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Characteristics Of Fuzzy Digraph With Generalized Modus Tollens Using Mathematical Models

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Abstract

In this work, we introduce the mathematical modelling in terms of fuzzy digraph by using fuzzy rule with generalized modus tollens method and define degree of a vertex in fuzzy digraph, Indegree of a vertex in fuzzy digraph, Out degree of a vertex in fuzzy digraph and generalized modus tollens are discussed.

Keywords: Mathematical Modelling, Generalized Modus Tollens, Fuzzy graph, Fuzzy digraph.

1.Introduction:

Graph theory was introduced 200-years ago. In 1736, Euler first introduced the concept of graph theory. The city of Konigsberg was located on the pregel river in Prussia. The river divided the city into four separate landmasses, including the island of Kneiphopf. In this history of mathematics, the solution given by Euler of the well known Konigsberg bridge problem is considered to be the first theorem of the graph theory.

The first definition of fuzzy graph was introduce by Haufmann in 1973, based on Zadeh's fuzzy relations in 1971. In 1975, Rosenfeld introduced the concept of fuzzy graphs. The fuzzy relations between fuzzy sets were also considered by Rosenfield and he developed the structure of fuzzy graphs using fuzzy relations, obtaining analogs of several graph theoretical concepts. During the same time Yeh and Bang have also introduced various connectedness concepts in fuzzy graph.

Mathematical Modelling have been seen in various other contexts, example, a toy model of a car, a model house a model village and so on. A mathematical model is of no difference to any of these; it will also mimic some objects, a process or a situation.

Simply speaking Mathematical Modeling is the art of using mathematical objects (e.g. equations, matrices as well as computer programs) to explain the dynamic or static behaviour of system/problems we encounter in our day-to-day life. Examples are

- Various biological systems (growth, behaviour, interactions among species, reactions, celluar behaviour
- Physical systems (e.g. travelling of light, waves formation of patterns in nature that we see in animals, universe, atoms, magneters, motion of planets).
- Economics (e.g. behaviour of financial market, growth of economics).
- Social behaviour (e.g. migration, conflicts, interactions)
- Behaviour of nature (e.g. formation of earthquakes, tsunamis, clouds, rain, snow, droughts)
- Industry (e.g. transportation, traffic control/analysis, manufacturing, production lines).

2.Basic Preliminaries:

2.1 Definition:

A graph G is defined as follows.

G = (V,E)

V: Set of vertices. A vertex is also called a node or element.

E: Set of edges. An edge is pair (x,y) of vertices in V.

2.1 Example:

Let G: (V,E) be a (4,5) graph.

Where $V(G) = \{v_1,v_2,v_3,v_4\}$ and

$E(G) = \{v_1v_2,v_2v_3,v_3v_4,v_4v_1,v_4v_2\}$.

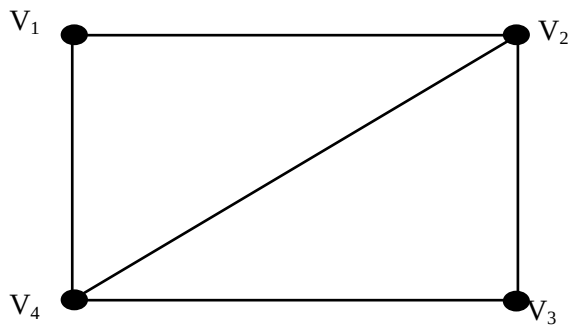

Fig1: G

2.2 Definition:

Let $X,Y \subseteq R$ be universel set, then $R = \{((x,y)),\mu_R(x,y)): (x,y) \in X\times Y\}$ is called a fuzzy relation from X to Y.

2.3 Definition:

$\tilde{G} = (V, \tilde{E})$

V: set of vertices
E: fuzzy set of edges between vertices.
We can think of set V which is a fuzzy set. In this case, we say this graph represents fuzzy relation of fuzzy nodes, and can be defined as following $\tilde{G} = (\tilde{V},\tilde{E})$.

2.3 Example:
The **Fig:2** shows an example of fuzzy graph represented as fuzzy relation matrix M_G.

M_G	b_1	b_2
a_1	0.7	0.5
a_2	0.2	0
a_3	0.8	0.3

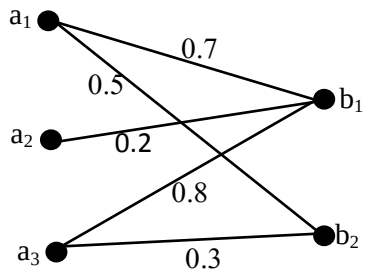

Fig2: Fuzzy Graph

2.4 Definition:
Mathematical model a self-contained set of formulas and/or equations based on an approximate quantitative description of real phenomena and created in the hope that the behavior it predicts will be consistent with the real behavior on which it is based.

- Is indispensable in many applications
- Is successful in many further applications
- Gives precision and direction for problem solution
- Enables a through understanding of the system method.
- Prepares the way for better design or control of a system.
- Allows the efficient use of modern computing capabilities.

2.4 Examples: Bacteria Culture Growth – A Mathematical Modelling
Problem:
A bacteria culture starts with 200 bacteria and grows at a rate proportional to its size. After 3 hours there are 8000 bacteria. Find the number of bacteria after 4 hours.

Discussion:
Identifying variables:
Let y stands for the bacteria culture and t stands for time passed. The first part of the problem "A bacteria culture starts with 200 bacteria, tells us that y(0) = 200". The second part of the problem "A bacteria grows at a rate proportional to its size (is the key for getting the mathematical model). Recall that the rate is the derivative and that is (proportional to … "corresponds to") equal to constant multiple of … "So, the equation relating the variables is

$\frac{dy}{dt}$ = ky.
The solution of this differential equation is
y = $y_0\, e^{kt}$.
Since y_0 = 200, it remains to determine the proportionality constant k. From the condition, after 3 hours there are 8000 bacteria. We obtain that
8000 = 200 e^{3k} which gives us that k= $\frac{1}{3}$In 40 = 1.2296.
Thus, the number of bacteria after t hours can be described by
y = 200 $e^{1.2296t}$.

Solution:
Applying mathematical modelling in the above problem we have obtained, the number of bacteria after 4 hours to be y(4) = 27359 bacteria.

2.5 Definition:
A directed fuzzy graph D is a pair (V,A), where V is a finite non-empty set and A is a subset of V×V- {(x,x)/xϵV}. The elements of V and A are respectively called vertices and arcs. If (u,v)ϵA then the arc (u,v) is said to have u as its initial vertex (tail) and v as its terminal vertex (head). Also the arc (u,v) is said to join u to v.

2.5 Example:
D = (V,A) where V = {$\sigma(v_1)$, $\sigma(v_2)$, $\sigma(v_3)$, $\sigma(v_4)$}
A = {$\mu(v_1v_2)$, $\mu(v_3v_1)$, $\mu(v_2v_3)$, $\mu(v_3v_2)$} is a fuzzy digraph.

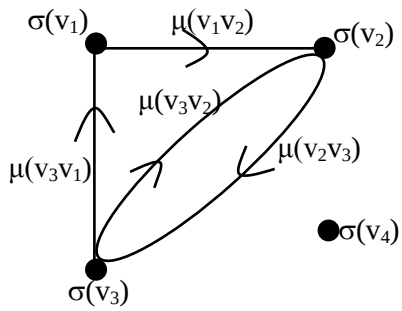

Fig3: Fuzzy Digraph

2.6 Definition:
The degree of any vertex $\sigma(v_i)$ of a fuzzy graph is sum of degree of membership of all those edges which are incident on a vertex $\sigma(v_i)$. And is denoted by $d[\sigma(v_i)]$.
In a fuzzy digraph the number of arcs directed away from the vertex σ(v) is called the outdegree of vertex, it is denoted by od[σ(v)]. The number of arcs directed to the vertex σ(v) is called indegree of vertex, it is denoted by id[σ(v)].
The degree of a vertex $\sigma(v_i)$ in a fuzzy digraph is defined to be degσ(v) = od[σ(v)] + id[σ(v)]. The ordered pair (od[σ(v)], id[σ(v)]) is called the degree pair of σ(v).

2.6 Example:

Vertex	id[$\sigma(v_i)$]	od[$\sigma(v_i)$]	Degree
$\sigma(v_1)$	0	0.9	0.9
$\sigma(v_2)$	0.8	0.1	0.9
$\sigma(v_3)$	0	0.8	0.8
$\sigma(v_4)$	1	0	1

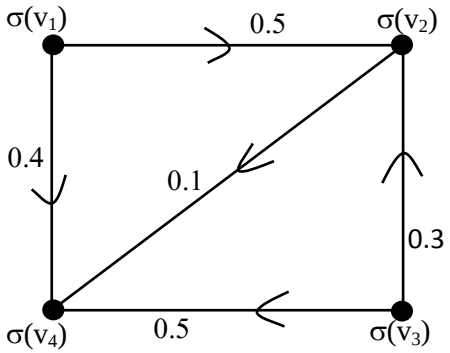

Fig4: G

2.7 Definition:
Two fuzzy digraphs $D_1 = (V_1,A_1)$ and $D_2 = (V_2,A_2)$ are said to be isomorphic ($D_1 \cong D_2$) if there exist a bijection $f : V_1 \rightarrow V_2$ such that $\mu(v_1,v_2) \in A_1$, iff $\mu(f(v_1),f(v_2)) \in A_2$. f is called isomorphism from D_1 to D_2.

2.7 Example:

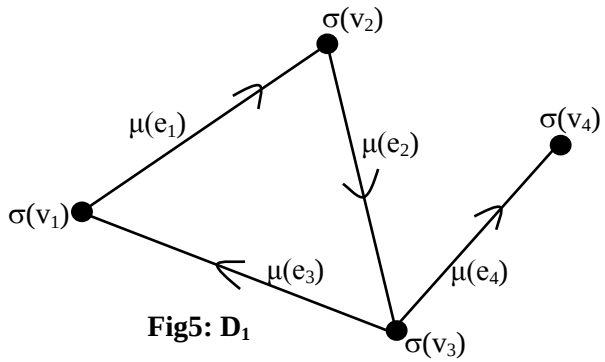

Fig5: D_1

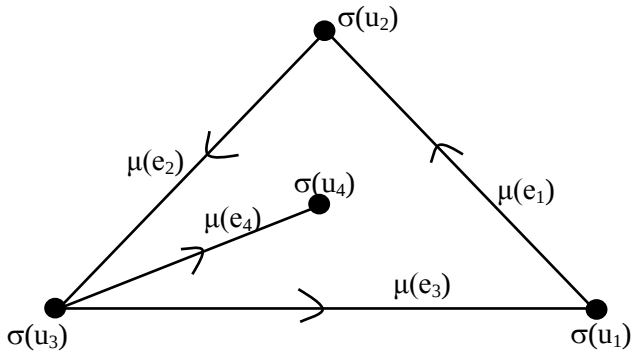

Fig5: D_2

The fuzzy digraphs are isomorphic under the mapping f where $f(\sigma(v_1)) = \sigma(u_1)$, $f(\sigma(v_2)) = \sigma(u_2)$, $f(\sigma(v_3)) = \sigma(u_3)$, $f(\sigma(v_4)) = \sigma(v_4)$.

2.8 Definition:
The set of all vertices adjacent to a vertex $\sigma(v)$ is called the neighborhood of $\sigma(v)$ and is denoted by $N(\sigma(v))$.

2.8 Example:
The neighborhood of $\sigma(v_2)$ is $\{\sigma(v_1), \sigma(v_4), \sigma(v_5)\}$ and $\mu(e_1)$, $\mu(e_4)$, $\mu(e_5)$ are incident with $\sigma(v_2)$.

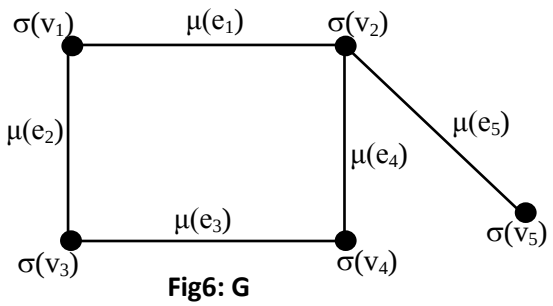

Fig6: G

2.9 Definition: A generalized modus tollens, is expressed by the following schema, such that
Rule: If x is A Then y is B
Fact: y is B'
Conclusion: x is A'
Where A' is close to A [i.e. A' = A] and B' is close to B [i.e. B' = B] are fuzzy sets of appropriate universes, the foregoing inference procedure is called approximate reasoning or fuzzy reasoning, it is also called Generalized Modus Tollens. The generalized modus tollens is in the backward one.

3. Characteristics of Fuzzy Digraph With Generalized Modus Tollens Using Mathematical Models

3.1 Theorem:
If two fuzzy digraphs are isomorphic then corresponding vertices have the same degree.

Proof:
Let $D_1 = (V_1,A_1)$ and $D_2 = (V_2,A_2)$ be isomorphic under an isomorphism f.
Let $\sigma(v) \in V_1$.
Let $N(\sigma(v)) = \{\sigma(w)/(\sigma(w) \in V_1$ and $\mu(\sigma(v),\sigma(w)) \in A_1\}$
And $N(f(\sigma(v))) = \{\sigma(w)/(\sigma(w) \in V_2$ and $[f(\sigma(v)),f(\sigma(w))] \in A_2\}$
Now, $\sigma(w) \in N(\sigma(v))$ iff $\mu(\sigma(v),\sigma(w)) \in A_1$
iff $(f(\sigma(u),r(\sigma(w))) \in A_2$
iff $f(\sigma(w)) \in N(f(\sigma(v)))$ [by definition of $N(f(v))$]
Hence $| N(\sigma(v)) | = | N(f(\sigma(v))) |$ (since f is a bijection)

Here the L.H.S and R.H.S are respectively the outdegree of $\sigma(v)$ and $f(\sigma(v))$.Hence $\sigma(v)$ and $f(\sigma(v))$ have the same outdegree.
Similarly, we can prove that $\sigma(v)$ and $f(\sigma(v))$ have the same indegree and hence $\sigma(v)$ and $f(\sigma(v))$ have the same degree pair.

3.1 Example:

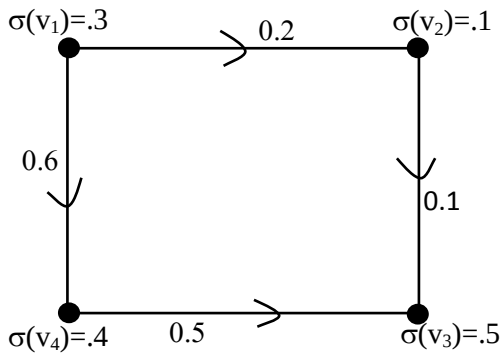

Fig7:D_1

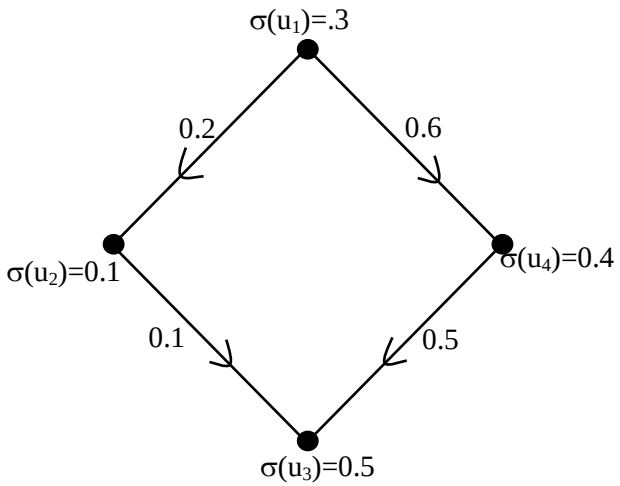

Fig7:D_2

Let $V_1 = \{\sigma(v_1), \sigma(v_2), \sigma(v_3), \sigma(v_4)\}$ and Let $V_2 = \{\sigma(v_1), \sigma(v_2), \sigma(v_3), \sigma(v_4)\}$ and $A_1 = \{\mu(v_1v_2), \mu(v_1v_3), \mu(v_2v_3), \mu(v_3v_2)\}$ $A_2 = \{\mu(v_1v_2), \mu(v_1v_3), \mu(v_2v_3), \mu(v_3v_2)\}$
$f(\sigma(v_1)) = \sigma(u_1)$, $f(\sigma(v_2)) = \sigma(u_2)$, $f(\sigma(v_3)) = \sigma(u_3)$, $f(\sigma(v_4)) = \sigma(v_4)$.
The indegree and outdegree of $\sigma(v_1)$ and $f(\sigma(v_1))$.

Vertices	Indegree	Outdegree
$\sigma(v_1)$	0	0.8
$f(\sigma(v_1))$	0	0.8

(i.e) The degree pair of $\sigma(v_1)$ is (0, 0.8) and the degree pair of $f(\sigma(v_1))$ is (0, 0.8).
Hence $\sigma(v_1)$ and $f(\sigma(v_1))$ have the same indegree and outdegree. Hence the vertices $\sigma(v_1)$ and $f(\sigma(v_1))$ have the same degree pair.

3.2 Result:
Characteristics of fuzzy digraph with generalized modus tollens method using mathematical models.

Proof:

For a communication networks, set up the corresponding transition probability matrix and find the importance of each member in the network. In a fuzzy digraph D, sum of the indegrees of all the vertices is equal to the sum of their outdegrees, each sum being equal to the number of arcs in D.

A fuzzy digraph can serve as a model for a communication network.Consider the network given in **fig8:**If an (edge) arc is directed from $\sigma(v_1)$ to $\sigma(v_2)$, it means that $\sigma(v_1)$ can communicate with $\sigma(v_2)$.

In the given networks $\sigma(v_5)$ can communicate directly with $\sigma(v_2)$, but $\sigma(v_2)$ can communicate with $\sigma(v_5)$ only indirectly through $\sigma(v_3)$ and $\sigma(v_4)$.

In the given networks

$\sigma(v_1)$ can communicate directly with $\sigma(v_2)$&$\sigma(v_3)$.

$\sigma(v_2)$ can communicate directly with $\sigma(v_1)$&$\sigma(v_3)$.

$\sigma(v_3)$ can communicate directly with $\sigma(v_1)$, $\sigma(v_2)$, &$\sigma(v_4)$.

$\sigma(v_4)$ can communicate directly with $\sigma(v_3)$ &$\sigma(v_5)$.

$\sigma(v_5)$ can communicate directly with $\sigma(v_2)$.

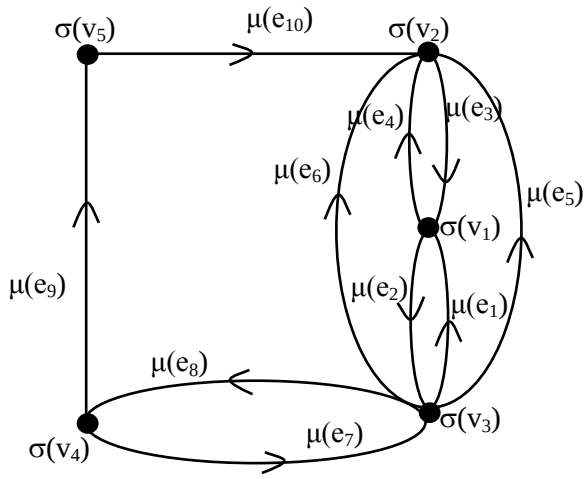

Fig8:Fuzzy Digraph

However, every individual can communicate with every other individual.

Our problem is to determine the importance of each individual in this network. The importance can be measured by the fraction of the messages on an average that pass through him.

In the absence of any other knowledge, we can assume that if an individual can send message direct to n-individuals, he will send a message to any one of them with probability $\frac{1}{n}$.

In the present example, the communication probability matrix is

$$\begin{array}{c} \\ \sigma(v_1) \\ \sigma(v_2) \\ \sigma(v_3) \\ \sigma(v_4) \\ \sigma(v_5) \end{array} \begin{array}{c} \begin{array}{ccccc} \sigma(v_1) & \sigma(v_2) & \sigma(v_3) & \sigma(v_4) & \sigma(v_5) \end{array} \\ \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Here no individual is to send a message to himself and so all diagonal elements are zero. Since all elements of the matrix are non-negative and the sum of elements of every row is unity, the matrix is a stochastic matrix and one of its eigenvalues is unity. The corresponding normalized eigenvector is $[\frac{11}{45}, \frac{13}{45}, \frac{3}{10}, \frac{1}{10}, \frac{1}{15}]$. In the long run, these fractions of messages will pass through $\sigma(v_1)$, $\sigma(v_2)$, $\sigma(v_3)$, $\sigma(v_4)$, $\sigma(v_5)$ respectively. Thus we can conclude that in this network, $\sigma(v_3)$ is the most important person.

Let B = sum of the indegrees of all the vertices in D.

C = sum of the outdegrees of all the vertices in D.

$\sum_i \mu(v_i, v_{i+1})$ = sum of the number of arcs in D.

Vertices	Indegree	Outdegree	Degree
$\sigma(v_1)$	0.2	0.9	1.1
$\sigma(v_2)$	1.6	0	1.6
$\sigma(v_3)$	1.3	1.4	2.7
$\sigma(v_4)$	0.2	0.9	1.1
$\sigma(v_5)$	0.4	0.5	0.9

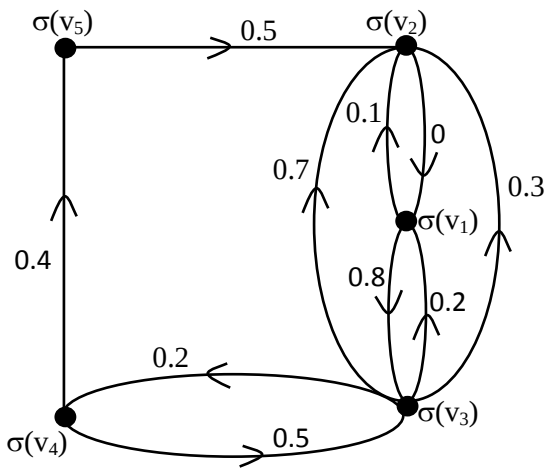

Fig8: Fuzzy Digraph

Rule1:
Rule : IF x is B THEN y is equal to C
Fact : y is 3.7
Conclusion **:** x is 3.7

Example:
B= $\sum_{v\epsilon V} id[\sigma(v)]$
= 0.2 + 1.6 + 1.3 + 0.2 + 0.4
= 3.7
C = $\sum_{v\epsilon V} od[\sigma(v)]$
= 0.9 + 0 + 1.4 + 0.9 + 0.5 =3.7

Rule2:
Rule : IF x is C THEN y is equal to $\sum_i \mu(v_i,v_{i+1})$
Fact : y is 3.7
Conclusion **:** x is 3.7

Example:
$\sum_i \mu(v_i,v_{i+1})$ = 0.2 + 0.8 + 0.1 + 0 + 0.7 + 0.3 + 0.2 + 0.5 + 0.4 + 0.5
= 3.7
Finally B = C = $\sum_i \mu(v_i,v_{i+1})$.
(i.e) 3.7 = 3.7 = 3.7

An arc ($\sigma(v_1)$, $\sigma(v_2)$) contributes one to the outdegree of $\sigma(v_i)$ and one to the indegree of $\sigma(v_i)$. Hence each arc contributes one to the sum B and one to the sum C.
Hence B = C = $\sum_i \mu(v_i,v_{i+1})$.
Hence proved.

Conclusion:

Finally, we have analyzed some concepts of fuzzy digraph, outdegree of fuzzy digraph, indegree of fuzzy digraph, and we have easily obtained fuzzy digraphs indegree and outdegree by mathematical model with generalized modus tollens method.

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