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Micro-inertial Effect in the Propagation of Micropolar Wave in Micropolar Medium

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Abstract

This paper concerns the effect of micro-inertia in the reflection of Micropolar wave from the free surface of Micropolar half-space. The Amplitude ratios and Energy ratios of the reflected waves are obtained. Numerical computation of amplitude and energy ratios for particular model to show the effect of micro-inertia in the assigned angle of incidence and assigned frequency.

AMS subject classification:
Keywords: Micropolar wave, micro-inertia, amplitude ratio, energy ratio.

1. Introduction

In classical elasticity the motion is described by displacement vector only, so that there exist three degrees of freedoms only. Eringen [1] introduced the linear theory of micropolar elasticity, he takes into account the micro-structural motions, in which the motion is described by not only displacement vector but also microrotation vector, so that there are six degrees of freedoms. Parfitt and Eringen [2] obtained the existence of four waves in micropolar elastic materials, they also studied the propagation of plane waves and their reflections from a stress free flat surface. Ariman [3] discussed wave propagation in an infinite micropolar elastic half space and the reflection of plane longitudinal displacement waves from a fixed flat surface of micropolar elastic half space. Eringen [4] obtained the dispersion relation for the transverse plane waves in linear non local micropolar elastic solid. Singh [5] investigated the plane wave propagation and reflection from a stress free boundary of an orthotropic micropolar elastic solid. Sharma and Kumar [6] investigated

Lamb waves in micropolar thermoelastic plates bordered by inviscid liquid layers with varying temperature on both sides.

Micropolar wave is one of the longitudinal wave in the micropolar medium. In this paper, we shall study the propagation of plane waves in the micropolar elastic medium using the linear theory developed by Eringen [1]. We shall investigate the effect of micro-inertia in the propagation of micropolar waves. The amplitude ratios and energy ratios of reflected waves are obtained when micropolar wave is made incident on the boundary using appropriate boundary conditions derived by Parffit and Eringen [2]. These amplitude and energy ratios are computed numerically for particular model to show the effect of micro-inertia.

2. Governing Equations

Following [1], the field equation for homogeneous and isotropic micropolar elasticity may be written as

$$(\mu+\kappa)\nabla^2\mathbf{u}+(\lambda+\mu)\nabla\nabla\cdot\mathbf{u}+\kappa\nabla\times\boldsymbol{\phi} = \rho\ddot{\mathbf{u}}, \tag{1}$$

$$(\gamma\nabla^2-2\kappa)\boldsymbol{\phi}+(\alpha+\beta)\nabla\nabla\cdot\boldsymbol{\phi}+\kappa\nabla\times\mathbf{u} = \rho J\ddot{\boldsymbol{\phi}}, \tag{2}$$

where λ, μ are lame's parameters; α, β, γ, κ are micropolar parameters. $\mathbf{u}$, and ϕ are respectively displacement and micropolar vectors; J is the micro-inertia tensor; ρ is the mass density.

Using Helmholtz decomposition theorem, we can write

$$\{\mathbf{u},\boldsymbol{\phi}\}=\nabla\{u',\phi'\}+\nabla\times\{\mathbf{u}'',\boldsymbol{\phi}''\},\ \nabla\cdot\{\mathbf{u}'',\boldsymbol{\phi}''\}=0, \tag{3}$$

where u' and ϕ' are the scalar potentials while $\mathbf{u}''$, $\boldsymbol{\phi}''$ are vector potentials.

Employing Eq. (3) into Eq. (1–2), we can have.

$$(\lambda+2\mu+\kappa)\nabla^2u' = \rho\ddot{u}', \tag{4}$$

$$(\alpha+\beta+\gamma)\nabla^2\phi'-2\kappa\phi' = \rho J\ddot{\phi}', \tag{5}$$

$$(\mu+\kappa)\nabla^2\mathbf{u}''+\kappa\nabla\times\boldsymbol{\phi}'' = \rho\ddot{\mathbf{u}}'', \tag{6}$$

$$(\gamma\nabla^2-2\kappa)\boldsymbol{\phi}''+\kappa\nabla\times\mathbf{u}'' = \rho J\ddot{\boldsymbol{\phi}}'', \tag{7}$$

It is clear that Eqs. (4)–(5) correspond to longitudinal waves while Eqs. (6)–(7) correspond to shear waves. Eq. (6) and Eq. (7) are couple in u'' and ϕ'' while Eq. (4) and Eq. (5) are not coupling. There are four waves in micropolar medium, two uncoupled longitudinal waves (say micropolar wave and longitudinal displacement wave) and two coupled shear waves (say shear micropolar wave and shear displacement wave). The micropolar wave and shear micropolar wave exist only when $\omega^2 > \dfrac{2\kappa}{\rho J}$, below which they degenerate into a distance decaying vibrations [2].

3. Reflection of micropolar wave

Let us consider the two dimensional plane (i.e. xz plane). By taking the displacement vector ($\mathbf{u}''$) and micro-rotation vector ($\boldsymbol{\phi}$) as follows:

$$\boldsymbol{\phi} = (\phi_1, 0, \phi_3), \ \mathbf{u}'' = (0, U, 0). \tag{8}$$

We shall consider the incident micropolar wave, satisfying the boundary conditions at the free surface $z = 0$, then the incident micropolar wave will give rise to micropolar wave, shear micropolar and shear displacement waves. Using Eq. (3), the stress and couple stresses at the free surface ($z = 0$) can be written as [2]

$$m_{zz} = (\alpha + \beta + \gamma)(\frac{\partial^2\phi'}{\partial z^2} + \frac{\partial^2\phi''}{\partial x\partial z}) + \alpha(\frac{\partial^2\phi'}{\partial x^2} - \frac{\partial^2\phi''}{\partial x\partial z}) = 0, \tag{9}$$

$$m_{zx} = \beta(\frac{\partial^2\phi'}{\partial x\partial z} + \frac{\partial^2\phi''}{\partial x^2}) + \gamma(\frac{\partial^2\phi'}{\partial x\partial z} - \frac{\partial^2\phi''}{\partial z^2}) = 0, \tag{10}$$

$$t_{zy} = \kappa\frac{\partial\phi'}{\partial x} - \kappa\frac{\partial\phi''}{\partial z} + \mu\frac{\partial^2 U}{\partial z^2} = 0, \tag{11}$$

where ϕ'' is the $y-$component of $\boldsymbol{\phi''}$.

The potentials ϕ' and U are taken in the form

$$\phi' = A_0 \exp\{\imath k_0(x\sin\theta_0 - z\cos\theta_0) - i\omega t\} + \sum_{r=1}^{3} A_r \exp\{\imath k_r(x\sin\theta_r + z\cos\theta_r) - i\omega t\}, \tag{12}$$

$$\{\phi'', U\} = \sum_{r=2}^{3}\{1, k_r^2\eta_r\}A_r \exp\{\imath k_r(x\sin\theta_r + z\cos\theta_r) - i\omega t\}, \tag{13}$$

where θ_0 is the angle of incident wave with the positive direction of the z-axis, and A_0 is amplitude of incident micropolar wave, and η_r is coupling parameters given as

$$\eta_r = k_r^{-2}(2\kappa + k_r^2\gamma - \omega^2\rho J)/\kappa$$

The reflected micropolar wave with amplitude A_1 make an angles θ_1, reflected shear micropolar wave with amplitude A_2 makes angle θ_2, and reflected shear displacement wave with amplitude A_3 makes angle θ_3, as shown in figure (1). The ratio of amplitudes of reflected waves to the amplitude of incident wave are represented as $Z_1(= A_1/A_0)$, $Z_2(= A_2/A_0)$, $Z_3(= A_3/A_0)$, From Snell's law, we have the relations between wave numbers and incident/reflected angles at the surface $z = 0$ given as

$$k_1\sin\theta_1 = k_2\sin\theta_2 = k_3\sin\theta_3 = k_0\sin\theta_0 \tag{14}$$

3.1. Amplitude ratios

Now, employing Eqs. (12)-(14) in the Eqs. (9)-(11), we have the following system of equations.

$$\sum_{r=1}^{3} a_{ij} Z_j = b_j,\ \ j = 1, 2, 3 \tag{15}$$

where,

$$a_{11} = (\alpha + (\beta + \gamma)\cos\theta_1)(k_1/k_0)^2,\ a_{12} = \{\alpha + (\beta + \gamma)\cos\theta_2 + (\beta + \gamma)\cos\theta_2 \sin\theta_2\}k_2^2,$$

$$a_{13} = \{\alpha + (\beta + \gamma)\cos\theta_3 + (\beta + \gamma)\cos\theta_3 \sin\theta_3\}k_3^2,\ b_1 = -\{\alpha + (\beta + \gamma)\cos^2\theta_0\}k_0^2,$$

$$a_{21} = \{(\beta + \gamma)\cos\theta_1 + \beta\sin\theta_1\}\sin\theta_1 k_1^2,\ a_{22} = \{(\beta + \gamma)\cos\theta_2\sin\theta_2 + \beta\sin^2\theta_2 - \gamma\cos^2\theta_2\}k_2^2,$$

$$a_{23} = \{(\beta + \gamma)\cos\theta_3\sin\theta_3 + \beta\sin^2\theta_3 - \gamma\cos^2\theta_3\}k_3^2,\ b_2 = \{(\beta + \gamma)\cos\theta_0 - \beta\sin\theta_0\}\sin\theta_0 k_0^2,$$

$$a_{31} = \kappa\sin\theta_1 k_1,\ a_{32} = \{\kappa\sin\theta_2 - \kappa\cos\theta_2 + k_2^2\mu\eta_2\cos^2\theta_2\}k_2,$$

$$a_{33} = \{\kappa\sin\theta_3 - \kappa\cos\theta_3 + k_3^2\mu\eta_3\cos^2\theta_3\}k_3,\ b_3 = -\kappa\sin\theta_0 k_0$$

The Eq. (15) gives the formulae for the Amplitude ratio of various reflected waves at a plane free surface $z = 0$ for incident micropolar wave.

3.2. Energy ratios

Let us consider the energy partitioning between various reflected waves at the free surface. The energy communication per unit area at the free surface ($z = 0$) of micropolar elasticity is given as

$$E^* = \langle m_{zz}, \dot{\phi}_z\rangle + \langle m_{zx}, \dot{\phi}_x\rangle + \langle t_{zy}, \dot{u}_2\rangle \tag{16}$$

Now, the energy ratio of reflected waves to incident waves can be calculated with the help of Eqs. (12)–(14) into Equation (16). The energy of incident wave is given by

$$E_{inc} = \{(\alpha + \beta + \gamma)\cos\theta_0 - \beta\sin^3\theta_0\}A_0^2\omega k_0^3 \tag{17}$$

The following expression gives the energy ratios.

$$E_i = E_i^*/E_{inc},\ i = 1, 2, 3 \tag{18}$$

where, $E_1^* = \{(\alpha + \beta + \gamma)\cos\theta_1 + \beta\sin^3\theta_1\}A_1^2 k_1^3\omega$,
$E_2^* = \{(\alpha + \beta + 2\gamma - \kappa/k_2^2 - \mu\eta_2\cos\theta_2)\cos\theta_2 + (\alpha + \beta + \kappa/k_2^2)\sin\theta_2\}A_2^2 k_2^3\omega$,
$E_3^* = \{(\alpha + \beta + 2\gamma - \kappa/k_3^2 - \mu\eta_3\cos\theta_3)\cos\theta_3 + (\alpha + \beta + \kappa/k_3^2)\sin\theta_3\}A_3^2 k_3^3\omega$.

Here, E_1 correspond to energy ratios of micropolar wave, E_2 and E_3 correspond to shear micropolar and shear longitudinal waves.

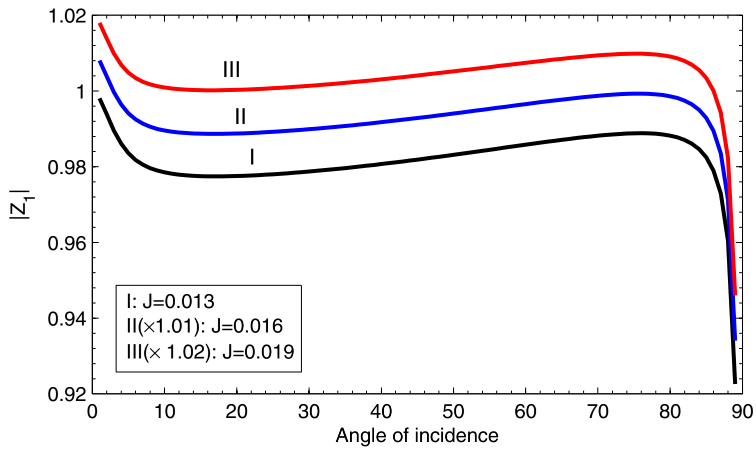

Figure 1: Variation of $|Z_1|$ with angle of incidence (θ_0) at different values of micro-inertia.

4. Numerical results

In this section, the amplitude ratios and the energy ratios of reflected waves are computed numerically to show the effect of micro-inertia. Following Gauthier [7], the modified values of physical constants for micropolar elasticity are considered as

$$\lambda = 7.59 \times 10^{10}\ N/m^2,\ \mu = 18.9 \times 10^9\ N/m^2,\ \kappa = 0.0149 \times 10^{10}\ N/m^2,$$

$$\rho = 2.19 \times 10^3\ Kg/m^3,\ \alpha = 0.7 \times 10^5\ N,\ \beta = 0.01 \times 10^5\ N,\ \gamma = 2.68 \times 10^5\ N.$$

The graphs are plot for different values of micro-inertia,

$$J = \{I : 0.013,\ II : 0.016,\ III : 0.019\} \times 10^{-4}\ m^2.$$

The variations of amplitude ratios and energy ratios of reflected waves for incident micropolar wave with respect to angle of incidence (θ_0) are depicted in the Figures 2-5, for this purpose the value of frequency (ω) is assigned $5s^{-1}$. Similarly, the variations of amplitude and energy ratios for incident micropolar waves with respect to frequency (ω) is also depicted respectively in Figures 6-9, for this purpose the angle of incidence (θ_0) is kept at 60^0.

In Figures 2 and 4, the values of $|Z_1|$ and $|E_1|$ start decrease from a certain values up to $\theta_0 = 13^0$ and then they increases up to $\theta_0 = 80^0$, thereafter they decrease to minimum values at $\theta_0 = 90^0$. The minimum effect of micro-inertia is found at $\theta_0 = 90^0$ and the values of $|Z_1|\&|E_1|$ increases as in creasing micro-inertia. In Figure 3, the values of $|Z_2|$ (Curves I, II, III) and $|Z_3|$ (Curves IV, V, VI) starts increase from a certain values up to $\theta_0 = 10^0$ and then they decreases as increasing angle of incidence. The values of $|Z_2|$ and $|Z_3|$ increase and decrease respectively with increasing micro-inertia J, and the minimum effect observed at the normal and grazing angle of incidence. The similar nature of $|E_2|$ and $|E_3|$ with $|Z_2|$ are found in Figure 5.

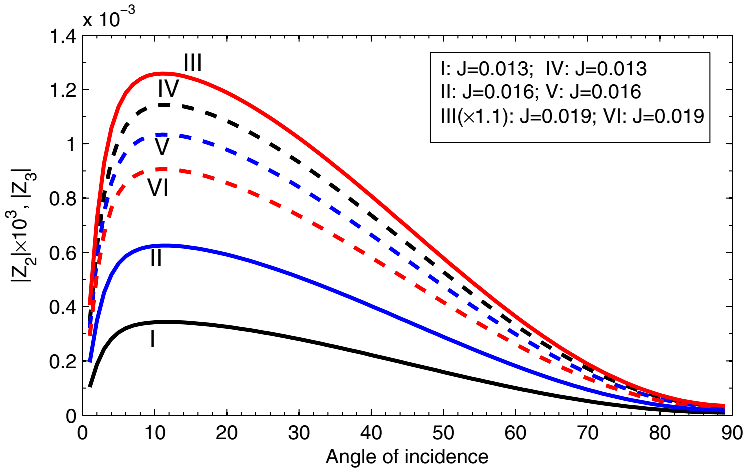

Figure 2: Variation of $|Z_2|\&|Z_3|$ with angle of incidence (θ_0) at different values of micro-inertia.

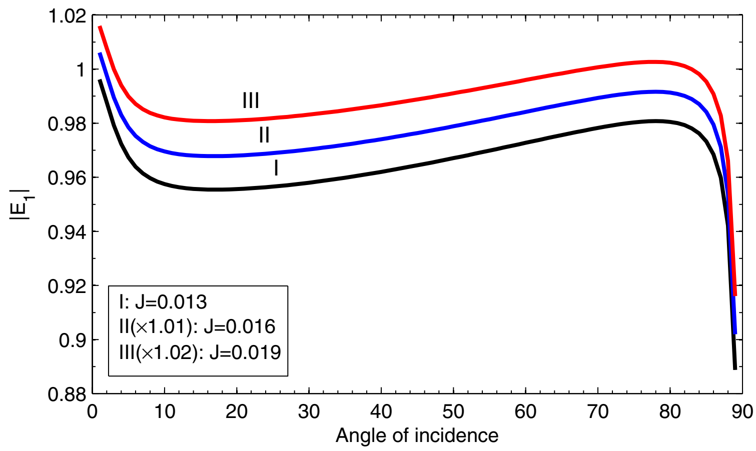

Figure 3: Variation of $|E_1|$ with angle of incidence (θ_0) at different values of micro-inertia.

Figure 6 show the similar natures of $|Z_1|$ (Curves I, II, III) and $|Z_2|$ (Curves IV, V, VI) with respect to frequency, they increase as increasing frequency throughout the ranges $(2-4.5)s^{-1}$, and their values both increase as increasing micro-inertia (J). The values of $|Z_3|$ decreases as increasing frequency shown in Figure 7 while the value of $|E_1|$ in Figure 8 increase with increasing frequency. We see that $|Z_3|\&|E_1|$ both increase with increasing micro-inertia. In Figure 8, $|E_2|$ (Curves I, II, III) increase and $|E_3|$ (Curves IV, V, VI) decrease as increasing frequency, they both increase with increasing micro-inertia in the range $(2-4.5)s^{-1}$. The minimum and maximum effect of micro-inertia is found at $\omega = 2s^{-1}$ and $\omega = 4.5s^{-1}$ for the ratios $|Z_1|$, $|Z_2|$, $|E_1|\&|E_2|$ while for $|Z_3|\&|E_3|$, maximum effect is at $\omega = 2s^{-1}$ and maximum effect is at $\omega = 4.5s^{-1}$. It is noted that

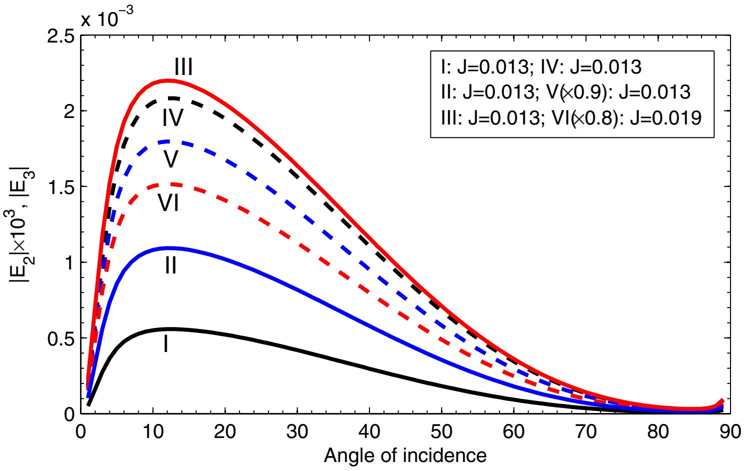

Figure 4: Variation of $|E_2|\&|E_3|$ with angle of incidence (θ_0) at different values of micro-inertia.

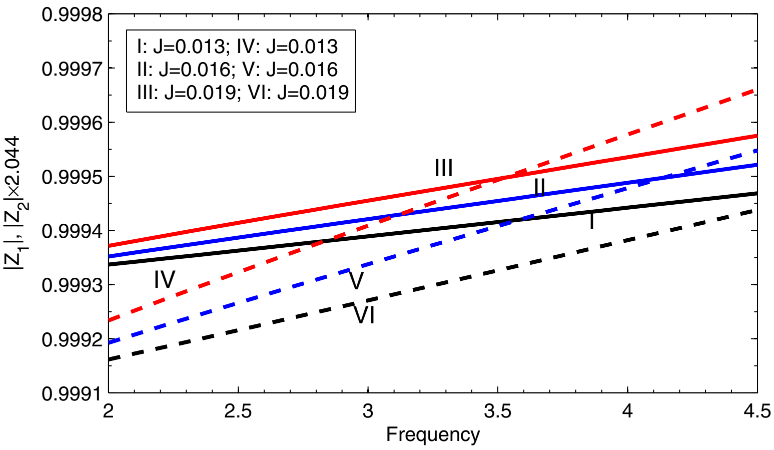

Figure 5: Variation of $|Z_1|\&|Z_2|$ with frequency (ω) at different values of micro-inertia.

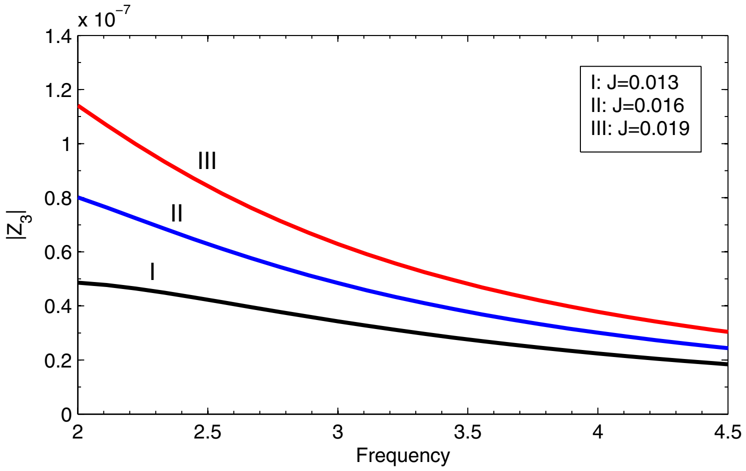

Figure 6: Variation of $|Z_3|$ with frequency (ω) at different values of micro-inertia.

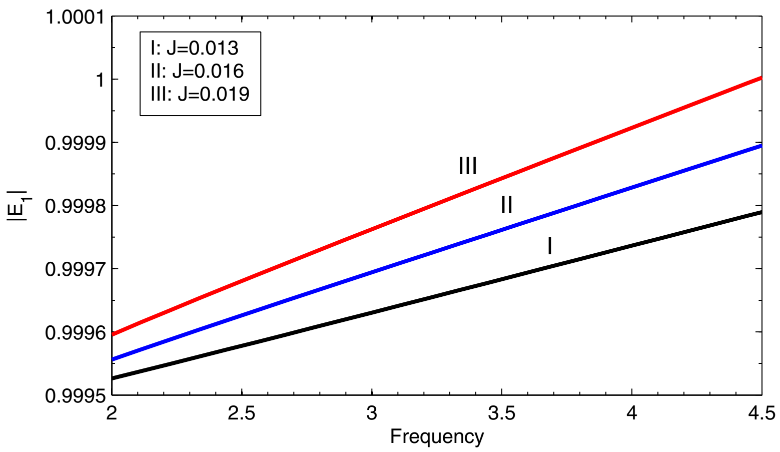

Figure 7: Variation of $|E_1|$ with frequency (ω) at different values of micro-inertia.

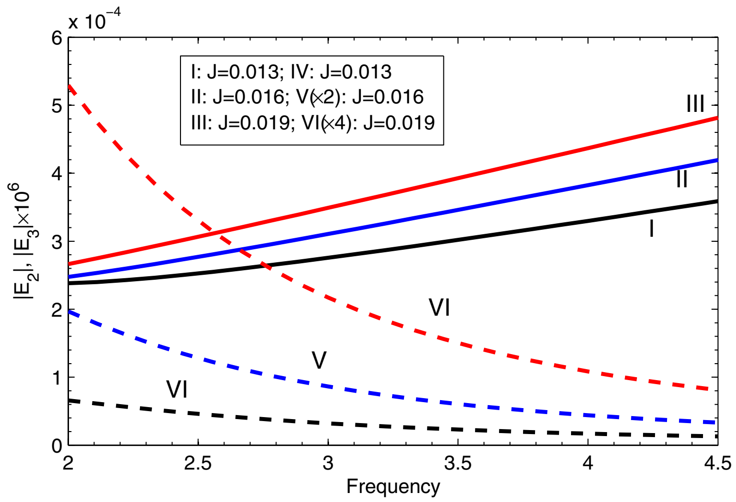

Figure 8: Variation of $|E_2|\&|E_3|$ with frequency (ω) at different values of micro-inertia.

sum of energy ratios is closed to unity in both the cases of variation of angle of incidence and frequency.

5. Conclusions

In micropolar elastic medium there are four plane waves, two uncoupled longitudinal waves and two coupled shear waves. When the micropolar wave is made incident on the boundary there are three reflected waves, reflected micropolar wave and two shear waves. The effects of micro-inertia on amplitude and energy ratios of reflected waves are calculated numerically, and depicted graphically with respect to angle of incidence and frequency. The following numerical results may be summarized as

1. The Amplitude ratios $|Z_1|$, $|Z_2|$ and the Energy ratios $|E_1|$, $|E_2|$ are increases as increasing micro-inertia at the assigned frequency ($\omega = 5s^{-1}$).
2. The Amplitude ratios $|Z_3|$ and the Energy ratio $|E_3|$ are decreases as increasing micro-inertia at the assigned frequency ($\omega = 5s^{-1}$).
3. All the Amplitude and the Energy ratios are increases as increasing micro-inertia at the assigned angle of incidence ($\theta_0 = 45^0$).
4. The sum of energy ratios of reflected waves are found to be unity for the assigned angle of incidence and for the assigned frequency.

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