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Fuzzy Optimal Transportation Problems by Improved Zero Suffix Method via Robust Rank Techniques

P. Jayaraman[1] and R. Jahirhussian[2]

[1]Department of Mathematics,
Bharathidasan University Constituent College, Srirangam T.K, Trichy
E-mail:jrmsathya@gmail.com
[2]P.G. research Department of mathematics
Jamal Mohamed College Trichy - Tamilnadu -India
E-mail: hssn_jhr@yahoo.com

Abstract

The purpose of this paper is to find the least transportation cost of some commodities through a capacitated network, when the supply and the demand of nodes and the capacity and cost edges are representing fuzzy numbers. Here, we proposed a ranking technique for solving fuzzy transportation problem, where fuzzy demand and supply are in form of Triangle Squared and Trapezoidal Squared fuzzy numbers. Here, simple algorithm is proposed for computed fuzzy transportation problem and finally feasibility of the proposed study is checked with a numerical example.

Keywords: Fuzzy set, convex set, Triangular fuzzy number, Trapezoidal fuzzy number, improved zero suffix method, robust ranking method.

Section-1 Introduction

The transportation problem is one of the earliest applications of linear programming problems. The occurrence of randomness and imprecision in the real work is inevitable owing to some unexpected situations. There are cases that the cost coefficients and the supply and demand quantities of a transportation problem may be uncertain due to some uncontrollable factors some of the researches are studied in [3,7] . To deal with imprecise information in making decisions, Zadeh [13] introduced the notion of fuzziness . A fuzzy transportation problems is a problem in which the transportation cost, supply and demand quantities are fuzzy quantities. The objective of the fuzzy transportation is to determine the shipping schedule that minimizes the total fuzzy transportation cost while satisfying fuzzy supply and demand limits. The basic transportation was originally developed by Hitchcock. In real world applications, all the parameters of the transportation problems may not be known precisely due to uncontrollable factors. Fuzzy numbers introduces by Zadeh[13] may represent this data. So, fuzzy decision making method is needed here. Zimmermann [14] showed that solutions obtained by fuzzy linear programming are always efficient. Zimmermann's [14] fuzzy linear programming has developed into seven fuzzy optimization methods for solving transportation problems.. O'heigeartaigh [7] proposed an algorithm for solving transportation problems where the capacities and requirements are fuzzy sets with linear are triangular membership functions.

Chanas and Kuchta [1] proposed the concept of the optimal solution for the transportation problem with fuzzy coefficients expressed as fuzzy numbers, and developed an algorithm for obtaining the optimal solution. Saad and Abbas[11] discussed the solution algorithm for solving the transportation in fuzzy environment. Gani and Razak [4] presented a two stage cost minimizing fuzzy transportation in which supplies and demands are trapezoidal fuzzy numbers. Dinagor and Palanivel[2] investigated fuzzy transportation problem with the aid of trapezoidal fuzzy numbers and proposed fuzzy modified distribution method to find the optimal solution in terms of fuzzy numbers. Pandian and Natarajan [9] proposed a new algorithm namely , fuzzy zero point method for finding fuzzy optimal solution for a fuzzy transportation problem , where the transportation cost , supply and demand are represented by trapezoidal fuzzy numbers .R.Nagarajan and A.Solairaju [6] presented an algorithm for solving fuzzy assignment problems using Robust ranking technique with fixed fuzzy numbers. In this paper. we proposed a ranking technique for solving fuzzy transportation problem, where fuzzy demand and supply are in form of Triangle Squared and Trapezoidal Squared fuzzy numbers. Here, simple algorithm is proposed for computed fuzzy transportation problem and finally feasibility of the proposed study is checked with a numerical example.

Section-2 Preliminaries

Zadeh [13] in 1965 first introduced Fuzzy set as a mathematical way of representing impreciseness or vagueness in everyday life.

2.1 Definition: A fuzzy set is characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval [0, 1]. (i,e) $A = \{(x, \mu_A(x) ; x \in X\}$, Here $\mu_A: X \rightarrow [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \varepsilon X$ in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

2.2 Definition : A fuzzy set A of the universe of discourse X is called a *normal* fuzzy set implying that there exist at least one $x \varepsilon X$ such that $\mu_A(x) = 1$.

2.3 Definition ; The fuzzy set A is *convex* if and only if, for any x_1, x_2 Є X, the membership function of A satisfies the inequality $\mu_A\{\lambda x_1+(1-\lambda)x_2\} \geq \min\{\mu_A(x_1), \mu_A(x_2)\}$. $0\leq\lambda\leq1$.

2.4 Definition (***Triangular fuzzy number***) : For a triangular fuzzy number A(x), it can be represented by A(a,b,c ; 1) with membership function μ(x) given by

$$\mu(x) = \begin{cases} (x-a)/(b-a), & a \leq x \leq b \\ 1, & x=b \\ (c-x)/(c-b), & c\leq x\leq d \\ 0, & \text{otherwise.} \end{cases}$$

2.5 Definition: (***Trapezoidal fuzzy number***): For a trapezoidal number A(x), it can be represented by A (a,b,c,d;1) with membership function μ(x) given by

$$\mu(x) = \begin{cases} (x-a)/(b-a), & a \leq x \leq b \\ 1, & b \leq x\leq c \\ (d-x)/(d-c), & c \leq x \leq d \\ 0, & \text{otherwise} \end{cases}$$

2.6 Definition: (***α-cut of a trapezoidal fuzzy number***): The α-cut of a fuzzy number A(x) is defined as $A(\alpha) = \{x : \mu(x) \geq \alpha, \alpha \in [0,1]\}$

Addition of two fuzzy numbers can be performed as

$(a_1,b_1,c_1) + (a_2,b_2,c_2) = (a_1+a_2, b_1+b_2, c_1+c_2)$

Addition of two trapezoidal fuzzy numbers can be performed.

Section- 3 Robust's Ranking Techniques

Robust's ranking technique [6] which satisfies compensation, linearity, and additivity properties and provides results which are consistent with human intuition. Give a convex fuzzy number ã, the Robust's Ranking Index is defined by

$$R(\tilde{a}) = \int_0^1 0.5\,(a_\alpha^L, a_\alpha^U)\,d\alpha$$, where (a_α^L, a_α^U) is the α – level cut of the fuzzy number ã.

In this paper we use this method for ranking the objective values. The Robust's ranking index R (ã) gives the representative value of the fuzzy number ã. It satisfies the linearity and additive property:

3.2 IMPROVED ZERO SUFFIX METHOD- ALGORITHM

We now introduce a new method called the improved zero suffix method for finding on optimal solution to the transportation problem.

- **Step 1**: Construct the transportation table.
- **Step2**: Subtract reach row entries of the transportation table form the corresponding row minimum after that subtract each column entries of the

transportation table from the corresponding column minimum.

- **Step 3**: In the reduced cost matrix there will be the at least one zero in each row and column, then find the suffix value all the zero's in the reduced cost matrix by following simplification.
- **Step 4**: Choose the maximum of S, if it has one maximum value then first supply to that demand corresponding to the cell. If it has more equal values then select $\{a_i,b_j\}$ and supply to that demand maximum possible.
- **Step 5**: After the above step, the exhausted demand (column) or supplies (row) to be trimmed. The resultant matrix must possess at least one zero is each row and column, Else repeat step 2.
- **Step 6**: Report step 3 to step 5 until the optimal solution is obtained.

Section -4 Numerical Example (Triangular square fuzzy numbers)

The fuzzy transportation cost for unit quantity of the product from i^{th} source j^{th} destination if c_{ij} where

$$(C_{ij})_{3x3} = \begin{bmatrix} (1, 4, 9) & (16, 25, 36) & (9, 36, 49) \\ (16, 25, 64) & (36, 64, 81) & (4, 49, 64) \\ (4, 25, 81) & (25, 36, 64) & (49, 64, 81) \end{bmatrix}$$

Fuzzy availability of the product at the source are (4 ,25, 36) (16, 36 ,49) (25, 49 ,81) and the fuzzy demand of the product at destination are (16, 25 ,36) (4 ,49, 81) (25 ,36 ,49).

Solution:

The fuzzy Transportation problems are given in Table-1

Step-1

Table-1

Sources	D_1	D_2	D_3	Supply
S_1	(1 , 4 , 9)	(16 ,25, 36)	(9, 36 ,49)	(4 ,25, 36)
S_2	(16, 25, 64)	(36, 64 ,81)	(4, 49 ,64)	(16, 36 ,49)
S_3	(4 ,25 ,81)	(25, 36, 64)	(49,64 ,81)	(25, 49 ,81)
Demand	(16, 25 ,36)	(4 ,49, 81)	(25 ,36 ,49)	

In Conformation to model the fuzzy transportation problem can be formulated in the following mathematical form

Min Z= R (1, 4, 9) x_{11} + R (16, 25 ,36)x_{12} + R (9, 36, 49)x_{13} + R (16, 25, 64)x_{21} + R (36, 64, 81)x_{22} + R (4 ,49 ,64)x_{23} + R (4 ,25, 81)x_{31} + R (25, 36, 64)x^{32} + R(49, 64 ,81)x_{33}

$$R(\bar{a}) = \int_0^1 (0.5)\,(a^L_{\alpha},\, a^U_{\alpha})\,d\alpha$$

Where $(a^L_{\alpha},\, a^U_{\alpha}) = \{(b-a)\,\alpha + a,\ c - (c-b)\alpha\}$
$R(1, 4, 9) = \int(0.5)\,(-2\alpha + 10)\,d\alpha = 4.5$

Similarly,
R (16, 25, 36) = 25.5, R (9, 36, 49) =32.5, R (16, 25, 64) = 32.5,
R (36, 84, 81) = 61.25, R (4 ,49 ,64) = 41.5, R(4 ,25, 81) = 33.75,
R (25, 36 ,64) = 38.25 , R (49 ,64, 81) = 64.5.

Rank of all supply: R (4 ,25 ,36) = 22.5, R (16 ,36 ,49) = 34.25, R (25, 49, 81) = 51.

Rank of all Demand: R (16 ,25 ,36) = 25.5, R (4, 49, 81) = 45.75, R (25, 36 ,49) = 36.5

Table-2 (After Ranking)

Sources	Demand D_1	Demand D_2	Demand D_3	Supply
S_1	4.5	25.5	32.5	22.5
S_2	32.5	61.25	41.5	34.25
S_3	33.75	38.25	64.5	51
Demand	25.5	45.75	36.5	

Table-3

Sources	Demand D_1	Demand D_2	Demand D_3	Supply
S_1	4.5 **22.5**	25.5	32.5	22.5
S_2	32.5 **3**	61.25	41.5 **31.25**	34.25
S_3	33.75	38.25 **45.75**	64.5 **05.25**	51
Demand	25.5	45.75	36.5	

Table-4

Sources	Demand D_1	Demand D_2	Demand D_3	Supply
S1	**(4, 25, 36)** (1 , 4, 9)	(16,25,36)	(9,36,49)	(4,25,36)
S2	**(12,0,0)** (16, 25,64)	(36,64,81)	**(4,36,49)** (4,49,64)	(16,36,49)
S3	(4,25,81)	**(4,49,81)** (25,36,64)	**(21,0,0)** (49,64,81)	(25,49,81)
Demand	(16,25,36)	(4,49,81)	(25,36,49)	

In this example, it has been that the total optimal cost obtained by our methods remains as that obtained by defuzzifying the total fuzzy optimal cost by applying the Robust's ranking method. For the fuzzy transportation problem with fuzzy objective function Min Z =(4.5) (22.5) + (32.5) (3) + (41.5) (31.25) + (38.25) (45.75) + (64.5) (5.25) =3584.19

Numerical Example: (Trapezoidal Squared fuzzy numbers)

A company has four sources S_1, S_2, S_3, S_4 and destinations D_1, D_2, D_3, D_4. The fuzzy transportation cost for unit quantity of product from i^{th} sources j^{th} destinations is C_{ij}

$$\text{where } C_{ij} = \begin{pmatrix} (1,4,9,16) & (4,9,16,25) & (9,16,25,36) & (16,25,36,49 \\ (4,9,16,25) & (9,16,25,36) & (16,25,36,49) & (25,36,49,64) \\ (9,16,25,36) & (16,25,36,49) & (25,36,49,64) & (36,49,64,81) \\ (16,25,36,49) & (25,36,49,64) & (36,49,64,81) & (25,36,49,81) \end{pmatrix}$$

Fuzzy availability of the product at source are (36 49 64 81) (16 25 36 49)
(25 36 49 64) (4 16 25 36) and there Fuzzy demand of the product and destination are (16 25 36 49) (4 16 25 36) (25 36 49 64) (36 49 64 81)

Solution:

The fuzzy Transportation problems are given in Table-1

Table-1

Sources	D_1	D_2	D_3	D_4	Supply
S_1	(1, 4 , 9, 16)	(4, 9,16,25)	(9,16,25,36)	(16,25,36,49)	(16,25,36,49)
S_2	(4, 9,16, 25)	(9,16,25,36)	(16,25,36,49)	(25,36,49,64)	(36,49,64,81)
S_3	(9,16,25,36)	(16,25,36,49)	(25,36,49,64)	(36,49,64,81)	(25,36,49,64)
S_4	(16,25,36,49)	(25,36,49,64)	(36,49,64,81)	(25,36,49,81)	(4,16,25,36)
Demand	(36,49,64,81)	(4,16,25,36)	(25,36,49,64)	(16,25,36,49)	

In Conformation to model the fuzzy transportation problem can be formulated in the following mathematical form

Min Z= R(1, 4 ,9 ,16) X_{11}+R (4 ,9 ,16, 25)X_{12} +R (9 ,16, 25, 36)X_{13} +R (16 ,25 ,36 ,49)X_{14} + R (4, 9 ,16, 25) X_{21} +R (9, 16 ,25 ,36)X_{22} +R(16 ,25, 36, 49)X_{23} + R (25, 36, 49 ,64)X_{24} + R (9 ,16, 25, 36)X_{31}+ R (16 ,25, 36 ,49) X_{32} + R(25, 36, 49, 64)X_{33} + R (36, 49, 64, 81) X_{34} + R (16, 25, 36, 49) X_{41} +R (25, 36, 49, 64) X_{42} +R (36, 49, 64, 81) X_{43} + R (25, 36, 49, 81) X_{44}

$$R(\bar{a}) = \int_0^1 (0.5)\,(a^L_{\alpha}, a^U_{\alpha})\,d\alpha$$

Where $(a^L_{\alpha}, a^U_{\alpha}) = \{(b\text{-}a)\,\alpha + a,\ d - (d\text{-}c)\alpha\}$

$$R(1, 4, 9, 16) = \int_0^1 (0.5)(-4\alpha + 17)\, d\alpha = 7.5$$

Similarly, R(4, 9, 16, 25) = 13.5, R(9, 16 ,25, 36) =21.5, R(16 ,25 ,36 ,49) = 31.5, R(4, 9, 16, 25) = 13.5, R(9, 16, 25, 36) = 21.5, R(16, 25, 36 ,49)=31.5, R(25, 36, 49, 64)= 33.5 , R(9, 16, 25, 36) = 21.5, R(16, 25, 36, 49) =31.5, R(25, 36, 49 ,64) = 33.5, R(36, 49, 64, 81) = 57.5, R(16, 25, 36, 49) = 31.5, R(25, 36, 49, 64) = 33.5, R(36, 49, 64, 81) = 57.5, R(16, 25, 36, 49) = 31.5, R(25, 36, 49, 64) = 33.5, R(36, 49, 64, 81) = 57.5, R(25, 36, 49, 81) = 47.5

Rank of all supply :R(36,49,64,81) = 57.5, R(16,25,36,49) = 31.5, R(25,36,49,64) = 33.5, R(4,16,25,36) = 20.25

Rank of all Demand: R(16,25,36,49) = 31.5, R(4,16,25,36) = 20.25, R(25,36,49,64) = 33.5, R (36,49,64,81) = 57.5.

Table-2

Sources	Demand D_1	Demand D_2	Demand D_3	Demand D_4	Supply
S_1	7.5	13.5	21.5	31.5	31.5
S_2	13.5	21.5	31.5	33.5	57.5
S_3	21.5	31.5	33.5	57.5	33.5
S_4	31.5	33.5	57.5	47.5	20.25
Demand	31.5	20.25	33.5	31.5	

Table-3(After Ranking)

Sources	Demand D_1	Demand D_2	Demand D_3	Demand D_4	Supply
S_1	**31.5** 7.5	13.5	21.5	31.5	31.5
S_2	**26.0** 13.5	**20.25** 21.5	**11.25** 31.5	33.5	57.5
S_3	21.5	31.5	**22.25** 33.5	**11.25** 57.5	33.5
S_4	31.5	33.5	57.5	**20.25** 47.5	20.25
Demand	57.5	20.25	33.5	31.5	

Table-4

Sources	Demand D_1	Demand D_2	Demand D_3	Demand D_4	Supply
S_1	**(16, 25, 36, 49)** (1, 4, 9, 16)	(4, 9, 16, 25)	(9, 16, 25, 36)	(16, 25, 36, 49)	(16, 25, 36, 49)
S_2	**(20, 24, 28, 32)** (4, 9, 16, 25)	**(4, 16, 25, 36)** (9, 16, 25, 36)	**(12, 9, 11, 13)** (16, 25, 36, 49)	(25, 36, 49, 64)	(36, 49, 64, 81)
S_3	(9, 16, 25, 36)	(16, 25, 36, 49)	**(13, 27, 38, 51)** (25, 36, 49, 64)	**(12, 9, 11, 13)** (36, 49, 64, 81)	(25, 36, 49, 64)
S_4	(16, 25, 36, 49)	(25, 36, 49, 64)	(36, 49, 64, 81)	**(4, 16, 25, 36)** (25, 36, 49, 81)	(14, 16, 25, 36)
Demand	(36, 49, 64, 81)	(4, 16, 25, 36)	(25, 36, 49, 64)	(16, 25, 36, 49)	

In this example it has show that the total optimal cost obtains by hour's method demands so that obtained by defuzzying the total four optimal solutions by applying Robust Ranking Method for the fuzzy transportation problem with fuzzy optimal solution

Min Z = (7.5) (31.5) + (13.5) (26) + (21.5) (20.25) + (31.5) (11.25) + (33.5) (22.25)+ (57.5) (11.25) + (47.75) (20.25) =3736.19

Conclusion

In this paper, the transportation cost is considered as imprecise fuzzy numbers. Here, the fuzzy transportation problem has been transformed into crisp transportation using Robust ranking Method. We can have the optimal solution from crisp and fuzzy optimal total cost of given example. Moreover, using Robust ranking Method one can conclude that the solution of fuzzy problem obtained more effectively.

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