

Analysis of Electrical Lines and Dual Core Optical Fiber Models Using Bifurcation and Optical Solitary Waves

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Abstract

The dual-core optical fiber and electrical lines has significant applications in optical electronics for long-wave propagation, especially in telecommunication fibers. The aim of this project is to study of the parametric effects on solitary wave propagation and characteristic aspects of long-wave traveling through optical fibers by establishing some standard and wide-spectrum solutions via Bifurcation analysis along with Hamiltonian system. Additionally, we find out the crucial point of the particles in nature consisting of the stated model. Furthermore, we demonstrate the equilibrium point in the graphical representation to analyze the stability of the signal by describing the saddle point and center of the system. Thus the novelty of this investigation is that the attained solutions of the stated model deliver several types of waves such as periodic, lump, rogue, breather, dark, and bright soliton depending on time and wave displacement parameters that are depicted in 3D and 2D figures. We also analyze the nature of the wave based on two diffusion constants of the governing model and describe the effect of these parameters on the wave profile. As can stated the bifurcation analysis and the two implemented method is very useful and informative to describe the mathematical model in future studies like this investigation and many more. Also, the investigated solitary wave solutions are ascertained as an integration of hyperbolic, exponential, rational and trigonometric functions and can be extensively applicable in optics. The physical significance of the solutions attained is illustrated for the definite values of the included parameters through depicting the 3D profiles. The solitons profile represents different types of waves associated with the free parameters which are related to the wave number and velocity of the

solutions. It turns out that the obtained solutions through both the methods are potential and might be used in further works to interpret the various fields in telecommunication fiber which can reduce casualties that ensue in essence.

Keyword: **electrical lines**, Dual-core optical fibers; soliton; exact solutions; Bifurcation Analysis.

Introduction

Researchers are constantly exploring the various phenomena that occur in nature in the language of mathematics called mathematical models by successfully presenting a well-thought-out research plan. Among these, scholars have preferred the nonlinear partial differential equation as an important means of expressing the most widely accepted. In that context, scientists have been solving the problems by explaining those mathematical models through various methods [1-31] have been exposed to solve the mathematical problems, the solutions from each schemes of which have given various solutions and research on these is ongoing. Among the above methods, very detailed results are found in bifurcation analysis [32, 33] which can be further enriched by explaining our research.

On the other side, communication is an essential value to human evolution nowadays. The invention and development of amplifiers erbium-doped fiber [35] and optical directional couplers generate the revolution in telecommunications fiber. This new-fangled all-optical equipment, which associates the norm of issue inspired in erbium with the controlling assets of the fiber, allows, without the conversion steps optical-electronic and electronic-optical growth rates of transmission [36]. The fiber can be an optical amplifier, an optical switch converter wavelength, solitons in a source, a compressor noise, an optical memory etc. Breeding in a directional coupler is modeled by the system of equation of coupled nonlinear Schrödinger equation (NLSE) [37] and the dual core optical fibers equation [38-42] is one of them. As per our knowledge, few researchers has been studied the dual core optical fibers equation via different schemes namely, Younis et al. [38] through the (G'/G) -expansion method, Abdelrahman and Moaaz [39] used Riccati-Bernoulli sub-ODE method, Nair et al. [40] via modulational instability (MI) analysis, Baskonus et al. [41] via the extended sinh-Gordon equation expansion method, Shamseldeen et al. [42] via linear stability analysis. Nevertheless, the stated equation has not been inspected through the IBSEF method and the NAE method. Therefore, the objective of this study is to establish wide-ranging and adequate definitive soliton solutions to the dual core optical fibers equation through setting in use the suggested methods. Besides, we analyze the various types of soliton like solutions for the different value free parameters of the obtained solutions illustrated in 3D plot via Matlab and marked out the significant role of the value of wave number and velocity of the solutions to changing the nature of the soliton in wave profile.

Application

The dual core optical fiber model

The dual core optical fiber equations which is of the following form:

$$\begin{aligned} p \phi_{tt} + i(\phi_x + q\psi_t) + r|\phi|^2\phi + s\psi &= 0, \\ p \psi_{tt} + i(\psi_x + q\phi_t) + r|\psi|^2\psi + s\phi &= 0. \end{aligned}$$

transformations :

$$\begin{aligned} \phi(x, t) &= u(\eta) e^{i(\sigma_1 x - \sigma_2 t)}, \\ \psi(x, t) &= v(\eta) e^{i(\sigma_1 x - \sigma_2 t)}, \\ \text{where } \eta &= \gamma x - \delta t. \end{aligned}$$

where p, q, r, s are free constants.

Based on the linear stability analysis [41], we attaining the traveling wave velocity for non-oscillatory soliton-form solutions. Let us consider

$$\begin{aligned} \phi &\rightarrow \bar{\phi}(z, t), \\ \psi &\rightarrow \bar{\psi}(z, t), \end{aligned}$$

where $(z, t) = (x - lt, t)$ is the travelling wave coordinates and l is the vector speed. Therefore, the equations (1) tends to its homogenous steady state appears as $\phi, \psi \rightarrow 0$. The partial derivatives for traveling wave coordinates are:

$$\begin{aligned} \frac{\delta}{\delta x} &\rightarrow \frac{\delta}{\delta z} \\ \frac{\delta}{\delta t} &\rightarrow \frac{\delta}{\delta t} - l \frac{\delta}{\delta z}, \end{aligned}$$

then the system (1) is linearized near its steady state in the coordinates and performs as

$$\begin{aligned} \omega(\bar{\phi}, \bar{\psi}) &= 0, \\ \omega(\bar{\psi}, \bar{\phi}) &= 0, \end{aligned}$$

where $\bar{\phi}$ and $\bar{\psi}$ are the perturbation of ϕ and ψ close the stable state and

$$\omega(K_1, K_2) = p \left(\frac{\delta}{\delta t} - l \frac{\delta}{\delta z} \right)^2 K_1 + i \left(\frac{\delta}{\delta z} K_1 + q \left(\frac{\delta}{\delta t} - l \frac{\delta}{\delta z} \right) K_2 \right) + \delta K_2.$$

Choosing the Fourier form of the modifying traveling wave perturbation be $\bar{\phi} = \bar{\psi} = \epsilon e^{(\xi z - i\mu t)}$, where ξ is complex parameter μ is real modulating frequency constant and $\epsilon \ll 1$ and the characteristic equation is of the form

$$pl^2\xi^2 + i(1 - ql + 2pl\mu)\xi + s + q\mu - p\mu^2 = 0.$$

If we consider $1 - ql + 2pl\mu = 0$, the eigenvalues of ξ are only real. Consequently, system (1) supports a soliton like solutions with a velocity $l_p = 1/(q - 2p\mu)$ is the transition speed.

To convert the equation (3.1) into nonlinear differential equation, the wave transformation is used as follow

$$\begin{aligned}\phi(x, t) &= u(\eta)e^{i(\sigma_1 x - \sigma_2 t)}, \\ \psi(x, t) &= v(\eta)e^{i(\sigma_1 x - \sigma_2 t)},\end{aligned}\tag{2} \text{ where}$$

$$\eta = \gamma x - \delta t.\tag{3}$$

For real parameters $\sigma_1, \sigma_2, \gamma, \delta$. By replacing (2) into system (1) gives

$$\begin{aligned}\theta(u(\eta), v(\eta)) &= 0, \\ \theta(v(\eta), u(\eta)) &= 0,\end{aligned}\tag{4}$$

Where

$$\theta(K_1, K_2) = p\delta^2 K_1'' + i(\gamma + 2p\delta\sigma_2)K_1' - iq\delta K_2' + rK_1^3 - (\sigma_1 + p\sigma_2^2)K_1 + (s + q\sigma_2)K_2.$$

By setting the complex parts in system (4) to zero, we attain

$$\begin{aligned}(\gamma + 2p\delta\sigma_2)u' - q\delta v' &= 0, \\ (\gamma + 2p\delta\sigma_2)v' - q\delta u' &= 0,\end{aligned}\tag{5}$$

This implies that

$$\begin{pmatrix} \gamma + 2p\delta\sigma_2 & -q\delta \\ -q\delta & \gamma + 2p\delta\sigma_2 \end{pmatrix} \begin{pmatrix} u' \\ v' \end{pmatrix} = 0.$$

By integrating, system (5) gives

$$\begin{pmatrix} \gamma + 2p\delta\sigma_2 & -q\delta \\ -q\delta & \gamma + 2p\delta\sigma_2 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = 0,\tag{6}$$

where the integration constants are zero and this procedure does not affect the generality, Therefore, when

$$\det \begin{pmatrix} \gamma + 2p\delta\sigma_2 & -q\delta \\ -q\delta & \gamma + 2p\delta\sigma_2 \end{pmatrix} = 0,\tag{7}$$

i.e., $\gamma + 2p\delta\sigma_2 = q\delta$, the system (6) has a nontrivial envelope solution $u = v$. Thus, by means of (5), the system (4) becomes

$$p\delta^2 u'' + ru^3 - Lu = 0,\tag{8}$$

$$\text{where } L = (\sigma_1 + p\sigma_2^2) - (s + q\sigma_2). \quad (9)$$

Besides, we get the relation from (7) as

$$\frac{\delta}{\gamma} = \frac{1}{q-2p\sigma_2} = l. \quad (10)$$

Therefore, in order to locate the solitary wave solution of the considered equation (1), it would be sufficient to examine the equation (8).

Phase portrait analysis of the Optical model

To continue using the approach in ref. (Kazmi et al. 2023; Infal et al. 2025), we can express Eq. [1] as a system of planar dynamics by selecting $V = X$ and $V' = Y$, thus reformulating the equation as follows:

$$\begin{cases} X' = Y = f(X, Y) \\ Y' = \frac{1}{p\delta^2} (LX - rX^3) = g(X, Y). \end{cases} \quad (11)$$

where

$$L = (\sigma_1 + p\sigma_2^2) - (s + q\sigma_2).$$

The Hamiltonian function for the planar Hamiltonian system given by equation [11] is

$$H(X, Y) = \frac{Y^2}{2} + \frac{rX^4}{4p\delta^2} - \frac{LX^2}{2p\delta^2}.$$

Where

$$L = (\sigma_1 + p\sigma_2^2) - (s + q\sigma_2).$$

The system exhibits three equilibrium points, which are $E_0^*: (X^*, Y^*) = (0, 0)$,

$$E_1^*: (X^*, Y^*) = \left(-\sqrt{\frac{L}{r}}, 0\right) \text{ and } E_2^*: (X^*, Y^*) = \left(\sqrt{\frac{L}{r}}, 0\right). \text{ The linearization of the}$$

system described by equation [1] at the points $E_i^* (i = 0, 1, 2)$ leads to the variational equation $\Delta x' = J\Delta x$, where $x = (X, Y)^T$, $\Delta x = x - x^*$, and J is the Jacobian matrix, which has the following form:

$$J(X, Y) = \frac{\partial(f, g)}{\partial(X, Y)} = \begin{pmatrix} \frac{\partial f}{\partial X} & \frac{\partial f}{\partial Y} \\ \frac{\partial g}{\partial X} & \frac{\partial g}{\partial Y} \end{pmatrix}_{|(X^*, Y^*)} = \begin{pmatrix} 0 & 1 \\ \frac{L}{p\delta^2} - \frac{3rX^2}{p\delta^2} & 0 \end{pmatrix}.$$

The eigenvalues of J are given by $|J - \lambda I| = 0$ which implies,

$$\lambda^2 - \text{tr}(J)\lambda + |J| = 0,$$

where $\text{tr}(J) = 0$ and $|J| = \frac{3rX^2}{p\delta^2} - \frac{L}{p\delta^2}$.

1. **Case 01.** Stability at $x^* = (0,0)^T$: In this scenario, the characteristic roots are $\lambda = \pm \sqrt{\frac{L}{p\delta^2}}$. Here, the following cases arise:

- a. When $L = 0$ i.e. $p\sigma_2^2 - q\sigma_2 - s + \sigma_1 = 0$ implies $\lambda = 0$. Here, the eigenvalues of the system have no imaginary parts and are equal to zero; therefore, the system is considered unstable. This scenario can be seen as a simple example of a complex eigenvalue with a zero component. The phase diagram of the two-dimensional system [1] is plotted with the parameter values $\delta = 1, p = 1, 1 = 1, s = 1, r = 2, \sigma_1 = 1, \sigma_2 = 1$. Figure 1 illustrates the paths followed by the system, showcasing isoclines and nullclines.
- b. When $L < 0$ and $p\delta^2 < 0$ or $L > 0$ and $p\delta^2 > 0$ then :

$$\lambda_1 = +\sqrt{\frac{L}{p\delta^2}} \quad \text{and} \quad \lambda_2 = -\sqrt{\frac{L}{p\delta^2}}$$

are real and have opposite signs. This indicates that the equilibrium point $x^* = (0,0)^T$ behaves as an unstable saddle point, as illustrated in Figures 2

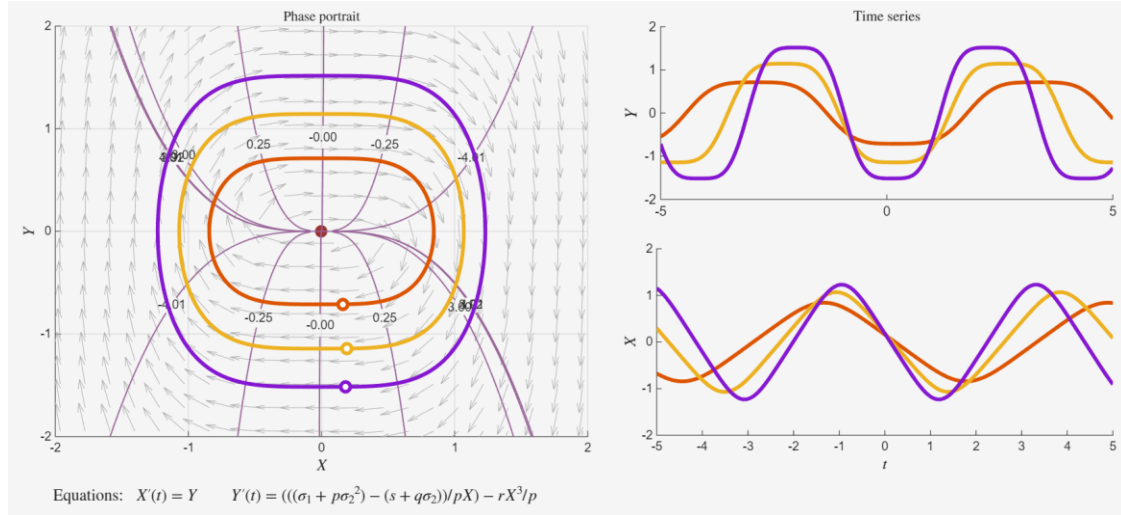
- c. When $L < 0$ and $p\delta^2 > 0$ or $L > 0$ and $p\delta^2 < 0$ then :

$$\lambda_1 = +i\sqrt{\frac{L}{p\delta^2}} \quad \text{and} \quad \lambda_2 = -i\sqrt{\frac{L}{p\delta^2}}$$

are purely imaginary and complex conjugates, which means the equilibrium point $x^* = (0,0)^T$ is a stable center, as shown in Figure 3.

2. **Case 02.** Stability at $x^* = \left(\sqrt{\frac{L}{r}}, 0\right)^T$: the characteristic roots are $\lambda = \pm \sqrt{\frac{-2L}{p\delta^2}}$
 - a. If $L = 0$ i.e. $p\sigma_2^2 - q\sigma_2 - s + \sigma_1 = 0$ then it is similar to the case 01 (a)
 - b. If $L < 0$ and $p\delta^2 > 0$ or $L > 0$ and $p\delta^2 < 0$ then it corresponds to the case 01(b)
 - c. If $L < 0$ and $p\delta^2 < 0$ or $L > 0$ and $p\delta^2 > 0$ then it represents case 01(c)
3. **Case 03.** Stability at $x^* = \left(-\sqrt{\frac{L}{r}}, 0\right)^T$: In this case, the characteristic roots are the same as those in Case 2. As a result, the nature of the eigenvalues is also identical to that of Case 1, since the behavior in Case 2 mirrors that of Case 1. Therefore,

the stability of the equilibrium point $x^* = \left(-\sqrt{\frac{L}{r}}, 0\right)^T$ will not be discussed further here.



The phase diagram of the two-dimensional system [\[eq:5.0.1\]](#) is generated using the parameter values $L = 0$ with $\delta = 1, p = 1, 1 = 1, s = 1, r = 2, \sigma_1 = 1, \sigma_2 = 1$. Figure [1\(a\)](#) displays the trajectories followed by the system, highlighting isoclines and nullclines.

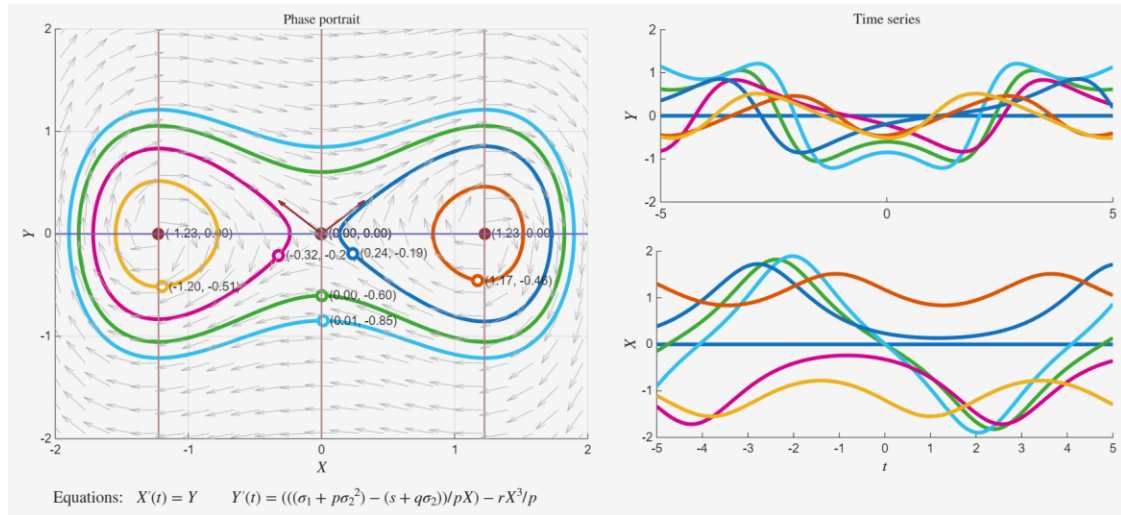


fig-2: The phase diagram of the two-dimensional system [\[eq:5.0.1\]](#) is plotted using the parameter values where $L < 0$ and $p\delta^2 < 0$ or $L > 0$ and $p\delta^2 > 0$, specifically $\delta = 1, p = 3, 1 = 1, s = 1, r = 2, \sigma_1 = 2, \sigma_2 = 1$. Figure [2](#) showcases the trajectories followed by the system, highlighting isoclines and nullclines.

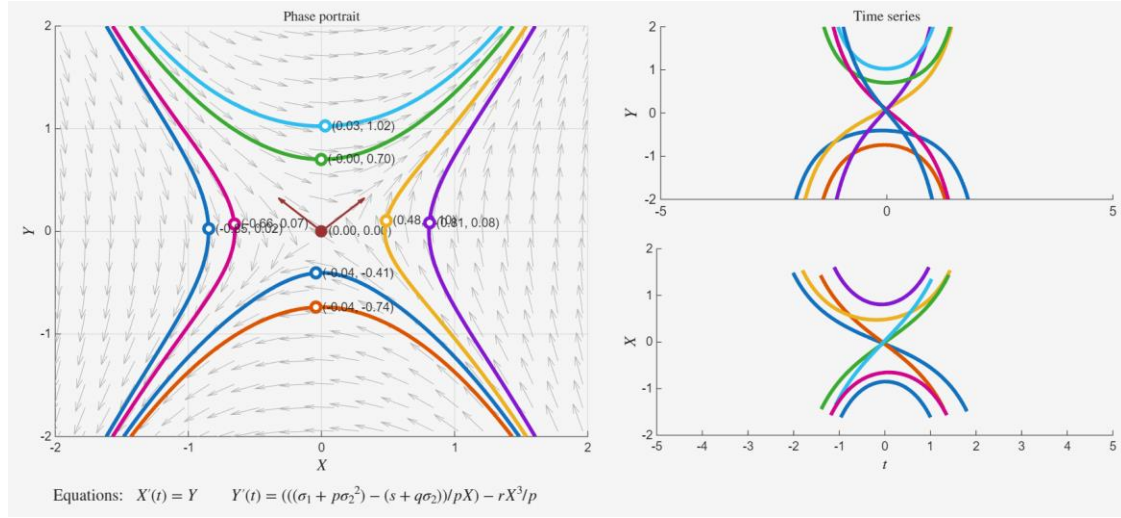


fig-3: The phase portrait of the two-dimensional system [\[eq:5.0.1\]](#) is plotted with the parameter values $L < 0$ and $p\delta^2 > 0$ or $L > 0$ and $p\delta^2 < 0$ for $\delta = 1, p = -3, 1 = 1, s = 1, r = 2, \sigma_1 = 2, \sigma_2 = 1$. Figure 3 illustrates the paths followed by the system, showcasing isoclines and nullclines.

Analysis of Solitary and Traveling Waves in the Optical Model

Let us consider a stochastic model with a continuous solution represented by $V(\xi)$, where $\xi \in \mathbb{R}$. If the following condition is satisfied:

$$\lim_{\xi \rightarrow -\infty} V(\xi) = \lim_{\xi \rightarrow \infty} V(\xi),$$

then the solution $V(\xi)$ corresponds to a solitary wave.

At the equilibrium points E_i (where $i = 0, 1, 2$), the energy levels are given by:

$$h_{E_0} = H(0, 0) = 0,$$

$$h_{E_1} = H\left(\sqrt{\frac{L}{r}}, 0\right) = \frac{L^2}{4\delta^2 p r} - \frac{L^2 \delta^2 p}{2r},$$

$$h_{E_2} = H\left(-\sqrt{\frac{L}{r}}, 0\right) = \frac{L^2}{4\delta^2 p r} - \frac{L^2 \delta^2 p}{2r},$$

respectively.

Using Eqs. [\[eq:3.5\]](#) and [\[eq:5.0.1\]](#) with $V = X$ and $V' = Y$, we obtained solution functions for h_{E_0} as

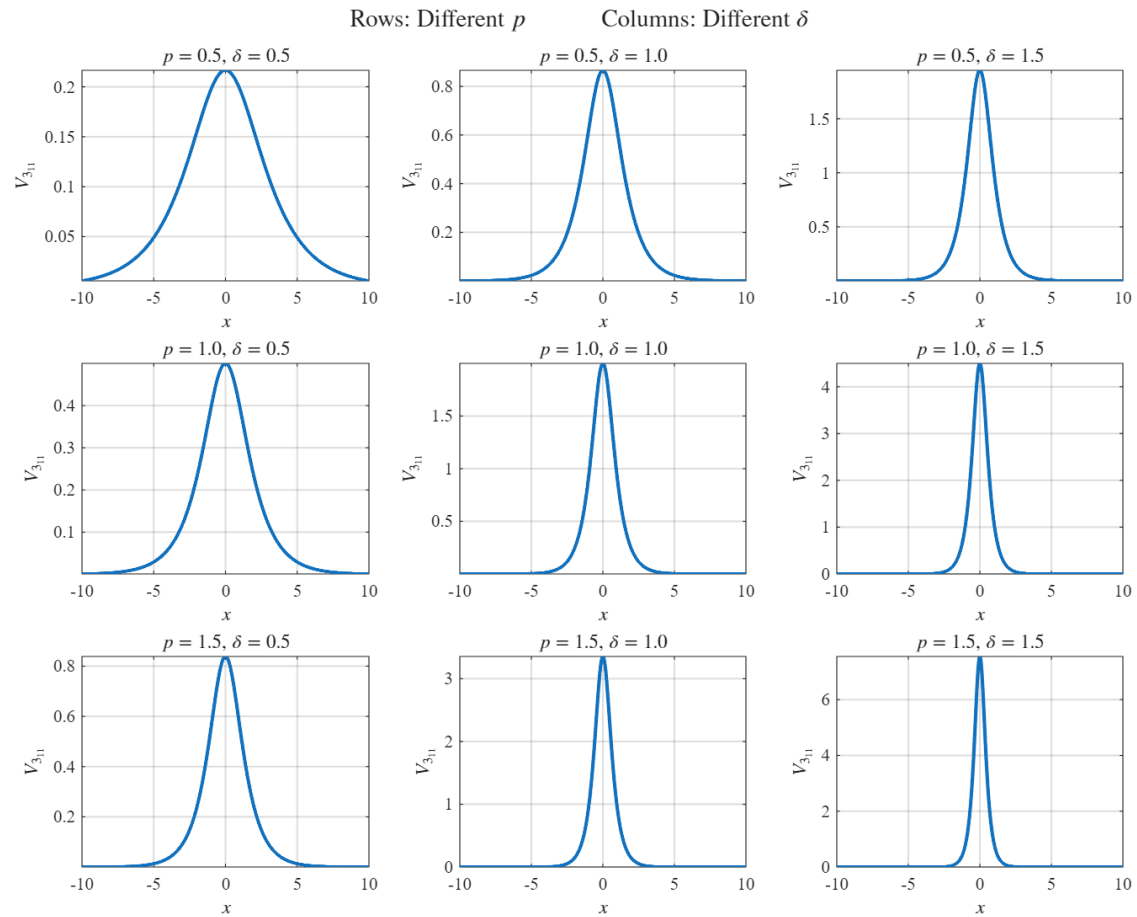
$$V_{311} = \frac{\sqrt{2} \delta^2 p \sqrt{-L r \left(\tanh(\sqrt{L} \delta \sqrt{p} \xi)^2 - 1 \right)}}{r},$$

where $L = p \sigma_2^2 - q \sigma_2 - s + \sigma_1$, and

Also, using Eqs. [\[eq:5.0.1\]](#) and [\[eq:3.6\]](#) or [\[eq:3.7\]](#) with $V = X$ and $V' = Y$, we obtained solution functions for h_{E_1} or h_{E_2} as

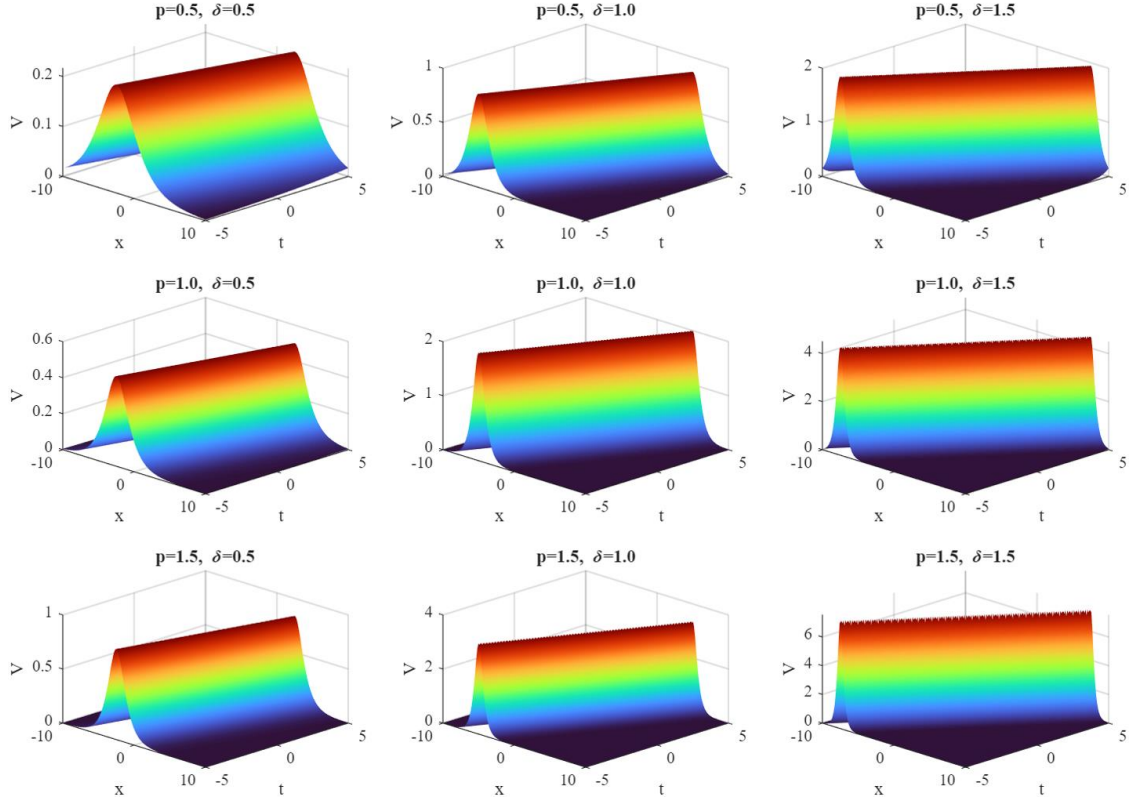
$$d\xi = -\int \frac{\sqrt{2} \delta \sqrt{p} \sqrt{r}}{\sqrt{X^2 r - L} \sqrt{-r X^2 + 2 L \delta^4 p^2 - L}} dX,$$

where $L = p \sigma_2^2 - q \sigma_2 - s + \sigma_1$, Unable to find explicit solution.



2d figure for [\[eq:3.5\]](#)

3D Profiles of V_{311} : Rows = p , Columns = δ



3d figure for [\[eq:3.5\]](#)

The Electrical Line model

We consider an nonlinear electrical transmission lines equation as

$$\frac{\partial^2 V(x, t)}{\partial t^2} + \alpha \frac{\partial^2 V^2(x, t)}{\partial t^2} - \gamma \frac{\partial^2 V^3(x, t)}{\partial t^2} + \beta^2 \frac{\partial^2 V^2(x, t)}{\partial x^2} + \frac{\beta^4}{12} \frac{\partial^4 V(x, t)}{\partial x^4} = 0.$$

Consider traveling wave transform for Eq. [\[eq:3.1\]](#) as

$$V(x, t) = W(\zeta), \quad \zeta = \lambda x + \mu t.$$

With the help of Eq. [\[eq:2.5\]](#), we obtain the NLODE of Eq. (1) as

$$-12\gamma\mu^2 W^3 + 12\alpha\mu^2 W^2 + \beta^4 \lambda^4 W'' + 12W(\beta^2 \lambda^2 + \mu^2) = 0.$$

Phase portrait analysis of the Electrical line model

To continue using the approach in ref. (Kazmi et al. 2023; Infal et al. 2025), we can express Eq. [\[eq:3.3\]](#) as a system of planar dynamics by selecting $V = X$ and $V' = Y$, thus reformulating the equation as follows:

$$\begin{cases} X' = Y = f(X, Y) \\ Y' = \frac{12}{\lambda^4 \beta^4} (12\gamma\mu^2 X^3 - \alpha\mu^2 X^2 - (\beta^2 \lambda^2 + \mu^2)) = g(X, Y). \end{cases}$$

The Hamiltonian function for the planar Hamiltonian system given by equation [\[eq:3.4\]](#) is

$$H(X, Y) = \frac{Y^2}{2} + \frac{1}{\beta^4 \lambda^4} (4\alpha\mu^2 X^3 - 3\gamma\mu^2 X^4 + 6(\beta^2 \lambda^2 + \mu^2) X^2).$$

The system exhibits three equilibrium points, which are $E_0^*: (X^*, Y^*) = (0, 0)$,

$$E_1^*: (X^*, Y^*) = \left(\frac{\alpha\mu + \sqrt{4\beta^2 \gamma \lambda^2 + \alpha^2 \mu^2 + 4\gamma \mu^2}}{2\gamma\mu}, 0 \right) \text{ and } E_2^*: (X^*, Y^*) =$$

$\left(\frac{\alpha\mu - \sqrt{4\beta^2 \gamma \lambda^2 + \alpha^2 \mu^2 + 4\gamma \mu^2}}{2\gamma\mu}, 0 \right)$. The linearization of the system described by equation [\[eq:5.0.1\]](#) at the points $E_i^* (i = 0, 1, 2)$ leads to the variational equation $\Delta x' = J \Delta x$, where $x = (X, Y)^T$, $\Delta x = x - x^*$, and J is the Jacobian matrix, which has the following form:

$$\begin{aligned} J(X, Y) &= \frac{\partial(f, g)}{\partial(X, Y)} = \begin{pmatrix} \frac{\partial f}{\partial X} & \frac{\partial f}{\partial Y} \\ \frac{\partial g}{\partial X} & \frac{\partial g}{\partial Y} \end{pmatrix}_{|(X^*, Y^*)} \\ &= \begin{pmatrix} 0 & 1 \\ \frac{12}{\beta^4 \lambda^4} (3\gamma\mu^2 X^2 - 2\alpha\mu^2 X^2 - (\beta^2 \lambda^2 + \mu^2)) & 0 \end{pmatrix}. \end{aligned}$$

The eigenvalues of J are given by $|J - pI| = 0$ which implies,

$$p^2 - \text{tr}(J)p + |J| = 0,$$

where $\text{tr}(J) = 0$ and $|J| = -\frac{12}{\beta^4 \lambda^4} (3\gamma\mu^2 X^2 - 2\alpha\mu^2 X^2 - (\beta^2 \lambda^2 + \mu^2))$.

1. **Case 01.** Stability at $x^* = (0, 0)^T$: In this scenario, the characteristic roots are $p = \pm \frac{2\sqrt{3}\sqrt{\beta^2 \lambda^2 + \mu^2}}{\beta^2 \lambda^2}$. Here, the following cases arise:
 - a. When $\beta^2 \lambda^2 + \mu^2 = 0$ implies $p = 0$. Here, the eigenvalues of the system have no imaginary parts and are equal to zero; therefore, the system is considered unstable.
 - b. When $\beta^2 \lambda^2 + \mu^2 > 0$ then :

$$p_1 = \sqrt{\beta^2 \lambda^2 + \mu^2} \quad \text{and} \quad p_2 = -\sqrt{\beta^2 \lambda^2 + \mu^2}$$

are real and have opposite signs. This indicates that the equilibrium point $x^* = (0,0)^T$ behaves as an unstable saddle point, as illustrated in Figures 6 and 7

2. **Case 02.** Stability at $x^* = \left(\frac{\alpha\mu + \sqrt{4\beta^2\gamma\lambda^2 + \alpha^2\mu^2 + 4\gamma\mu^2}}{2\gamma\mu}, 0 \right)^T$: the characteristic roots are

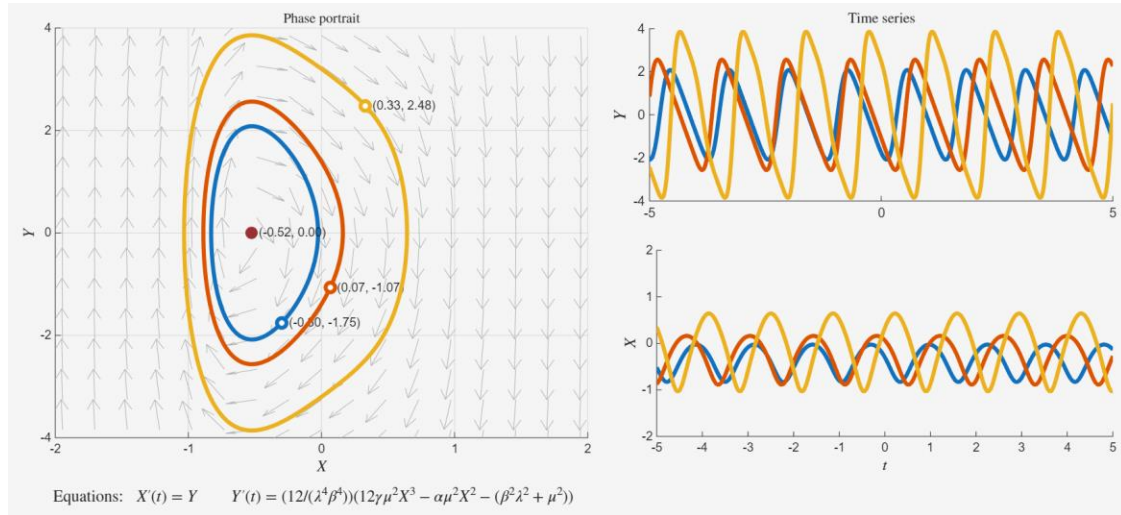
$$p = \pm 2\sqrt{3} \sqrt{-\frac{\mu^2 - \frac{3\sigma_1}{4\gamma} + \beta^2\lambda^2 + \frac{\alpha\sigma_1}{2\gamma^2}}{\beta^4\lambda^4}},$$

where

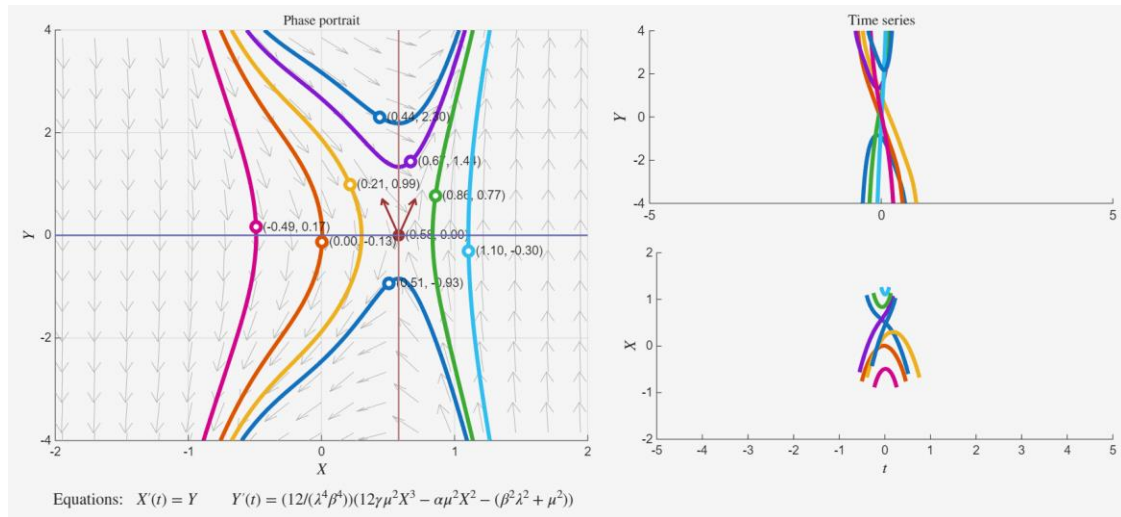
$$\sigma_1 = \left(\alpha\mu + \sqrt{\alpha^2\mu^2 + 4\gamma\beta^2\lambda^2 + 4\gamma\mu^2} \right)^2.$$

3. **Case 03.** Stability at $x^* = \left(\frac{\alpha\mu - \sqrt{4\beta^2\gamma\lambda^2 + \alpha^2\mu^2 + 4\gamma\mu^2}}{2\gamma\mu}, 0 \right)^T$: In this case, the characteristic roots are the same as those in Case 2. As a result, the nature of the eigenvalues is also identical to that of Case 1, since the behavior in Case 2 mirrors that of Case 1.

Therefore, the stability of the equilibrium point $x^* = \left(\frac{\alpha\mu - \sqrt{4\beta^2\gamma\lambda^2 + \alpha^2\mu^2 + 4\gamma\mu^2}}{2\gamma\mu}, 0 \right)^T$ will not be discussed further here.



The phase diagram of the two-dimensional system [\[eq:5.0.1\]](#) is generated using the parameter values $\beta^2\lambda^2 + \mu^2 > 0$ with $\alpha = -1, \beta = 2, \gamma = -1, \lambda = 1, \mu = 2$. Figure 6 displays the trajectories followed by the system, highlighting isoclines and nullclines.



The phase diagram of the two-dimensional system [eq:3.4] is plotted using the parameter values where $\beta^2 \lambda^2 + \mu^2 > 0$, specifically $\alpha = 1, \beta = 2, \gamma = 1, \lambda = 1, \mu = 2$. Figure 7 showcases the trajectories followed by the system, highlighting isoclines and nullclines.

Conclusion

In this article, the parametric effects on solitary wave propagation and characteristic aspects of long-wave propagation in the dual-core optical fibers and electrical lines have been studied by establishing lots of illustrative, advanced and broad-ranging solitary wave solutions via bifurcation analysis. It has been clearly explained, how the free parameters in the solutions obtained are inextricably linked with the wave profile. We have also discussed the impact of the change of the velocity and wave number in the wave propagation. The physical significance of the solutions attained has been illustrated for the definite values of the included parameters through depicting the 3D profiles. These wave profiles have a wide range of applications in optics and telecommunications. The methods introduced are straightforward, simple, and efficient, and they can be extended to a wide range of nonlinear models in optics and telecommunications.

CRedit authorship contribution statement

Md. Azizur Rahman: Methodology, Formal analysis, Data curation, Writing - review & editing, Supervision, Project administration. **Arjuman Ara:** Visualization, Validation, Resources, Investigation. Software, Conceptualization, Writing - original draft.

Declaration of Competing Interest

The authors declare that there is no conflict of interests regarding the publication of this article.

Data availability statement

All data generated or analyzed during this study are included in this published article.

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