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Analysis of Jenkins Model Ferro Flow for Magnetic Field Dependent Viscosity with Porosity

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The effect of magnetic field dependent viscosity on a steady axi-symmetric ferrofluid flow over a rotating disk with porosity is investigated through Jenkins model. The governing equations are reduced to ordinary differential equations and are solved through asymptotic expansion. In this system we analyze the effect of magnetic field dependent viscosity (m), material constant $(\bar{\beta})$ and porosity (β) on the velocity profiles and pressure. The results obtained are represented in the form of graphs by varying one of the parameter with fixed values of other two parameters. The results reveals that the parameters has significant effect on radial velocity and pressure whereas negligible effect on tangential and axial velocity.

Keywords: Ferrofluid, Material constant, Porosity, Magnetic field dependent viscosity, Rotating disk.

1 Introduction

Ferrofluids, which are also known as magnetic nanofluids are colloidal suspensions of nano sized magnetic particles in a base fluids like water, oil etc (polar or non-polar [2]). The particles changes to super paramagnetism from ferromagnetism when the size of it is in nanometer level. The properties of ferrofluid are excelled in the presence of an external magnetic field. The history of ferrofluid began in 1963, when Steve Papell at NASA invented a suitable liquid rocket fuel with an intention of the fuel to be attracted towards the inlet of pump using external magnetic field in a zero gravity environment [1].

Magnetorheological fluids are different from the ferrofluids. In magnetorheological fluids, as viscosity increases, the fluid behaves as a quasi-solid, but in the case of ferrofluid, they remain in its fluid state even when the stronger magnetic field is applied. Based on this fact, ferrofluids have attracted researchers in the field of engineering and medicine. In engineering, ferrofluid finds its applications in magnetic seals like feed throughs, tandem seals and exclusion seals since ferrofluid seals are reliable and durable. They are used in loud speakers for heat transfer to dissipate the excess heat from the coil to the surrounding and also ferrofluid damps the resonances which produce an unpleasant noise in stepper motors and in printing system. Ferrofluids modelled through Jenkins Model have wear resisting and better friction reduction and hence used in mechanical engineering for bearing and lubrications purposes, which leads to increase in effective performance of the system. In thermal engineering ferrofluids are used for the enhancement of heat flow.

In the case of drug targeting, ferrofluids with an appropriate drug is injected into the artery and then guided to the target cell using strong external magnets. In MRI scans ferrofluids are used as a contrasting agent which in turn calculates the relaxation time through which a healthy cells can be differentiated from a tumor cells. In the case of MPI scans, ferrofluids used provides a high-contrast image of human or small animal angiography in which, the technique exploits an iron oxide nanoparticles to develop an image whose resolution is dependent on the nanoparticle and the applied magnetic field etc. [3, 4, 5].

Magnetoviscous effects are explained by the formation of chain of larger particles in a small fraction in the fluid. Stefan Odenbach [6] in his study on effects of magnetoviscosity of ferrofluids showed that in the collinear case of magnetic field and vorticity, the influence of viscosity on the flow does not appear. Where in the case of vorticity and field direction are perpendicular, the magnetic torque produced exhibits an increased viscosity by increasing the flow resistance. Further Odenbach explains that in fluid the smaller particles forms an upper fraction, which expresses the weak magnetoviscous effects and behaves as a Newtonian fluid whereas the lower fraction formed includes particles of large size and thus shows a remarkable reaction in change of magnetoviscosity on increasing magnetic field strength. McTague [7] Showed that viscosity depends magnetic field strength and its orientation to the flow which in turn depends on the suspended particles rotation. In support to this Rosensweig et al. [8] showed the viscosity can be measured as the ratio of hydrodynamic stress to magnetic stress. He predicts that ratio value in the range of 10^{-6} to 10^{-4}, the viscosity depends on the magnetic field and rate of shear. Further increase in the ratio leads the viscosity to be independent of the field and shear rate.

Verma and Singh [9] and Joseph [10] analyzed the system in porous medium. Paras Ram et al. [11, 12] analyzed the system of ferrofluid revolving on a rotating disk with and without porous medium and reduced the evaluated set of non-linear partial differential equations to non-linear ordinary differential equations using Von Karman [13] transformation. The equations are solved through Cochran [14] asymptotic expansions.

Rosensweig and Odenbach [15, 6] gave the detailed explanation of magnetic field dependent viscosity effects on ferrofluid. Paras Ram et al. [16, 17] studied the effect of magnetic field dependent viscosity on ferrofluid flow in which they analyzed that the increase in the magnetic field dependent viscosity has a converse effect on radial velocity and pressure whereas negligible effect on tangential velocity and axial velocity. Paras Ram et al. [18] analyzed the system through Neuringer-Rosensweig model and studied the effect of magnetic field dependent viscosity on velocity components and pressure. Bhandari [19] studied the impact of magnetic field dependent viscosity on the velocity profiles and pressure in the presence of a stationary disk.

Vaidyanathan [20] in his study on ferroconvection, analysed that the magnetic field dependent viscosity stabilizes the system which in turn delays the onset of convection. Nanjudappa et al. [21] showed that the effect of magnetic field dependent viscosity for cases of free-free, free-rigid and rigid-rigid surfaces and showed that increase in magnetic field dependent viscosity increases the critical Rayleigh number respectively and hence stabilizes the system. Nanjudappa et al. [22] studied the effect of coriolios force and magnetic field dependent viscosity on a Benard-Marangoni convection and analyzed that with an increase in magnetic field dependent viscosity, delays the ferroconvection and with the decrease of the parameter results in convection cells size reduction. Sunil et al. [23, 24] showed that the presence of magnetic field dependent viscosity parameter stabilizes the system in his study on thermal convection and thermosolutal convection in a ferrofluid contained in a porous medium.

Sunil et al. [25, 26] studied the effects of magnetic field dependent viscosity, dust particles and buoyancy on the system in which ferromagnetic fluid is heated from below and observed that the magnetic field viscosity delays the onset of ferroconvection whereas the dust particles hasten the onset of convection, further buoyancy may have stabilizing or destabilizing effect on the system in the presence of magnetic field dependent viscosity. Vaidhyanathan [27, 28] in his study on ferroconvection with porous medium analyzed that the increase in the magnetic field dependent viscosity increases the critical Rayleigh number and in turn stabilizes the system.

Jyoti Prakash et al. [29, 30] studied the effect of magnetic field dependent viscosity on a complex growth rates in convection of ferromagnetic fluid with and without porous medium. Jyoti Prakash et al. [31] showed that the effect of magnetic field dependent viscosity on the rotating medium of ferromagnetic convection and the magnetic field dependent viscosity has a destabilizing and stabilizing effect in the case of stationary and oscillatory mode. Ramanathan and Suresh [32] studied the effect of the magnetic field dependent viscosity on an isotropy porous medium and analyzed that the presence of magnetic field dependent viscosity stabilizes the system where as anisotropic porous medium destabilizes the system.

Soto-Aquino and Rinaldi [33] studied the effect of magnetic field dependent viscosity on a Newtonian fluid with a dilute suspensions of magnetic nano particles under applied constant magnetic field through rotational Brownian dynamic simulation. Nidhi Andhariya [34] studied the effect of field induced rotational viscosity on a ferrofluid through a capillary placed in a magnetic field and showed the capillary diameter is inversely propotional to field induced rotational viscosity. Pinho et al. [35] showed that in his study, the effect of magnetic field dependent viscosity is more in the case of steady flow in comparison to oscillating shear flow. In this paper mathematical formulation of the problem is in section 2, the solution procedure is in section 3, section 4 contains results and discussion and section 5 contains graphs.

2 Mathematical Formulation

A flow of electrically non-conducting, incompressible, steady symmetric ferrofluid on an infinite disk which is placed at z = 0 is analyzed. The disk is rotating about an axis perpendicular (z-axis) to the plane with constant angular velocity ω occupied with porosity where Darcy law is considered. A constant magnetic field is applied to the system. The system is analyzed through Jenkins model in which the applied magnetic field and magnetization are collinear and co-rotational derivative of Magnetization is involved. The geometry of the system is represented in Figure 1.

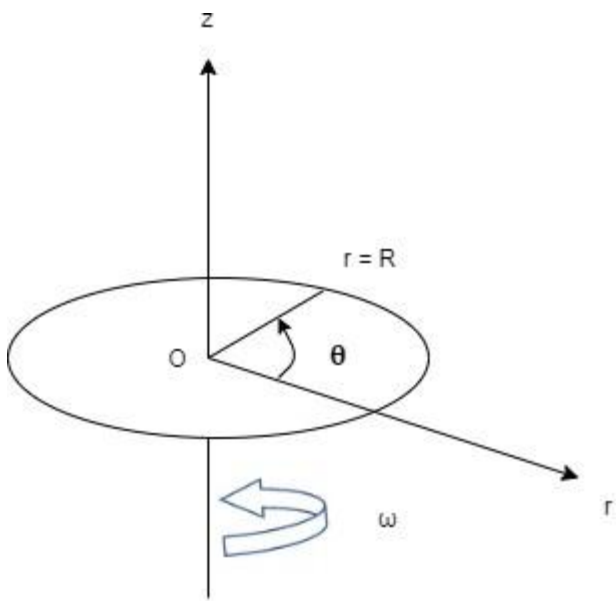

Figure 1: Ferrofluid flow on rotating disk

Equation of continuity for incompressible fluid is,

$$\nabla \cdot \vec{q} = 0, \tag{1}$$

Equation of motion of ferrofluid for Jenkins model is,

$$\rho\left((\vec{q}.\nabla)\vec{q}\right) = -\nabla p + \mu_0\left(\overrightarrow{M}.\nabla\right)\vec{H} + \eta_M \nabla^2 \vec{q} - \frac{\eta_M}{K_0}\vec{q} + \frac{\rho\beta^2}{2}\nabla \times \left(\frac{\vec{M}}{M} \times (\nabla \times \vec{q}) \times \vec{M}\right), \tag{2}$$

Maxwell's Equations are

$$\nabla \times \vec{H} = 0, \tag{3}$$

$$\nabla.\left(\vec{H} + 4\pi\vec{M}\right) = 0, \tag{4}$$

$$\vec{M} = \bar{\mu}\vec{H}, \tag{5}$$

$$\vec{M} \times \vec{H} = 0 \tag{6}$$

where $\vec{q}$ is velocity of ferrofluid, ρ the density, $\vec{H}$ is magnetic field intensity, $\vec{M}$ is magnetization, p is fluid pressure, $\eta_M = \eta\left(1 + \vec{\delta}.\vec{B}\right)$ is magnetic field dependent viscosity, η is the viscosity of ferrofluid in the absence of magnetic field, $\vec{\delta}$ is the variation coefficient of viscosity, μ_0 is the magnetic permeability, $\vec{B}$ is the magnetic induction and K_0 is the permeability of porous medium.

3 Methodology

Let $\vec{q} = (v_r, v_\theta, v_z)$, where v_r, v_θ and v_z represents the radial, tangential and axial velocity. The equations (1) and (2) for a constant magnetic field (H_0, 0, 0) in the form of cylindrical components reduces to

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z} = 0, \tag{7}$$

$$-\frac{\partial p}{\partial r} + \frac{H_0\beta^2\bar{\mu}}{2}\left[\frac{\partial^2 v_r}{\partial z^2} - \frac{\partial^2 v_z}{\partial z\partial r}\right] +$$

$$\eta_B\left[\frac{\partial^2 v_r}{\partial r^2} + \frac{\partial}{\partial r}\left(\frac{v_r}{r}\right) + \frac{\partial^2 v_r}{\partial z^2}\right] - \frac{\eta_B}{K_0}v_r = \rho\left[v_r\frac{\partial v_r}{\partial r} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right], \tag{8}$$

$$\frac{H_0\beta^2\bar{\mu}}{2}\left[\frac{\partial^2 v_\theta}{\partial r^2} + \frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right] + \eta_B\left[\frac{\partial}{\partial r}\left(\frac{v_\theta}{r} + \frac{\partial^2 v_\theta}{\partial r^2} + \frac{\partial^2 v_\theta}{\partial z^2}\right)\right]$$
$$-\frac{\boldsymbol{\eta_B}}{K_0}v_\theta = \rho\left[v_r\frac{\partial v_\theta}{\partial r} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right] \tag{9}$$

$$\frac{\partial p}{\partial z} + \frac{H_0\beta^2\bar{\mu}}{2}\left[\left(\frac{\partial^2 v_r}{\partial r\partial z} - \frac{\partial^2 v_z}{\partial r^2}\right) + \frac{1}{r}\left(\frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial z}\right)\right]$$

$$+\eta_B\left[\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r} + \frac{\partial^2 v_z}{\partial z^2}\right] - \frac{\eta_B}{K_0}v_z = \rho\left[v_r\frac{\partial v_z}{\partial r} + v_z\frac{\partial v_z}{\partial z}\right]. \tag{10}$$

The flow is subjected to following boundary conditions in case of a rotating disk

at $z = 0$; $v_r = 0, v_\theta = r\omega, v_z = 0,$

at $z = \infty$; $v_r = 0, v_\theta = 0, v_z = -c.$ (11)

The equations (7) to (11) will be reduced to nonlinear ordinary differential equations with boundary conditions by Von Karman transformations [13] for rotating disk

$$v_r = r\omega E(\alpha), v_\theta = r\omega F(\alpha), v_z = \sqrt{\nu\omega}G(\alpha), p = \rho\omega\nu P(\alpha), \alpha = \sqrt{\tfrac{\omega}{\nu}}z. \quad (12)$$

where ω is the rotation, ν is the kinematic viscosity, α is the dimensionless parameter of z.

$$G' + 2E = 0, \quad (13)$$

$$\left(m - \bar{\beta}\right)E'' - E^2 + F^2 - GE' - m\beta E = 0, \quad (14)$$

$$mF'' - GF' - 2EF - m\beta F = 0, \quad (15)$$

$$P' - mG'' - 2\beta E' + GG' + m\beta G = 0, \quad (16)$$

$$E(0) = 0, F(0) = 1, G(0) = 0, P(0) = P_0, E(\infty) = 0, F(\infty) = 0$$

and

$G(\infty) = -c$, where c is a finite negative value. (17)

The solution to the equations (13) to (16) are obtained through Cochran asymptotic expansion [14], which are in the form of power series as mentioned below

$$E \approx \sum_{n=1}^{\infty} A_n e^{-(c\alpha n)}, \quad (18)$$

$$F \approx \sum_{n=1}^{\infty} B_n e^{-(c\alpha n)}, \quad (19)$$

$$G \approx \sum_{n=1}^{\infty} C_n e^{-(c\alpha n)}, \quad (20)$$

$$P - P_0 \approx \sum_{n=1}^{\infty} D_n e^{-(c\alpha n)} \quad (21)$$

Assuming $E'(0) = a$ and $F'(0) = b$ and the conditions given in (17) we get values of higher derivatives in E, F, G and P at zero, the initial values are given by

$$\begin{cases} E(0) = 0;\ E'(0) = a;\ E''(0) = -\frac{1}{(m-\bar{\beta})};\ E'''(0) = \frac{am\beta-2b}{(m-\bar{\beta})}; \\ F(0) = 1;\ F'(0) = b;\ F''(0) = \beta; F'''(0) = \frac{m\beta b-2a}{m}; \\ G(0) = 0;\ G'(0) = 0;\ G''(0) = -2a; G'''(0) = \frac{2}{(m-\bar{\beta})}; \\ P(0) = 0;\ P'(0) = 2a(\bar{\beta}-m);\ P''(0) = 2;\ P'''(0) = 4b; \end{cases} \tag{22}$$

Using the numerical values of $a = 0.54$ and $b = -0.62$ and $c = 0.886$ in (22) and approximating the series up to the fourth order we obtain the coefficients A_i, B_i, C_i and D_i where i = 1, 2, 3 and 4 which are listed in the Table 1.

Table 1: Coefficients of variables for $\beta = 1$ and $\bar{\beta} = 0.5$

Coefficients	m = 1	m = 1.1	m = 1.2	m = 1.3
A_1	-0.32750	0.18883	0.55764	0.83425
A_2	1.84178	0.50511	-0.44966	-1.16574
A_3	-2.09159	-0.96723	-0.16412	0.43822
A_4	0.57731	0.27329	0.05614	-0.10673
B_1	2.47111	2.49463	2.51424	2.53083
B_2	-3.22558	-3.29616	-3.35498	-3.40475
B_3	2.33807	2.40865	2.46747	2.51724
B_4	-0.58360	-0.60712	-0.62673	-0.64332
C_1	2.43883	2.27907	2.16496	2.07938
C_2	-2.68839	-2.20912	-1.86679	-1.61004
C_3	1.60429	1.12502	0.78269	0.52594
C_4	-0.46873	-0.30897	-0.19486	-0.10928
D_1	0.58630	0.05808	-0.47013	-0.99835
D_2	-2.61819	-1.46018	-0.30216	0.85585
D_3	2.86800	2.01473	1.16146	0.30818
D_4	-0.83611	-0.61264	-0.38916	-0.16568

4 Results and Discussion

In this system we analyze the effect of magnetic field dependent viscosity (m), material constant ($\bar{\beta}$) and porosity (β) for a ferrofluid flow on a rotating disk with porosity. The graphs are plotted by varying one of the parameter with fixed values of other two parameters.

In figures 2 − 5, by varying magnetic field dependent viscosity parameter and for fixed values of material constant and porosity we get variations in velocity profiles and pressure. In figure 2 it is observed that the increase in the magnetic field dependent viscosity parameter increases the radial velocity. In figures 3 and 4 we observe that the magnetic field dependent viscosity parameter has a negligible effect on tangential velocity and axial velocity. In figure 5 it is observed that increase in the magnetic field dependent viscosity parameter decreases the pressure.

Figures 6-9 represents the variation in the velocity and pressure for varying values of material constant with m and β fixed. In figure 6 it can be depicted that the increase in the material constant, increases the radial velocity for small values of the material parameter and further decreases with increasing material parameter values. The figure 7 it is observed that the tangential velocity has no difference in its profile in the absence and presence of material constant. In figure 8, the axial velocity has negligible difference in the absence and presence of material constant. Figure 9 reveals that increase in the material constant parameter increases the pressure.

In Figures 10 − 13, we demonstrate the variation in the velocity profiles and pressure, with varying porosity parameter for the fixed value of magnetic field dependent viscosity and material constant. In figure 10 the radial velocity, retains in the positive region with increasing values of the porosity parameter. In the figure 11, the tangential velocity shows the slender variation in the absence of β and for increasing values of β. In figures 12 and 13 we see that the axial velocity and pressure has no variations in its profile with the increase in the porosity parameter.

5 Graphs

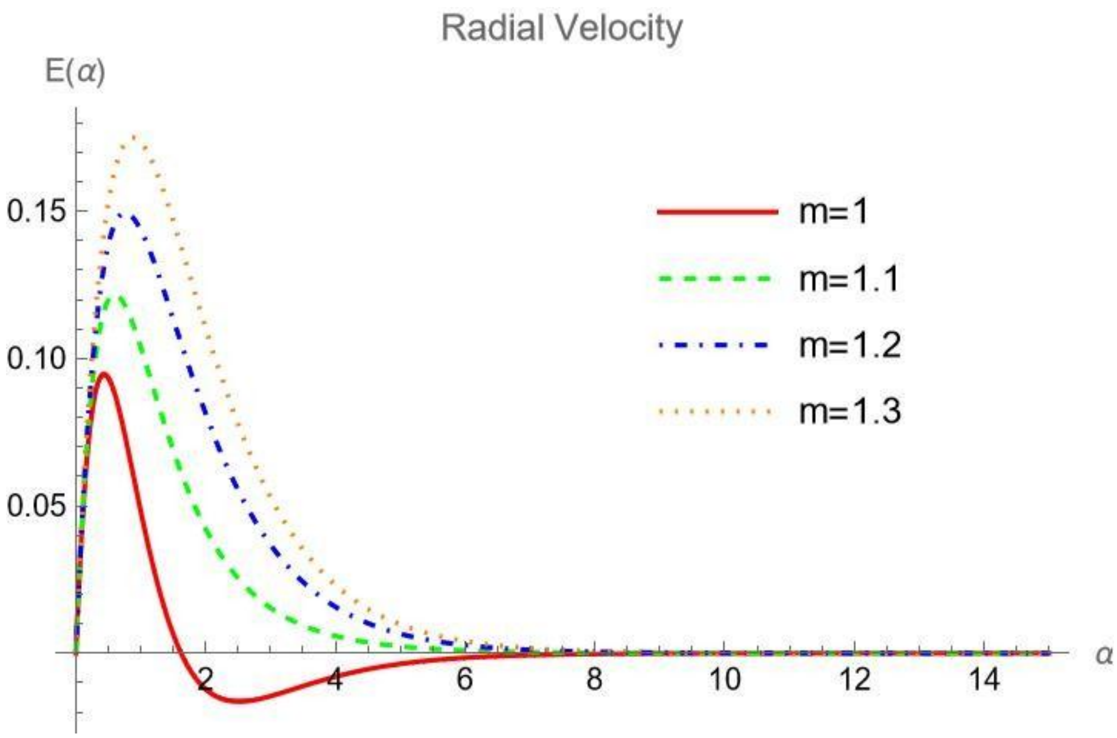

Figure 2: Radial velocity versus α for $\bar{\beta}$ = 0.5 and β = 1.

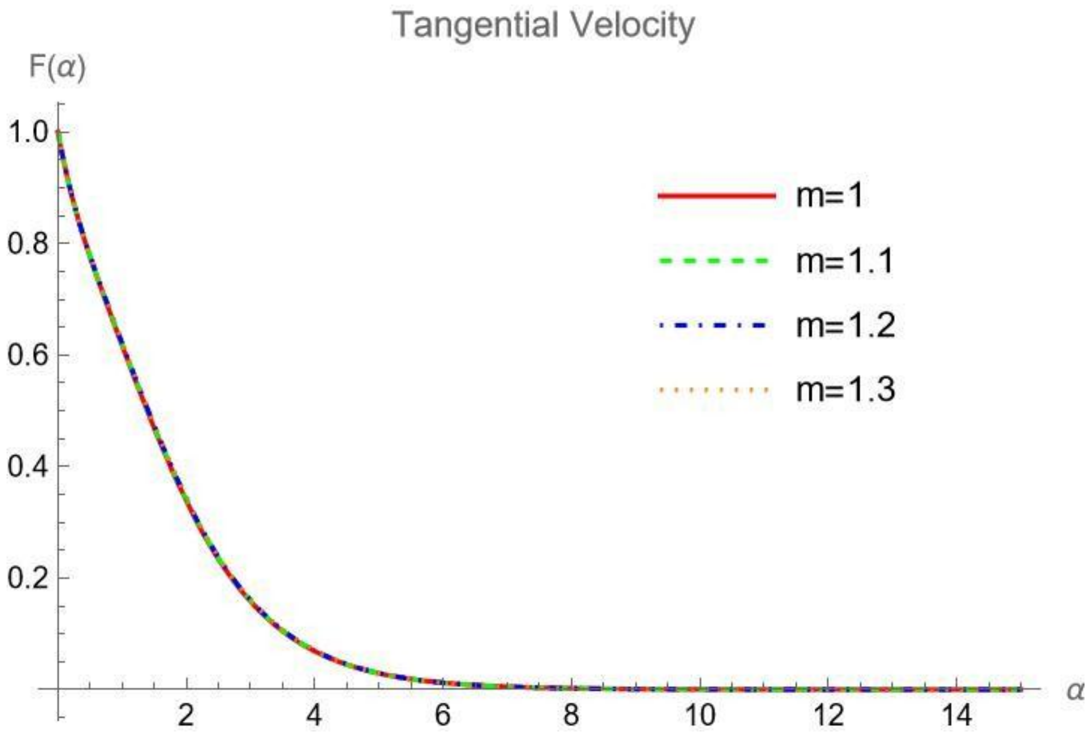

Figure 3: Tangential velocity versus α for $\bar{\beta}$ = 0.5 and β = 1.

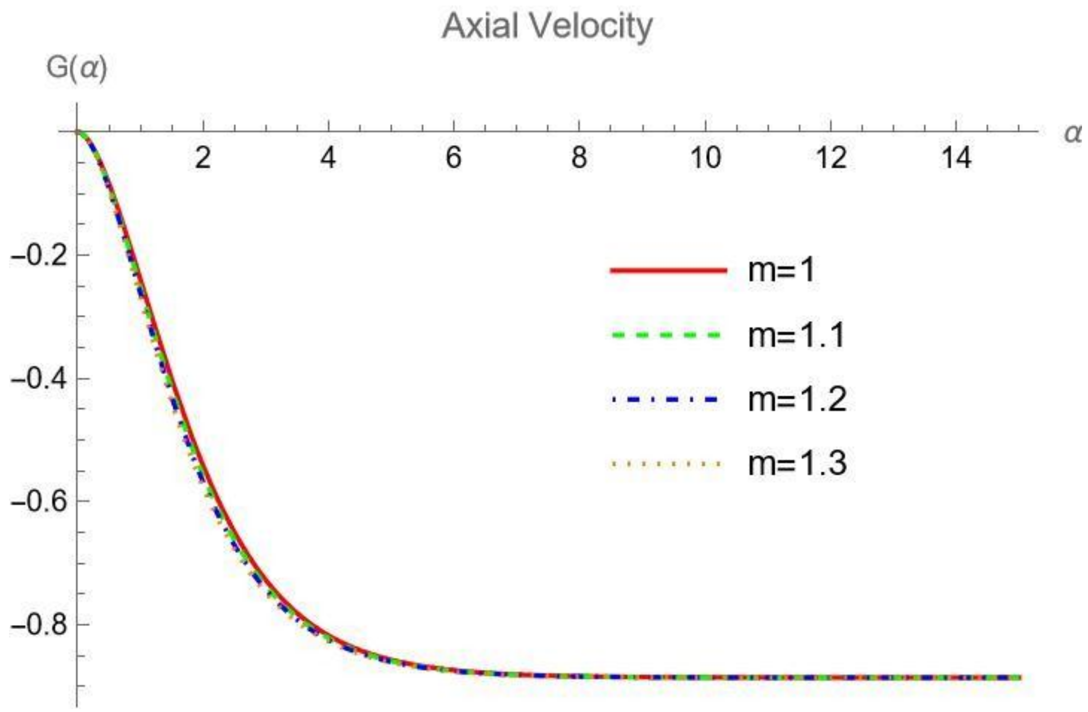

Figure 4: Axial velocity versus α for $\bar{\beta}$ = 0.5 and β = 1.

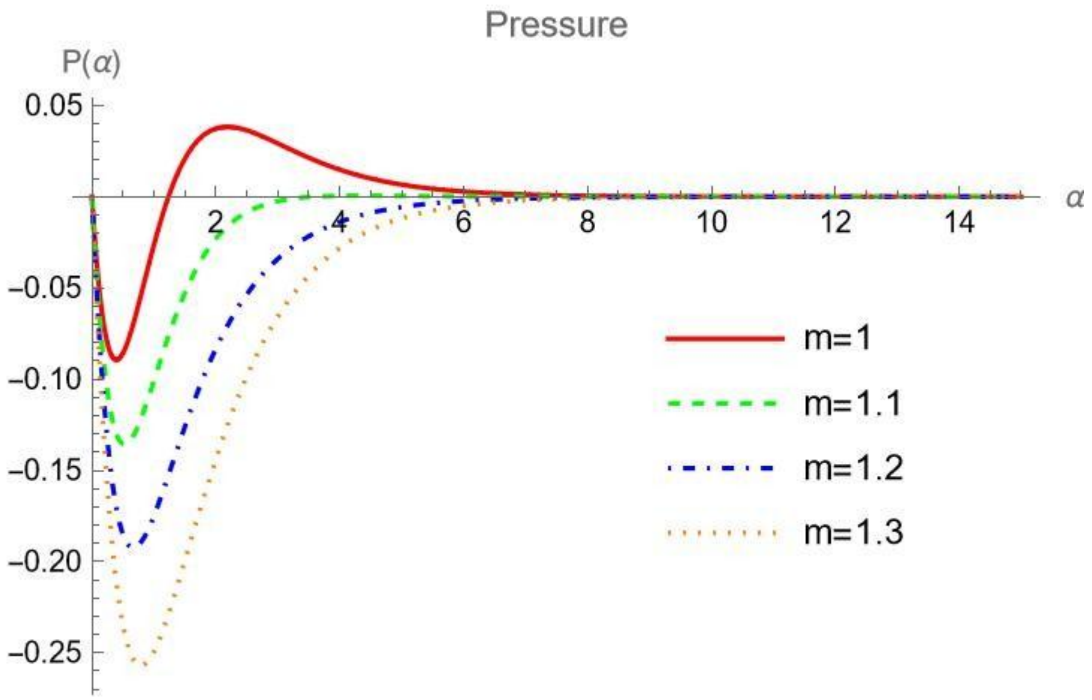

Figure 5: Pressure versus α for $\bar{\beta}$ = 0.5 and β = 1.

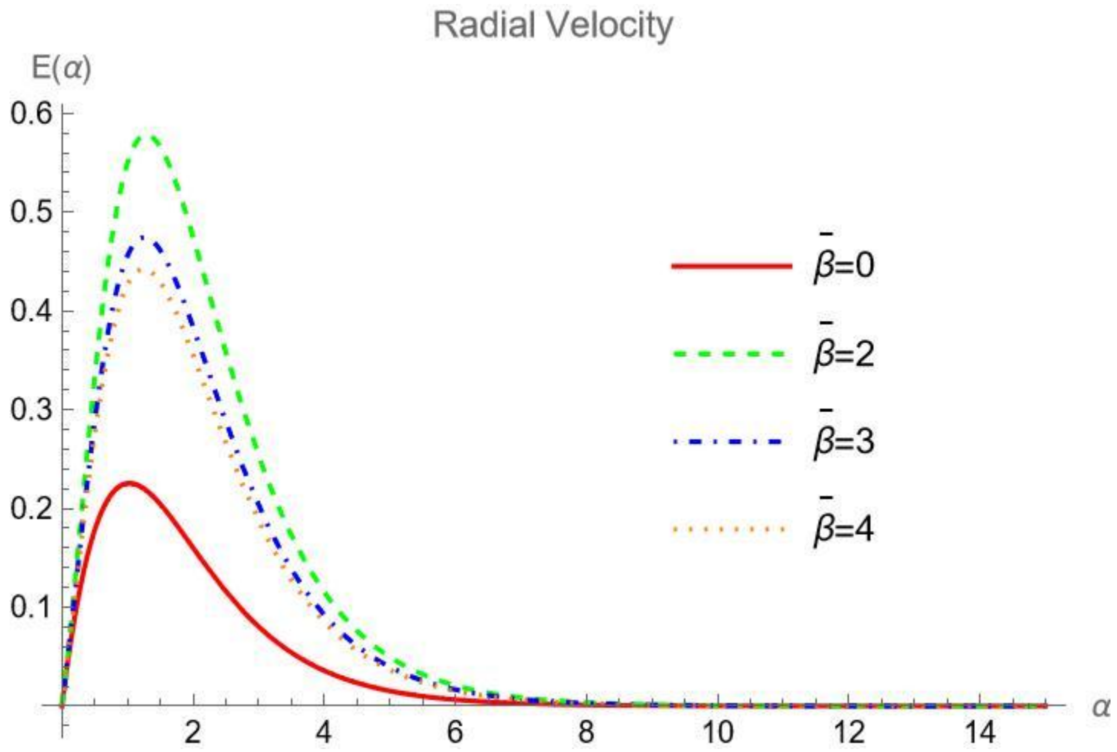

Figure 6: Radial velocity versus α for m = 1.1 and β = 1.

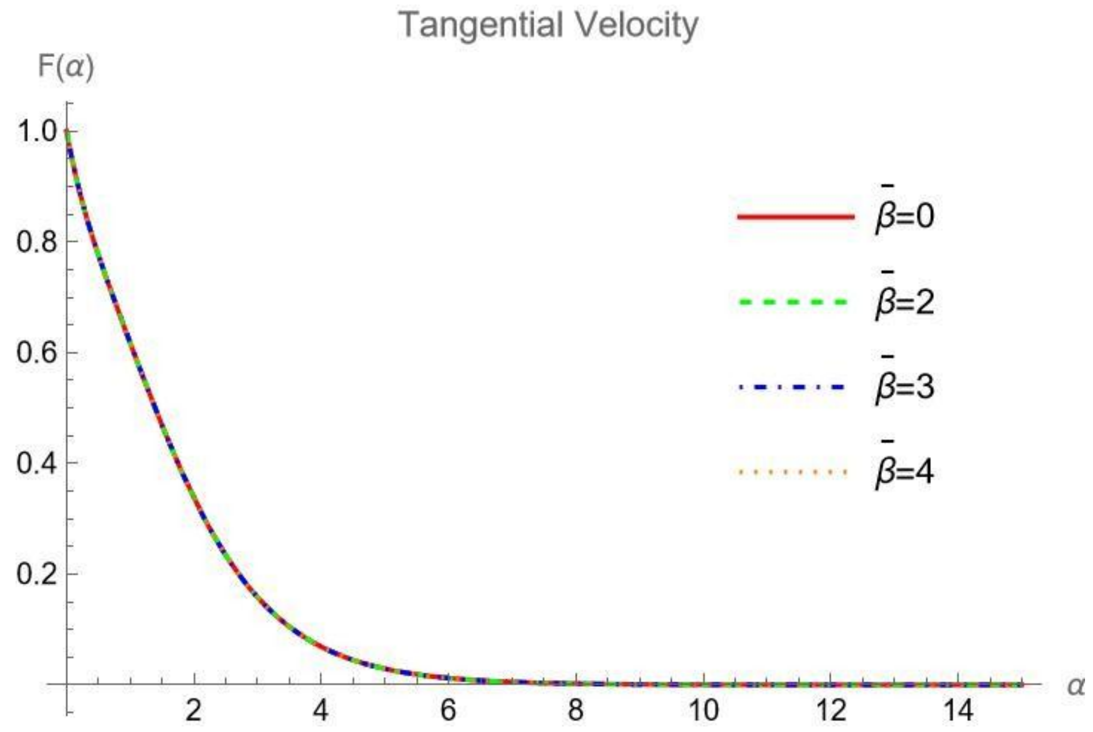

Figure 7: Tangential velocity versus α for m = 1.1 and β = 1.

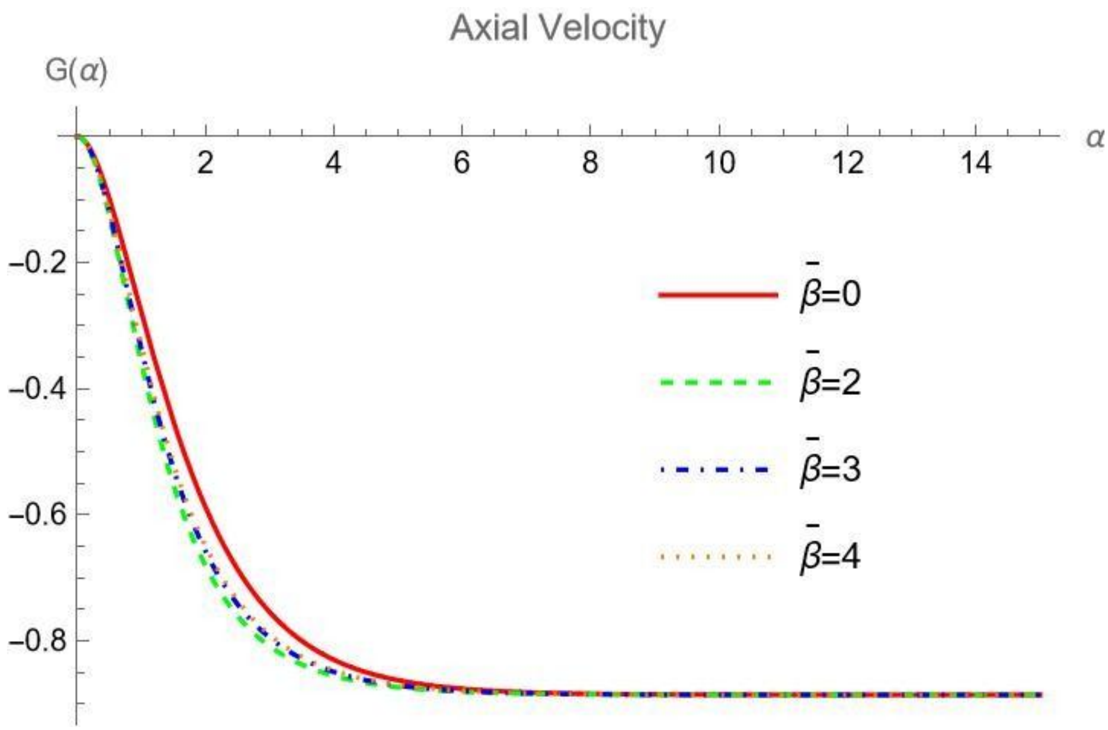

Figure 8: Axial velocity versus α for m = 1.1 and β = 1.

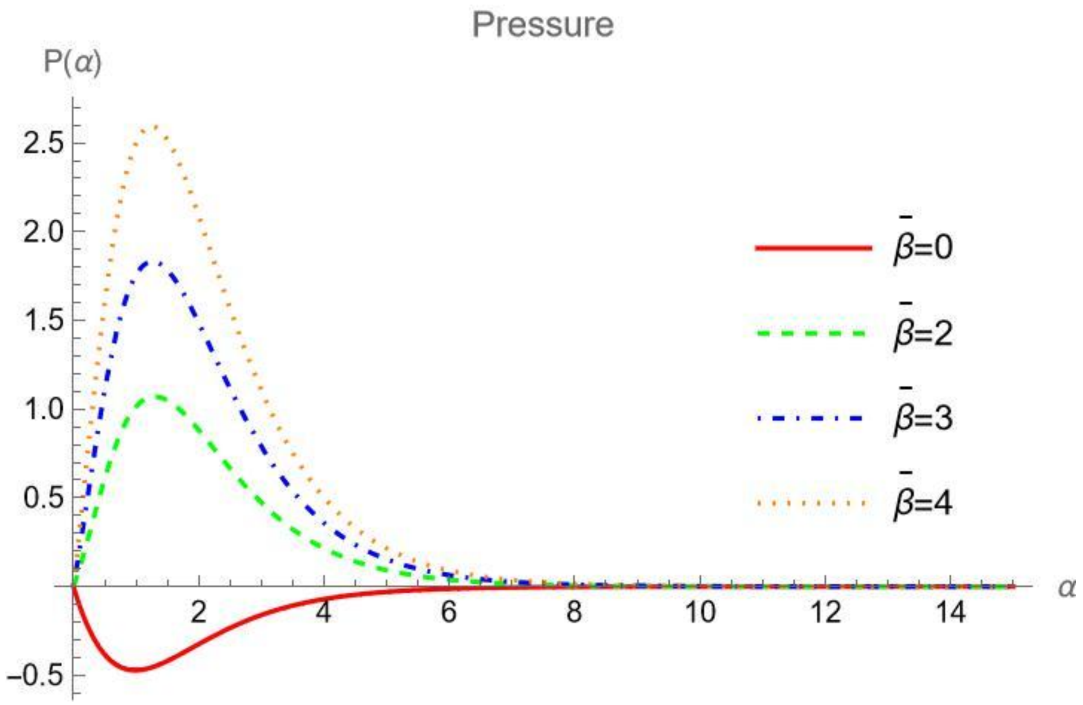

Figure 9: Pressure versus α for m = 1.1 and β = 1.

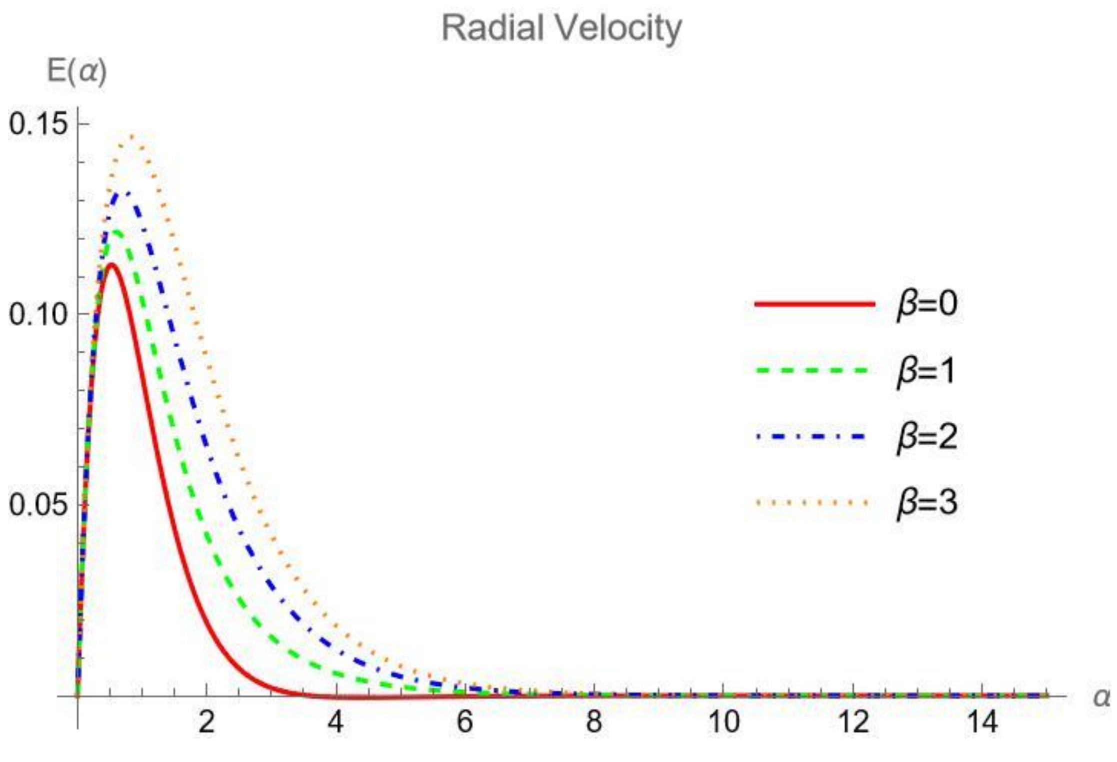

Figure 10: Radial velocity versus α for m = 1.1 and $\bar{\beta} = 0.5$

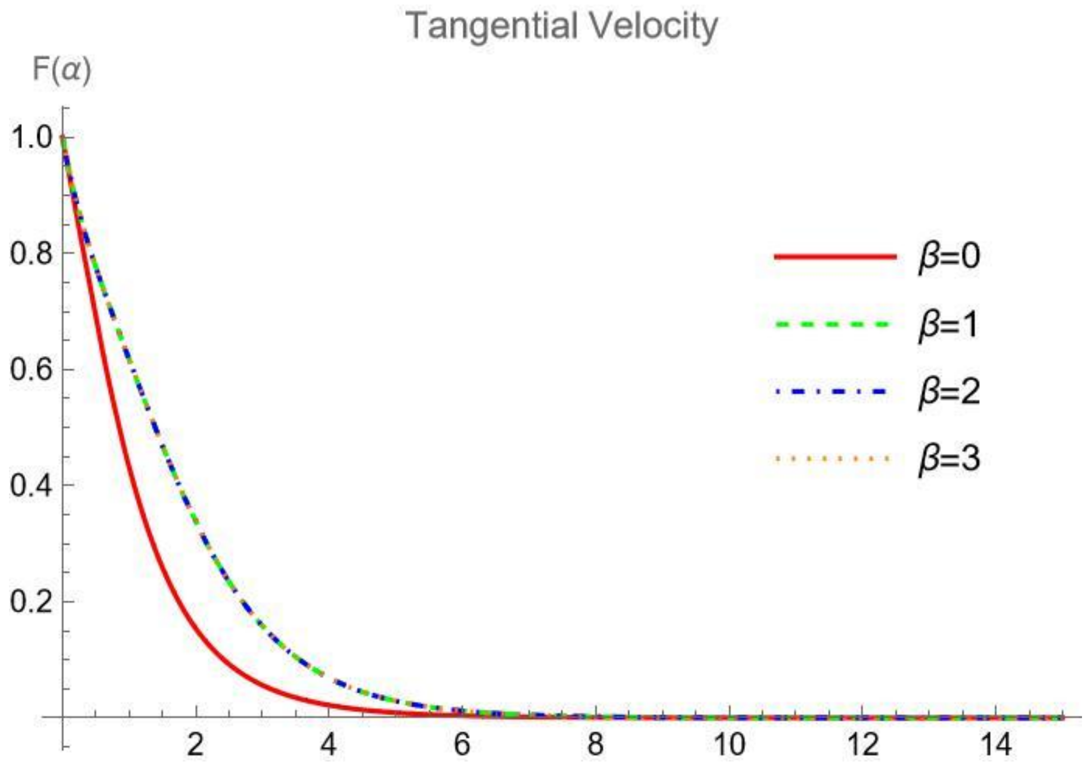

Figure 11: Tangential velocity versus α for m = 1.1 and β = 0.5.

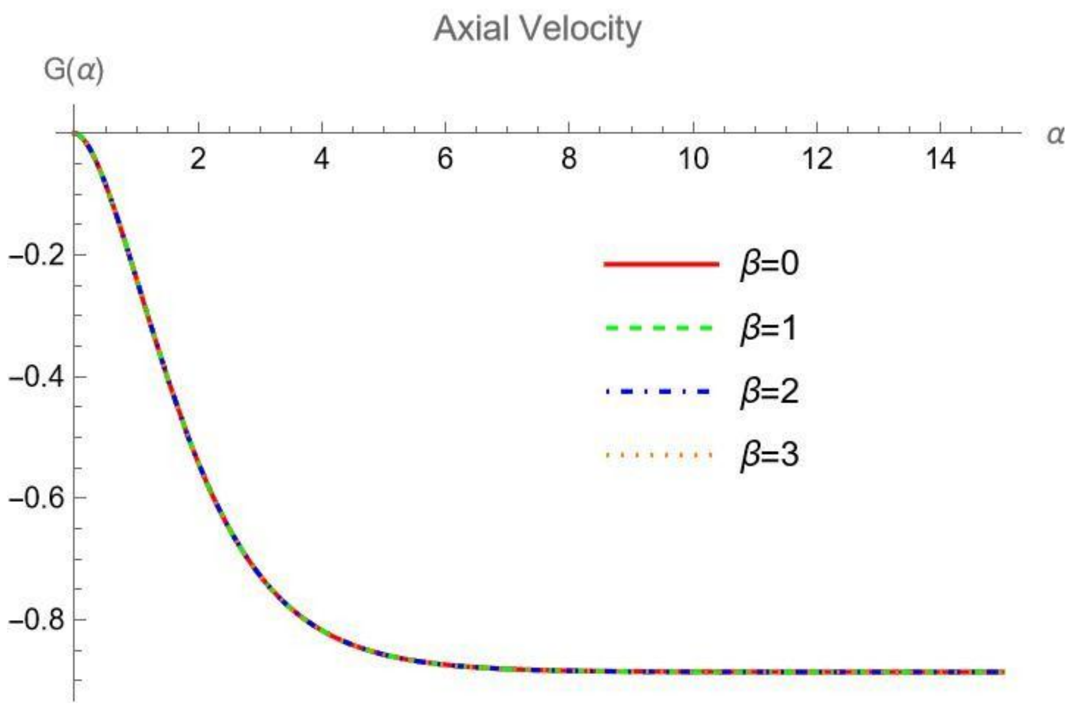

Figure 12: Axial velocity versus α for m = 1.1 and $\bar{\beta} = 0.5$.

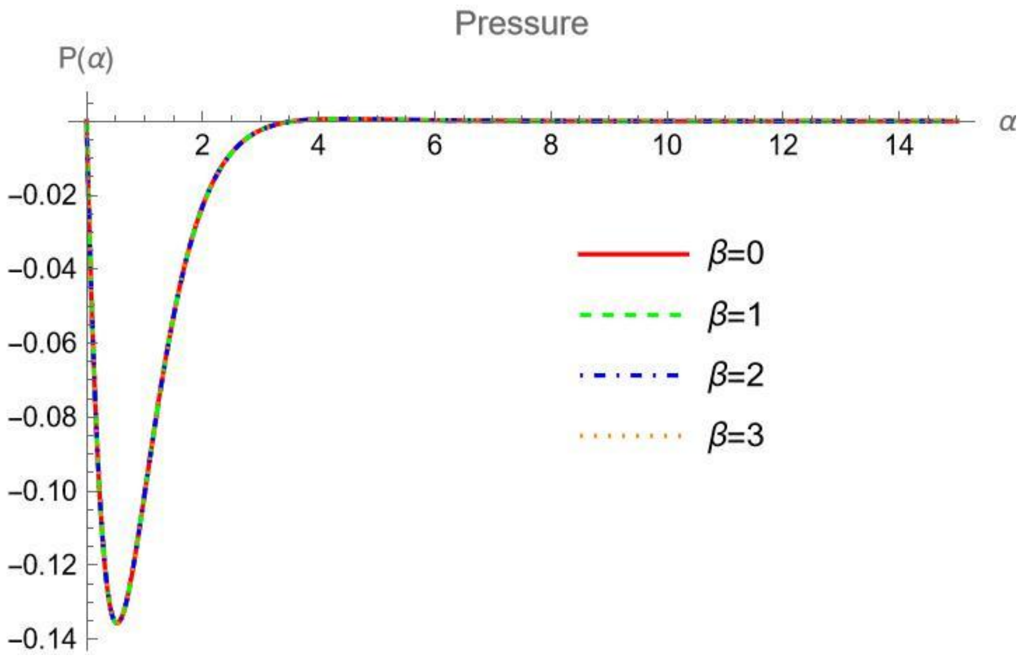

Figure 13: Pressure versus α for m = 1.1 and $\bar{\beta} = 0.5$.

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