose int (λ) ≠0. Then thereexists a non-zero fuzzy open set μ in (X, T) such thatμ ≤ λ. Then f (μ) ≤ f (λ) . Since μ is a non-zero fuzzy open set in (X, T) and f (μ) ≠0 and f is a somewhat fuzzy open function, there exists a non-zero fuzzy open set υ in (Y, S) such that υ ≤ f (μ) . Hence υ ≤ f (μ) ≤ f (λ) . That is, int[f (λ)] ≠0, which is a contradiction to int [f (λ)] = 0 . Hence int (λ) = 0 for a fuzzy first category set in (X, T) and therefore, by theorem 3. 1, (X, T) is a fuzzy Baire space.

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