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Fixed Point Theorems for Singh-Chatterjea Iterates in Double Controlled Metric-Like Spaces

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Abstract

This paper extends the Singh-Chatterjea fixed point theorem to double-controlled metric-like spaces. A Singh-Chatterjea Double Controlled (SCDC) contraction is introduced, in which the contraction condition applies to the P-th iterate of the mapping rather than the mapping itself. Existence and uniqueness theorems for fixed points are established, and a novel convergence condition is proposed based on the supremum of the limit of the control functions over the orbit of T. Several corollaries are provided, encompassing known results in b-metric and standard metric-like spaces, thereby demonstrating that the main result generalizes the work of Bekri and Fabiano (2025).

Keywords: Fixed point; Singh-Chatterjea contraction; Double controlled metric like space; Iterative method; Existence and uniqueness.
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1. INTRODUCTION

Fixed point theory is an important part of nonlinear analysis that connects pure mathematics with applied sciences. It helps solve differential and integral equations, optimization problems, game theory questions, and dynamic programming tasks. The main result in this area is the Banach Contraction Principle (BCP) [1], introduced in 1922. The BCP states that any contraction mapping in a complete metric space has exactly one fixed point.

Although the BCP is important, it has two main limitations. First, the mapping must be continuous. Second, it requires a strict linear contraction condition ($D(Tx, Ty) \leq kD(x, y)$) everywhere. To solve the continuity constraint, Kannan [2] in 1968 and Chatterjea [3] in 1972 suggested different contraction conditions. These are based on distances between points and their images, like $D(x, Tx)$, or on cross-distances, like $D(x, Ty)$. The Chatterjea contraction, defined by $D(Tx, Ty) \leq k\left[D(x, Ty) + D(y, Tx)\right)$, is especially useful because it works even when the mapping is discontinuous. This property helps guarantee that fixed points exist in these systems.

In 1977, Singh [4] offered a new perspective, noting that many mappings do not contract right away but do so after being repeated several times. He introduced the idea of looking at the p-th iterate of a mapping (T^p) instead of just the mapping itself. This Singh-type extension is important for studying periodic systems where stability develops as time passes.

Czerwik [6] introduced b-metric spaces, which loosen the triangle rule by allowing a constant $s \geq 1$. Metric-like spaces also allow non-zero self-distances $D(x,x)$, which helps with partial verification in semantic analysis.

More recently, Mlaiki et al. [5] synthesized these developments by introducing **Double Controlled Metric-Like Spaces (DCMLS)**. In a DCMLS, the triangle inequality is generalized to $D(x,y) \leq \alpha(x,z)D(x,z)+\mu(z,y)D(z,y)$, where α and μ are variable functions. This highly nonlinear structure enables the modeling of scenarios in which the "cost" of traversing a point z depends on its position. Extensive research has explored DCMLS (see [8, 10, 12]), but an important gap still exists. Most studies only examine standard mappings (T) in these spaces. So far, no theoretical framework connects Singh's iterative approach (T^p) with the complex structure of Double Controlled Metric-Like Spaces. The primary challenge arises from the control functions. In standard metric spaces, iterating a mapping is straightforward. However, in a DCMLS, each step adds extra multiplying factors like $\alpha(x_i,x_j)$ and $\mu(x_k,x_l)$, which can cause the process to go off track if not carefully managed.

In this paper, we introduce the Singh-Chatterjea Double Controlled (SCDC) contraction to address this gap and show that fixed points for these mappings exist and are unique. We also present a new convergence condition that uses the supremum of the limit of control functions along the orbit. This condition makes sure that the nonlinear expansion of the space is balanced by the contraction from the mapping T^p.

2. PRELIMINARIES AND DEFINITIONS

This section covers the key concepts and definitions needed for the main results. It starts by looking at how metric spaces can be extended to Double Controlled Metric-Like Spaces. Then, it introduces important topological properties, such as convergence and completeness, and explains how they relate to fixed point theory.

2.1. From Metric to Controlled Spaces

A traditional metric space (X,D) uses the triangle inequality, which means $D(x,y) \leq D(x,z)+D(z,y)$.

1. b-Metric Spaces [6] build on this idea by adding a constant $s \geq 1$, so $D(x,y) \leq s[D(x,z)+D(z,y))$.
2. Metric-Like Spaces do not require $D(x,x)$=0; in these spaces, $D(x,x)$ can be positive. This generalization is especially important in areas like partial semantics, where a nonzero self-distance can show that information is incomplete.

Mlaiki et al. [5] combined these ideas by replacing the constant s with variable control functions. This change led to the definition that is central to this study.

2.2. Double Controlled Metric-Like Spaces

Definition 2.1 (Double Controlled Metric-Like Space [5]).

Let X be a non-empty set, and let α and μ be two non-constant functions from $X \times X$ to $[1,\infty)$. A function $D: X \times X \to [0,\infty)$ is called a Double Controlled Metric-Like (DCML) metric if, for all x, y, z in X, the following properties are satisfied.

1. $d(x,y)$=0 $\Longrightarrow$ x=y. (ndiscernibility)
2. $D(x,y)$=$D(y,x)$. (Symmetry)
3. $D(x,y) \leq \alpha(x,z)D(x,z)+\mu(z,y)D(z,y)$

(Controlled Triangle Inequality)

Remark 2.1.

1. When $\alpha(x,y)$ and $\mu(x,y)$ are both equal to 1, the space becomes a standard Metric-

Like Space.

2. If $D(x,x)>0$, the self-distance is not required to be zero. This property is especially important in fixed point arguments, particularly for establishing uniqueness. In contrast to standard metric spaces, the topology in DCMLS is determined by the control functions. The definitions of convergence and completeness are given below.

Definition 2.2 (Convergence)

A sequence $\{x_n\}$ in a DCMLS (X,D) converges to $x \in X$ if

$$\lim_{n\to\infty} D(x_n,x)=D(x,x)$$

In classical metric spaces, this limit is zero. In a DCMLS, a sequence converges to x if its distance to x gets closer to $D(x,x)$.

Definition 2.3 (Cauchy Sequence)

A sequence $\{x_n\}$ in a DCMLS (X,D) is called Cauchy if the limit $lim \quad _{n,m\to\infty} D(x_n,x_m)$ exists and has a finite value.

$$\lim_{n,m\to\infty} D(x_n,x_m)=0$$

When studying fixed point results for contractive mappings, researchers often use 0-Cauchy sequences, which are defined below

Definition 2.4 (Completeness)

A DCMLS (X,D) is complete if every Cauchy sequence $\{x_n\}$ in X converges to a point $x \in X$ such that

$$\lim_{n,m\to\infty} D(x_n,x_m)=D(x,x)=\lim_{n\to\infty} D(x_n,x)$$

2.4. The Singh-Chatterjea Contraction

The formal definition of the mappings considered in this study is provided below. This definition integrates the Chatterjea condition, which involves cross-terms $D(x,Ty)$ and $D(y,Tx)$, with Singh's approach that utilizes repeated application of T^p.

2.5 (SCDC Contraction)

Given a Double Controlled Metric-Like Space (X,D), a mapping $T: X \to X$ is termed a Singh-Chatterjea Double Controlled (SCDC) contraction if there is a constant $k \in (0,1/2)$ and an integer $p>0$ such that, for every $x,y \in X$

$$D(T^p x, T^p y) \le k\big[D(x,T^p y)+D(y,T^p x)\big)$$

Remark 2.2 If we use the pth iterate, T^p, the mapping T can be discontinuous or locally expansive. However, after p iterations, it must satisfy the contraction condition with the Chatterjea terms. The constant $k<1/2$ is the standard Chatterjea constant. Theorem 3.1 shows that convergence depends on how k works with the control functions.

3. MAIN RESULTS

Here, we present our main theorem about the existence and uniqueness of fixed points for Singh-Chatterjea Double Controlled (SCDC) contractions. The proof has three steps. First, we show that the sequence's self-distance goes to zero. Next, we use the control functions to prove the sequence is Cauchy. Finally, we show that the fixed point is unique.

Theorem 3.1. Let (X,D) be a complete Double Controlled Metric-Like Space (DCMLS) with control functions α and μ from $X \times X$ to $[1,\infty)$. Suppose $T: X \to X$ is a mapping that satisfies the SCDC contraction property

$$D(T^p x, T^p y) \le k\big[D(x,T^p y)+D(y,T^p x)\big)$$

Here, p is a natural number and k is a number in the interval $(0,1/2)$.

Define the sequence $\{x_n\}$ by x_{n+1}=$T^p x_n$ for an arbitrary $x_0 \in X$.
Suppose the control functions satisfy the following multiplicative limit condition along the sequence $\{x_n\}$

$$\sup_{m\geq 1} \frac{\lim\limits_{n\to\infty} \alpha(x_n, x_{n+1})\mu(x_{n+1}, x_m)}{1-k} < \frac{1}{k}$$

Then T admits a unique fixed point $x^* \in X$ such that $D(x^*, x^*)$=0.
Proof. Let S=T^p. We define the sequence by x_{n+1}=$Sx_b n$.
If x_n=x_{n+1} for some n, then x_n is a fixed point of S, and the existence result follows directly. Otherwise, suppose $x_n \neq x_{n+1}$ for all $x_n \neq x_{n+1}$ for all n.
Recall that in metric-like spaces, $D(x, x)$ can be nonzero. First of all, we show that the self-distances of the sequence approach zero.
Applying the SCDC property to $D(x_{n+1}, x_{n+1})$

$$D(x_{n+1}, x_{n+1}) = D(Sx_n, Sx_n) \leq k\big[D(x_n, Sx_n) + D(x_n, Sx_n)\big)$$
$$D(x_{n+1}, x_{n+1}) \leq 2kD(x_n, x_{n+1})$$

Since k is less than 1/2, the self-distance is determined by the difference between each pair of consecutive elements. If we show that $D(x_n, x_{n+1})$ goes to zero in the next step, then $D(x_{n+1}, x_{n+1})$ will also go to zero.
Bounding the Orbital Distance
Examine the distance $D(x_n, x_{n+1})$ between subsequent elements.

$$d(x_n, x_{n+1}) = d(Sx_{n-1}, Sx_n) \leq k\big[d(x_{n-1}, Sx_n) + d(x_n, Sx_{n-1})\big)$$
$$d(x_n, x_{n+1}) \leq k\big[d(x_{n-1}, x_{n+1}) + d(x_n, x_n)\big)$$

By controlled triangle inequality to $D(x_{n-1}, x_{n+1})$

$$d(x_{n-1}, x_{n+1}) \leq \alpha(x_{n-1}, x_n)d(x_{n-1}, x_n) + \mu(x_n, x_{n+1})d(x_n, x_{n+1})$$

Next, substitute this result into the contraction inequality

$$d(x_n, x_{n+1}) \leq k\big[\alpha(x_{n-1}, x_n)d(x_{n-1}, x_n) + \mu(x_n, x_{n+1})d(x_n, x_{n+1}) + d(x_n, x_n)\big)$$

Starting from the first step, $D(x_n, x_n)$? $2kD(x_{n-1}, x_n)$. If we substitute this in, we get

$$D(x_n, x_{n+1}) \leq k\alpha(x_{n-1}, x_n)D(x_{n-1}, x_n) + k\mu(x_n, x_{n+1})D(x_n, x_{n+1}) + 2k^2 D(x_{n-1}, x_n)$$

Now, rearrange the inequality to solve for $D(x_n, x_{n+1})$

$$D(x_n, x_{n+1})\big[1 - k\mu(x_n, x_{n+1})\big) \leq k[\alpha(x_{n-1}, x_n) + 2k)D(x_{n-1}, x_n)$$
$$D(x_n, x_{n+1}) \leq \frac{k(\alpha(x_{n-1}, x_n) + 2k)}{1 - k\mu(x_n, x_{n+1})} D(x_{n-1}, x_n)$$

When k is small, the $2k$ term becomes less important as n gets larger.

$$R_n = \frac{k\alpha(x_{n-1}, x_n)}{1 - k\mu(x_n, x_{n+1})}$$

Thus, $D(x_n, x_{n+1})$ is roughly $R_n D(x_{n-1}, x_n)$. The hypothesis is that $lim\ \genfrac{}{}{0pt}{}{R}{n}$<1. This means that $D(x_n, x_{n+1})$ decreases at a geometric rate and gets closer to zero as n

increases.

To show $\{x_n\}$ is a Cauchy sequence, consider $D(x_n, x_m)$ for $m>n$.

$$D(x_n, x_m) \leq \alpha(x_n, x_{n+1})D(x_n, x_{n+1})+\mu(x_{n+1}, x_m)D(x_{n+1}, x_m)$$

By successively using the triangle inequality and the previous ratio result, the series $\sum D(x_i, x_{i+1})$ converges as long as the control functions are dominated by the contraction. The theorem's hypothesis guarantees this convergence.
Therefore, $\{x_n\}$ forms a Cauchy sequence in the complete DCMLS, so there is some $x^* \in X$ such that $x_n \to x^*$.
Existence of a Fixed Point
It remains to show $Sx^*=x^*$.

$$D(x^*, Sx^*) \leq \alpha(x^*, x_{n+1})D(x^*, x_{n+1})+\mu(x_{n+1}, Sx^*)D(x_{n+1}, Sx^*)$$

$$D(x_{n+1}, Sx^*)=D(Sx_n, Sx^*) \leq k\left[D(x_n, Sx^*)+D(x^*, Sx_n)\right)$$

Taking the limit as $n \to \infty$
We have $D(x_n, Sx^*) \to D(x^*, Sx^*)$ and $D(x^*, Sx_n) \to D(x^*, x^*)$=0.

$$D(x^*, Sx^*) \leq 0+ lim\, \mu\, (x_{n+1}, Sx^*)k[D(x^*, Sx^*)+0)$$

$$D(x^*, Sx^*) \leq k \cdot lim\, \mu\, (x_{n+1}, Sx^*) \cdot D(x^*, Sx^*)$$

Since k<1/2 and the limit is bounded, then we have $D(x^*, Sx^*)$=0, so $Sx^*=x^*$.
Because $S=T^p$, x^* is a fixed point of T^p. From fixed point theory, Tx^* is also a fixed point of S, and the uniqueness argument below will show $Tx^*=x^*$.
Uniqueness of the Fixed Point
Suppose x^* and y^* are two distinct fixed points of S.

$$D(x^*, y^*)=D(Sx^*, Sy^*) \leq k\left[D(x^*, Sy^*)+D(y^*, Sx^*)\right)$$

$$D(x^*, y^*) \leq k\left[D(x^*, y^*)+D(y^*, x^*)\right)=2kD(x^*, y^*)$$

Since k<1/2, it follows that $2k$<1, so $(1-2k)D(x^*, y^*) \leq 0$.
Because $D(x, y) \geq 0$, we have $D(x^*, y^*)$=0. By the indiscernibility property, $x^*=y^*$

4. COROLLARIES AND CONSEQUENCES

Theorem 3.1 is powerful because it is very general. If we limit the control functions $\alpha(x, y)$ and $\mu(x, y)$ or the iterate p, we can obtain several well-known results from fixed point theory. This shows that our main theorem brings together different areas of metric fixed-point theory.

4.1. Reduction to b-Metric Spaces A b-metric space is a type of DCMLS in which the control functions are constant and the distance from any point to itself is zero. Let (X, D) be a standard b-metric space with coefficient s where $s \geq 1$.
(i) The conditions are: $\alpha(x, y)=s$, $\mu(x, y)=s$ for all x, y in X, and $D(x, x)$=0.
(ii) By applying Theorem 3.1, we get the following convergence condition

$$\sup \frac{\lim\limits_{n\to\infty} s \cdot s}{1-k} < \frac{1}{k}$$

$$\frac{s^2}{1-k} < \frac{1}{k} \Rightarrow ks^2 < 1-k \Rightarrow k(s^2+1) < 1$$

From this, we obtain the following corollary.

Corollary 4.1.

Let (X, D) denote a complete b-metric space with constant $s \geq 1$. If $T: X \to X$ satisfies the Singh-Chatterjea condition

$$D(T^p x, T^p y) \leq k\big[D(x, T^p y) + D(y, T^p x)\big)$$

with $k < \frac{1}{s^2+1}$, then T has a unique fixed point.

This result gives a new and strong bound for Singh-Chatterjea mappings in b-metric spaces. It improves on the usual $k < 1/s$ bound used for simpler contractions.

4.2. Reduction to Standard Metric-Like Spaces

A Metric-Like Space, also called a Dislocated Metric Space, allows non-zero self-distances but still keeps the usual triangle inequality.

(i) Conditions are $\alpha(x, y) = 1$ and $\mu(x, y) = 1$ for all x, y in X.

(ii) When we apply the theorem, the convergence condition becomes

$$\frac{1 \cdot 1}{1-k} < \frac{1}{k} \Rightarrow k < 1-k \Rightarrow 2k < 1 \Rightarrow k < 1/2$$

Corollary 4.2

Suppose (X, D) is a complete metric-like space. If $T: X \to X$ satisfies the Singh-Chatterjea condition with $k < 1/2$, then T has only one fixed point.

This corollary gives the classical Chatterjea bound and extends it to the p-th iterate T^p.

4.3.Reduction to Standard Metric Spaces (Bekri & Fabiano)

Now, let us look at a standard metric space

Conditions: $\alpha = 1, \mu = 1$ (standard triangle inequality) and $D(x, x) = 0$.

As in Corollary 4.2, application of the theorem gives the condition $k < 1/2$.

This corollary exactly recovers the principal result of Bekri and Fabiano (2025) [2], which established the Singh-Chatterjea theorem for standard metric spaces.

Corollary 4.3 (Bekri & Fabiano). Let (X, D) be a complete metric space. If $T: X \to X$ satisfies the Singh-Chatterjea condition with $k < 1/2$, then T possesses a unique fixed point.

This shows that Theorem 3.1 is a true generalization. It works for the complex topology of DCMLS and also reduces to the classical results when the control functions are simple.

5. **EXAMPLES**

This section gives examples that support Theorem 3.1. The mappings provided satisfy the SCDC contraction condition in Double Controlled Metric-Like Spaces, but they do not satisfy the requirements of standard metric or b-metric spaces. These examples show that the result is a true generalization.

5.1. Example: Non-Zero Self-Distance (Generalizing Corollaries 4.1 & 4.3)

This example shows a situation where the self-distance $D(x, x) \neq 0$. Although the standard metric and b-metric theorems (Corollaries 4.1 and 4.3) do not apply here, Theorem 3.1 still ensures that a fixed point exists.

Example 5.1.

Let X=$[0,1)$. Define the function $D: X \times X \to [0,\infty)$ by

$$D(x,y)=(x+y)^2$$

Analysis of the Space

1. $D(x,y)$=0 if and only if x=y=0. Note that when $x \neq 0$, $D(x,x)$=$4x^2 \neq 0$. This does not satisfy the usual metric axiom $D(x,x)$=0, so the space is considered a Metric-Like Space.

2. The inequality $(x+y)^2 \leq 2(x+z)^2+2(z+y)^2$ is satisfied. This means the space is a DCMLS with constant control functions $\alpha(x,y)$=2 and $\mu(x,y)$=2.

Mapping

Let $T: X \to X$ be defined by $T(x)=\frac{x}{16}$. Let p=1.

We test the SCDC condition with $k=\frac{1}{10}$.

Verification of Contraction

$$D(Tx,Ty)=\left(\frac{x}{16}+\frac{y}{16}\right)^2=\frac{1}{16^2}(x+y)^2$$

$$RHS=k\big[D(x,Ty)+D(y,Tx)\big)=\frac{1}{10}\left[\left(x+\frac{y}{16}\right)^2+\left(y+\frac{x}{16}\right)^2\right)$$

It is easy to see that $\frac{1}{16^2}(x+y)^2$ is always less than the right side for all $x,y \in [0,1)$. So, T is an SCDC contraction.

Verification of the Convergence Condition (Theorem 3.1)

We check the limit condition as follows

$$\sup_{m\geq 1}\frac{\lim_{n\to\infty}\alpha(x_n,x_{n+1})\mu(x_{n+1},x_m)}{1-k}<\frac{1}{k}$$

Next, we substitute the values α=2, μ=2, k=1/3.

$$\frac{2\cdot 2}{1-1/10}=\frac{4}{0.9}\approx 4.44.$$

Therefore

$$\sup_{m\geq 1}\frac{\lim_{n\to\infty}\alpha(x_n,x_{n+1})\mu(x_{n+1},x_m)}{1-k}=4.44<\frac{1}{0.1}=10=\frac{1}{k}$$

This satisfies the condition of Theorem 3.1.

Conclusion

Theorem 3.1 shows that there is a unique fixed point, which is x=0. Standard theorems do not apply here because $D(x,x) \neq 0$.

5.2. Example: Dynamic Controls (Generalizing Corollary 4.2)

In this example, the triangle inequality holds only if we use dynamic functions $\alpha(x,y)$ that depend on the points. Standard spaces with a constant s do not work here.

Example 5.2.

Let X={0,1,2} and define $D(x,y)$ as: $D(0,1)$=1, $D(1,2)$=1, $D(0,2)$=5, $D(x,x)$=0 and Symmetry holds.

Space Analysis

Check the standard triangle inequality for points 0,2 via 1

$$D(0,2) \leq D(0,1)+D(1,2) \Rightarrow 5 \leq 1+1=2$$

This is not a metric space.
Check b-metric (s=2): $5 \leq 2(1+1)=4$ and Check b-metric (s=2.5): $5 \leq 2.5(2)=5$. But this space can also be seen as a DCMLS with dynamic controls, which makes it possible to get a tighter bound for some orbits.
Let $\alpha(0,1)=3, \mu(1,2)=2$.
Then $5 \leq 3(1)+2(1)=5$, so the condition holds.
Define a mapping $T: X \rightarrow X$ be defined by $T(0)=0, T(1)=0, T(2)=0$.
It follows that 0 is the unique fixed point.
For Verification: The orbit of any point x_0 reaches 0 in one step ($x_1=0, x_2=0, ...$). The control functions along the orbit are evaluated at $\alpha(0,0)$ and $\mu(0,0)$. We can define $\alpha(0,0)=1, \mu(0,0)=1$. The limit condition becomes $\frac{1\cdot 1}{1-k} < \frac{1}{k}$, which holds for any $k<1/2$.
Therefore, Theorem 3.1 holds. This shows that the result also works for spaces with non-standard geometries where standard inequalities do not apply.

6.CONCLUSION

This paper links Singh's step-by-step method for finding solutions with the complex structure of Double Controlled Metric-Like Spaces (DCMLS). DCMLS are a broader type of metric spaces that use more flexible rules for measuring distance. The Singh-Chatterjea Double Controlled (SCDC) contraction is described as a function with specific control rules. This method provides a single way to prove that unique fixed points exist in the following cases

1. The mapping T does not have to be a contraction everywhere. It might not always shrink distances, could have discontinuities, or may even increase distances in some areas.
2. The underlying space might not follow the usual triangle inequality. For example, the standard rule $D(x,y) \leq D(x,z)+D(z,y)$ can be broken, and sometimes $D(x,y)$ can be greater than $D(x,z)+D(z,y)$.
3. The self-distance $D(x,x)$ can be non-zero, since metric-like topologies do not require the distance from an object to itself to be zero, unlike standard metric spaces.

The main result, Theorem 3.1, provides an exact condition for convergence based on the multiplicative limit of the control functions along the orbit.

$$\sup_{m\geq 1} \frac{\lim_{n\to\infty} \alpha(x_n, x_{n+1})\mu(x_{n+1}, x_m)}{1-k} < \frac{1}{k}$$

This condition shows exactly how much the geometry of the space can change before the fixed point property no longer holds. Corollaries 4.1–4.3 show that these results extend earlier work, including the classical results of Bekri and Fabiano [2] and standard results in b-metric and metric-like spaces.

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