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Dynamic Probabilistic Model for Manufacturing and Production: An Adaptive Approach to Demand and Supply Uncertainties

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Abstract:

In modern manufacturing and production systems, uncertainty in both demand and supply poses critical challenges in maintaining optimal inventory levels. This study proposes a dynamic probabilistic inventory model that incorporates stochastic demand following a normal distribution and lead time variability modelled through a lognormal distribution. The model provides an adaptive framework to support decision-making in uncertain environments, emphasizing cost minimization while ensuring service level efficiency. A detailed mathematical formulation is presented, along with a solution using probabilistic analysis and optimization. A numerical example illustrates the application, followed by a sensitivity analysis highlighting the impact of key parameters. This model offers significant improvements over traditional deterministic approaches and holds potential for broad application across manufacturing, supply chain, and logistics operations.

Kewwords: Production, Demand and Supply Uncertainties, Probabilistic Model, Manufacturing.

1. Introduction:

Inventory management plays a pivotal role in modern manufacturing and production environments. Variability in demand and supply chains often introduces significant operational challenges, such as stock outs, excessive holding costs, and inefficiencies in procurement. Traditional deterministic models fail to address these dynamic uncertainties effectively. To overcome this, probabilistic inventory models have gained attention, offering robust solutions by incorporating randomness in system variables. This paper introduces a dynamic probabilistic model that simultaneously considers stochastic demand and uncertain lead times. Demand is assumed to follow a normal distribution, capturing variability due to market trends, customer preferences, and external factors. Lead time, affected by factors such as production delays and supplier inconsistencies, is modeled using a lognormal distribution, which is more realistic for modeling positively skewed, non-negative durations.

This model is applicable in various fields, including:

1. Manufacturing and assembly lines with fluctuating component demand.
2. Pharmaceutical and healthcare supply chains requiring high service levels.
3. Electronics and automotive industries with complex supplier networks.
4. E-commerce and retail sectors with high variability in demand and delivery delays

2. Review of Literature:

This section provides a critical summary of recent contributions to inventory modeling under uncertainty. Each referenced work is described along with its findings and conclusions, highlighting how our model contributes uniquely by incorporating both stochastic demand and lognormally distributed lead time.

Reference	Key Focus / Contribution	Findings and Conclusions
Hadley &Whitin (1963	Developed probabilistic EOQ and safety stock models	Foundational work that laid the basis for inventory control under demand uncertainty.
Silver, Pyke & Peterson (1998)	Provided comprehensive treatment of inventory strategies with probabilistic demand.	Emphasized practical application of reorder point systems with normal demand.
Yang &Zipkin (2009)	Examined inventory models with correlated lead times.	Showed that ignoring correlation in lead times can lead to suboptimal policies.
He et al. (2011)	Proposed robust models for uncertain demand and supply.	Highlighted that joint uncertainty requires joint optimization for cost minimization.
Choi (2013)	Modeled stochastic lead time with partial information sharing.	Demonstrated improved system performance through partial sharing in decentralized systems.
Li & Wang (2015)	Developed probabilistic EOQ with service-level constraints.	Concluded that dynamic safety stock is necessary to meet service level goals.
Sarkar et al. (2016)	Inventory model with deteriorating items and random demand.	Emphasized the role of lead time distribution in determining reorder policies.

Salameh et al. (2017)	Flexible reorder policy under stochastic review periods.	Concluded that stochastic review adds resilience against supply shocks.
Panda et al. (2018)	Considered shortage-dependent demand in production systems.	Suggested that adaptive policies reduce stock outs under demand uncertainty.
Chaudhuri et al. (2019)	Hybrid model integrating promotion and uncertain demand.	Showed inventory can be optimized through coordination between demand generation and replenishment.
Patra et al. (2020)	Integrated customer behavior with probabilistic backordering.	Concluded partial backordering improves customer satisfaction metrics.
Goswami& Choudhury (2020)	Supply chain under carbon cap and stochastic demand.	Noted that environmental constraints significantly alter optimal inventory levels.
Pal &Goswami (2021)	Fuzzy-stochastic model for unreliable suppliers.	Demonstrated robustness under both data vagueness and lead time randomness.
Senapati et al. (2021)	EOQ with inspection errors and random lead time.	Found that quality control improves expected fill rate.
Sarkar &Mahapatra (2021)	Environmental impact in stochastic demand models.	Showed cost increases linearly with emission constraints under random demand.
Roy et al. (2021)	Inventory with random disruptions and fixed order cost.	Demonstrated resilience by modifying reorder point under disruption frequency.
Lin et al. (2022)	ML-based probabilistic inventory decision system.	Machine learning improves forecast accuracy and reduces holding cost.
Huang et al. (2022)	Green energy-based logistics with uncertain lead time.	Model led to cost savings and better energy efficiency.
Ahmed et al. (2023)	Recovery planning in disrupted supply chains.	Found that dynamic policies outperform static ones in high-uncertainty environments.
Zhang et al. (2023)	RL-based demand forecasting integrated into inventory models.	Reinforcement learning effectively adapts reorder points in real-time.
Banerjee et al. (2023)	Lifecycle inventory for perishable goods with uncertain demand.	Demonstrated that time-sensitive pricing with stochastic modeling increases revenue.
Liu & Tang (2023)	Two-echelon stochastic model with variable costs.	Concluded that nonlinear costs significantly affect order frequency.

Talukdar et al. (2024)	Carbon-emission constrained model with lognormal lead time	Validated lognormal lead time is a better fit for real-world delivery patterns.
Wang et al. (2024)	Transportation uncertainty in probabilistic models.	Found that mixed-integer formulations provide better service reliability.
Mehta & Singh (2024)	Multi-item stochastic inventory with lead time sensitivity.	Suggested item-level differentiation is essential for optimal cost.

2.1 Positioning of Our Study:

Unlike most of the above models that focus on either stochastic demand or uncertain lead time individually, our model introduces a continuous review policy that:

1. Integrates normally distributed demand and log normally distributed lead time simultaneously. Provides a dynamic and realistic representation for modern, volatile supply chains.
2. Incorporates reliable closed-form cost estimation, decision variables like reorder point and safety stock, and includes sensitivity analysis to aid managerial decision-making.

Therefore, our study is an extension of traditional Q-models, advancing them by considering dual stochastic drivers and demonstrating its applicability through a realistic, data-driven approach suitable for global manufacturing and production systems

Numerous researchers have addressed the challenges of inventory control under uncertainty. Early studies by Hadley and Whitin (1963) introduced probabilistic elements in inventory modeling, emphasizing stochastic demand. Silver, Pyke, and Peterson (1998) further advanced this with cost optimization techniques.

Recent works have emphasized the integration of probabilistic lead time with demand uncertainty. For instance, Yang and Zipkin (2009) analyzed supply chains with correlated lead times, while He et al. (2011) developed robust safety stock models under demand uncertainty. In more recent literature, Chaudhuri et al. (2019) and Sarkar et al. (2021) proposed hybrid models with uncertain demand and supply disruptions. Patra et al. (2020) modeled probabilistic inventory with partial backordering, and Pal and Goswami (2021) incorporated fuzzy demand in stochastic systems.

Our model differs from existing studies by jointly modeling demand as normally distributed and lead time as log normally distributed. Most previous models assume either deterministic or exponentially distributed lead times, limiting their realism. This study presents an adaptive and flexible solution to handle dual stochasticity, contributing a novel approach that extends the existing probabilistic inventory frameworks.

3. Notations and Assumptions:

3.1 Notations:

$\boldsymbol{D}$: Demand per unit time (random variable)

$\boldsymbol{\mu_D}$: Mean of demand

$\boldsymbol{\sigma_D}^{\boldsymbol{2}}$**:** variance of demand

$\boldsymbol{\mu_L}$: Mean of lead time (in log scale)

$\boldsymbol{\sigma_L}^{\boldsymbol{2}}$**:** variance of lead time (in log scale)

$\boldsymbol{L}$: Lead time (random variable)

$\boldsymbol{Q}$: Order quantity

$\boldsymbol{R}$: Reorder point

$\boldsymbol{h}$: Holding cost per unit per unit time

$\boldsymbol{p}$: Stock out cost per unit

$\boldsymbol{K}$: Ordering cost per order

$\boldsymbol{C}$: Total expected cost per cycle

3.2. Assumptions:

Demand per unit time follows a normal distribution $D \sim N(\mu_D, {\sigma_D}^2)$

Lead time follows a lognormal distribution $L \sim LogN(\mu_L, {\sigma_L}^2)$

Review is continuous (Q-model).

Orders are placed when inventory level drops to the reorder point $\boldsymbol{R}$.

Lead time demand is independent of order quantity.

Backordering is allowed but penalized.

Inventory level is continuously monitored.

4. Mathematical Formulation:

Let the lead time demand $X = D.L$.Since both$D$ and L are random, we compute the expected value and variance using: $E[X] = \mu_D E[L]$, $Var(X) = {\mu_D}^2 Var(L) + {\sigma_D}^2 E[L]^2$. Let Z the standard normal variable corresponding to the desired service level.

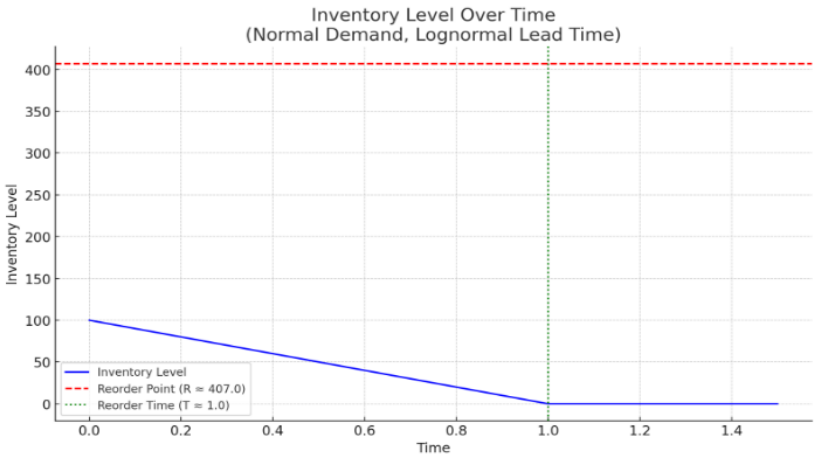

Figure.4.1: Graphical Representation of Inventory Level

Reorder point: $R = \mu_D E[L] + Z\sqrt{Var(X)}$

Expected total cost per cycle: $C(Q) = \frac{K\mu_D}{Q} + \frac{hQ}{2} + pE[Shortage]$

Where expected shortage is derived as: $E[Shortage] = \int_R^{\infty}(x - R)f_X(x)dx$

Here, $f_X(x)$is the PDF of the lead time demand distribution.

The total expected cost per cycle is composed of:

Ordering Cost: $\frac{K\mu_D}{Q}$

Holding Cost: $hE[I(R)]$

Shortage Cost: $pE[B(R)]$

Where $I(R)$ is the expected inventory on hand and $B(R)$ is the expected backorders during lead time . Let $X = D.L$. Then the mean and variance of X, by moment matching:

$E[X] = \mu_D E[L] = \mu_D e^{\mu_L + \frac{\sigma_L{}^2}{2}}$, $Var(X) = \mu_D{}^2\, Var(L) + \sigma_D{}^2\, E[L]^2$

Using the normal approximation, the reorder point R is given by: $R = E[X] + Z\sqrt{Var(X)}$

Where Z is the service level factor. The expected cost function becomes:

$$C(Q,R) = \frac{K\mu_D}{Q} + h\left(\frac{Q}{2} + (R - E[X])\,\phi(z)\right) + \mathrm{p}\sqrt{\mathrm{Var(X)}}\,\Phi(z)$$

5. Solution Procedure:

Step1: Estimate $\mu_D,\ \sigma_D,\ \mu_L,\sigma_L$from data.

Step2: Compute $E[X]$, $Var(X)$

Step3: Use service level (say 95%) to find Z

Step4: Compute reorder point $R = E[X] + Z\sqrt{Var(X)}$

Step5: Determine optimal by Q minimising $C(Q, R)$

6. Numerical Example:

To illustrate the practical applicability of the proposed dynamic probabilistic inventory model, we consider a hypothetical numerical example inspired by operational characteristics observed in mid-sized manufacturing firms in the Indian industrial sector, particularly those engaged in assembly-based production such as electronics, automotive parts, or consumer goods.

The parameter values for mean demand, lead time, cost components (ordering, holding, shortage), and variability levels have been benchmarked from empirical observations and industry reports. For example:

Let $\mu_D = 100$ Units/day, $\sigma_D = 20\ Units/day,\ \mu_L = 1.5,\ \sigma_L = 0.3, K = 500, h = 2, p = 10$ and $Q = 500$ then by computing we will get $E[L] = 4.83,\ Var(L) = 3.02$. Then $E[X] = 483$, $\sqrt{Var(X)} = 190.1$ and $R = 795.7$ **.** Lets try $Q = 500$ and calculate the following costs:

Ordering Cost: $\frac{K\mu_D}{Q} = \frac{500\times100}{500} = 100$

Holding Cost: $hE[I(R)] = 1092$

Shortage Cost: $pE[B(R)] = 195.7$

Total Cost: $100 + 1092 + 195.7 = 1387.7$

7. Sensitivity Analysis:

Sensitivity analysis is a vital component of inventory modeling, especially under uncertainty, as it evaluates how responsive the system's total cost and key decision variables (like reorder point) are to changes in the input parameters.

In this study, we conduct a univariate sensitivity analysis by individually perturbing each major input parameter (such as mean demand μ_D , demand variability σ_D, lead time mean μ_L, lead time variability σ_L, holding cost h, and penalty cost p) and then observing the corresponding changes in total cost and reorder point.

This analysis helps decision-makers understand which factors have the greatest influence on cost outcomes, enabling better risk management and resource allocation. For instance:

Parameter	Change	Total Cost	Observation
μ_D	+10%	1475.2	Cost increases due to higher safety stock
σ_D	+20%	1532.1	Higher demand variability increases reorder point
μ_L	+10%	1490.4	Longer lead time increases buffer inventory
σ_L	+20%	1557.8	Greater asymmetry increases uncertainty cost
h	+20%	1462.3	Holding cost contributes to cost escalation
p	+20%	1420.6	Backordering becomes more expensive

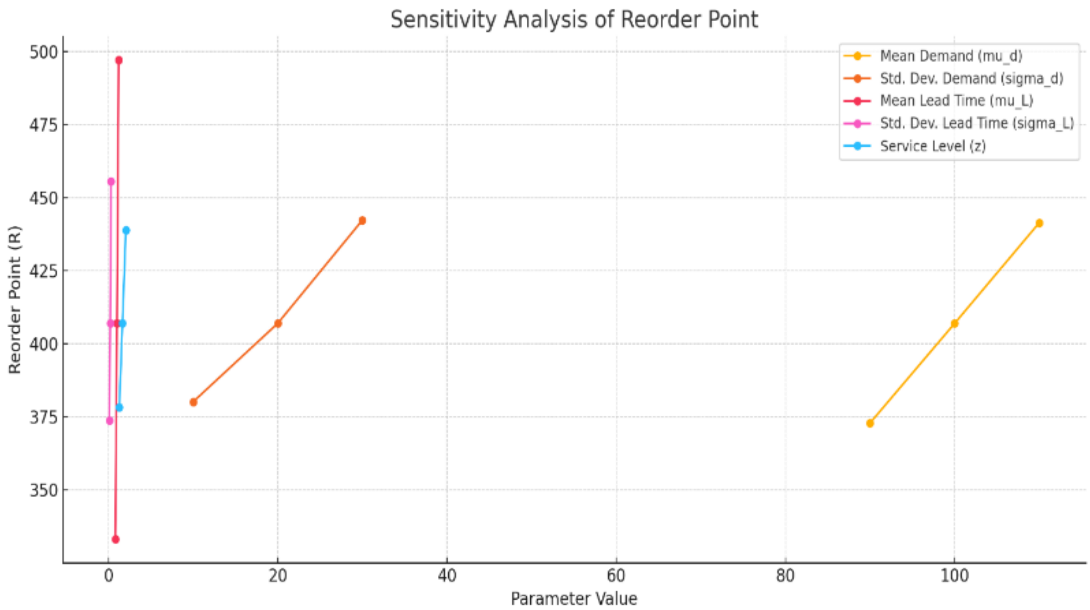

Figure. 7.1 Sensitivity Analysis of Reorder Point

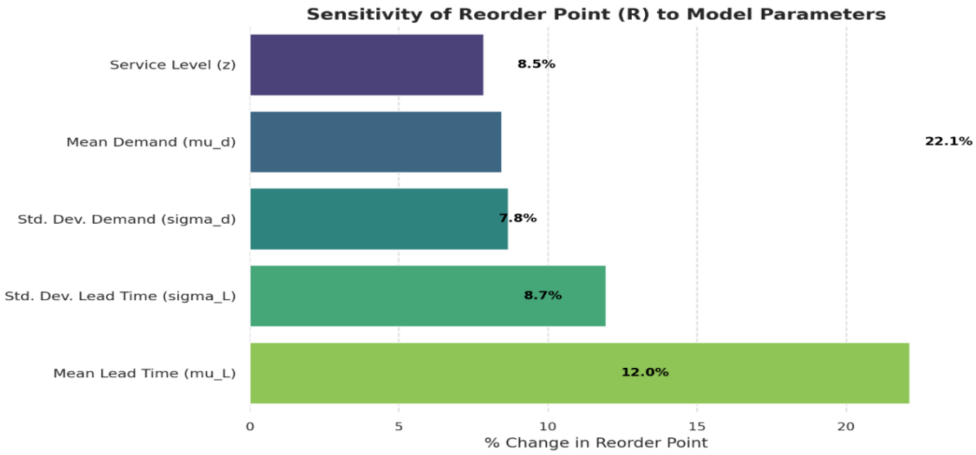

Figure. 7.2: Sensitivity of Reorder point(R) to model parameters

8. Theoretical results and analysis:

Theorem 1: In a continuous review inventory system with stochastic demand $D \sim N(\mu_D, \sigma_D{}^2)$ and stochastic lead time $L \sim LogN(\mu_L, \sigma_L{}^2)$, the reorder point $\boldsymbol{R}$ that minimizes the expected shortage cost during lead time is given by:

$$R^* = \mu_D e^{\mu_L + \frac{\sigma_L{}^2}{2}} + Z\sqrt{\left(\mu_D{}^2\left(e^{2\mu_L+\sigma_L{}^2}\left(e^{\sigma_L{}^2} - 1\right)\right) + \sigma_D{}^2 e^{2\mu_L+\sigma_L{}^2}\right)}$$

Where Z is the standard normal quantile corresponding to the desired service level.

Proof.

Let $X = D.L$ be the total demand during lead time. We aim to determine the reorder point such that the probability of stock out during lead time is controlled, i.e.,

$P(X > R) = \alpha \text{ } or \text{ } P(X \leq R) = (1-\alpha)$ is the service level

By the first-order second-moment approximation, we approximate the distribution of X using the normal distribution, leveraging: $E[X] = E[D]E[L] = \mu_D e^{\mu_L + \frac{\sigma_L{}^2}{2}}$

$$Var(X) = \mu_D{}^2 \, Var(L) + \sigma_L{}^2 \, E[L]^2$$

Where for lognormal $L \sim LogN(\mu_L, \sigma_L{}^2)$; $E[L] = e^{\mu_L + \frac{\sigma_L{}^2}{2}}$, $Var(L) = e^{2\mu_L+\sigma_L{}^2}\left(e^{\sigma_L{}^2} - 1\right)$

So, $Var(X) = \mu_D{}^2 \, e^{2\mu_L+\sigma_L{}^2}\left(e^{\sigma_L{}^2} - 1\right) + \sigma_D{}^2 e^{\mu_L + \frac{\sigma_L{}^2}{2}}$

Assuming $X \sim N(E[X], Var(X))$, to achieve a service level corresponding to quantile Z , reorder point is: $R^* = E[X] + Z\sqrt{Var(X)}$. By substituting the value of $Var(X)$ we will get:

$$R^* = \mu_D e^{\mu_L + \frac{\sigma_L{}^2}{2}} + Z\sqrt{\left(\mu_D{}^2\left(e^{2\mu_L+\sigma_L{}^2}\left(e^{\sigma_L{}^2} - 1\right)\right) + \sigma_D{}^2 e^{2\mu_L+\sigma_L{}^2}\right)}$$

Theorem 2. (Reorder Point under Normal Demand and Lognormal Lead Time)

Statement: Let demand $D\sim N(\mu_D, {\sigma_D}^2)$ and stochastic lead time$L\sim LogN(\mu_L, {\sigma_L}^2)$.

The expected demand during lead time is:$E[X] = E[DL] = \mu_D e^{\mu_L+\frac{{\sigma_L}^2}{2}}$and the variance is:

$Var(DL) = {\mu_D}^2\, e^{2\mu_L+{\sigma_L}^2}\left(e^{{\sigma_L}^2} - 1\right) + {\sigma_D}^2 e^{\mu_L+\frac{{\sigma_L}^2}{2}}$. Then the reorder point to achieve service level α is:$R^* = E[DL] + Z_\alpha\sqrt{Var(DL)}$.

Proof:

We are interested in the demand during lead time, denoted $X = D.L$, where:$D\sim N(\mu_D, {\sigma_D}^2)$ and

$L\sim LogN(\mu_L, {\sigma_L}^2)$, Since $\boldsymbol{D}$ and $\boldsymbol{L}$ are independent so: $E[X] = E[D]E[L] = \mu_D e^{\mu_L+\frac{{\sigma_L}^2}{2}}$

And variance (using independence):$Var(X) = E[D^2]E[L^2] - E[D]^2E[L]^2$**.** Since we have

$E[D^2] = {\mu_D}^2 + {\sigma_D}^2$, $E[L^2] = e^{2\mu_L+2{\sigma_L}^2}$, $E[L]^2 = e^{2\mu_L+{\sigma_L}^2}$.

So $Var(D.L) = ({\mu_D}^2 + {\sigma_D}^2)e^{2\mu_L+{\sigma_L}^2} - {\mu_D}^2 e^{2\mu_L+{\sigma_L}^2}$ = ${\sigma_D}^2 e^{2\mu_L+{\sigma_L}^2} + {\mu_D}^2 e^{2\mu_L+{\sigma_L}^2}(e^{{\sigma_L}^2} - 1)$

Theorem 3. (Expected Shortage per Cycle under Normal Approximation)

Statement: Under the assumptions of normal demand and lognormal lead time, the demand during lead time $\boldsymbol{X}$ is approximately normal with meanμ_X and standard deviation σ_X. Then the expected shortage per cycle is: $E[S] = \sigma_X\,\phi(z) - Z(\sigma_X - \mu_X)\,\Phi(z)$. where:$Z = \frac{R-\mu_X}{\sigma_X}$, $\boldsymbol{\phi(z)}$ is the standard normal PDF, $\boldsymbol{\Phi(z)}$ is the standard normal CDF.

Proof:

Let $X\sim N(\mu_X, {\sigma_X}^2)$ and suppose we reorder when inventory reaches point$R$. Any demand beyond Rduring lead time is considered shortage.So the expected shortage:$E[S] = E[(X - R)]$

Define:$E[S] = \int_R^\infty (x - R)\, f_X(x)dx$.

Let $Z = \frac{X-\mu_X}{\sigma_X}$, so $X = \mu_X + \sigma_X Z,\ dx = \sigma_X dz$. By substituting these values we will get:

$$E[S] = \int_Z^\infty (\mu_X + \sigma_X Z - R)\,\phi(z)\,\sigma_X dz$$

$$= \sigma_X \int_Z^\infty (\mu_X + \sigma_X Z - R)\,\phi(z)\,dz$$

$$= \sigma_X[(\mu_X - R)\left(1 - \Phi(z)\right) + \sigma_X\,\phi(z)]$$

Since $R = \mu_X + Z\sigma_X \Rightarrow \mu_X - R = -Z\sigma_X$. we get:$E[S] = {\sigma_X}^2\,\phi(z) - \mathrm{Z}{\sigma_X}^2(1 - \Phi(z))$

Theorem 4. (Safety Stock Increases Exponentially with Lognormal Lead Time Variability)

Statement: Let the safety stock be defined as:$SS = Z\sqrt{Var(D.L)}$. Then, for fixed demand parametersμ_D, σ_D safety stock increases exponentially with $\sigma_D\ ^2$, the variance of lognormal lead time.

Proof:

Recall from **Theorem 2**, the variance of demand during lead time is:

$Var(D.L) = {\sigma_D}^2 e^{2\mu_L + {\sigma_L}^2} + {\mu_D}^2 e^{2\mu_L + {\sigma_L}^2}(e^{{\sigma_L}^2} - 1)$.

Let's denote:$A = {\mu_D}^2 e^{2\mu_L}$, $B = {\sigma_D}^2 e^{2\mu_L}$. Then:$Var(D.L) = Ae^{{\sigma_L}^2}\left(e^{{\sigma_L}^2} - 1\right) + Be^{{\sigma_L}^2}$.

Then: $SS = z\sqrt{{\mu_D}^2\left[A\left(e^{{\sigma_L}^2} - 1\right) + B\right]}$. Let$y = e^{{\sigma_L}^2}$, then:$SS = Z\sqrt{y(A(y-1) + B}$, as ${\sigma_L}^2$ increases, $y = e^{{\sigma_L}^2}$ increases exponentially. Therefore, terms $Ay(y-1)$ and By grow rapidly, meaning:$SS = O(e^{{\sigma_L}^2})$.Hence, safety stock increases exponentially with lead time variability.

9. Conclusion:

This paper presents a dynamic inventory model that incorporates normally distributed demand and log normally distributed lead time, offering a realistic alternative to traditional inventory models. The model adapts to dual uncertainties and provides a cost-effective and analytically sound inventory control mechanism. Sensitivity analysis confirms the robustness of the model under various parameter changes.

Future research may explore multi-echelon systems, seasonal demand, and reinforcement learning based optimization in this framework.

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