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Construction of Quality Interval Skip Lot Sampling Plan-2 through Tangent Angle

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Abstract
Acceptance sampling systems are advocated when small sample sizes are necessary and desirable ***due to costly testing*** for product quality. In this paper, we introduce a new design for ***Quality Interval Skip-Lot Sampling Plan-2***. This paper ***develops*** a Skip-Lot Sampling Plan-2 based on trigonometric ratios and hypotenuse ratios, along with ***decision region (d_1)*** and ***probabilistic region (d_2)***, which ***are*** more applicable in practical situations. ***The Maximum Allowable Percent Defective (MAPD)*** is also considered for the selection of parameters for ***the sampling plan***, which is representative of ***the quality interests of all involved in production***. Tables ***are presented***, and examples ***are illustrated***.

Keywords: Skip Lot Sampling Plan-2, Quality Probability of Acceptance, Decision Regions, Operating Characteristic Curve, Trigonometric ratios, Hypotenuse ratios.

1. Introduction

To improve the quality for any product and services, it is customary to modernize the quality practices and simultaneously reduce the cost for inspection and quality improvement. An efficient quality improvement programme can be instrumental with increasing productivity at reduced cost. As a result of increasing customer quality requirements and development for new product technology, many existing quality assurance practices and techniques need to be modified.

The need for such statistical and analytical techniques in quality assurance is rapidly increasing owing to stiff competition in industry towards product quality improvement. Acceptance sampling is a tool for consumer to reject bad lots as well as producer to quicken the process control. In a progressive atmosphere of production with increased chances for occurrence of non-conforming item, statistical process control optimizes the process capability and acceptance sampling acts logically to prevent passing out nonconforming units.

Under continuous production, acceptance sampling schemes and systems are well used for maintaining quality. When the producer set up a strong process control mechanism, the acceptance control procedure is superfluous as they no longer needed but reinstated as and when it is necessary. Sampling plans do not estimate product parameters, but constitute an overall decision to maintain and improve existing quality for the product.

This paper introduces a method for Selection of Skip Lot Sampling Plan based on range of quality instead of point wise description of quality by invoking a novel approach called Quality Interval Sampling (QIS) plan. Divya (2009) has studied single sampling plan through Quality Region and constructed single sampling plan indexed with quality region involving QIS.

This method seems to be versatile and can be adopted in the elementary production process where the stipulated quality level is advisable to fix at later stage and provides a new concept for Selection of Skip Lot Sampling Plan involving quality levels.

The sampling plan provides both vendor and buyer decision rules for the product acceptance to meet the present product quality requirement. Due to rapid advancement of manufacturing technology, suppliers require their products to be of high quality with very low fraction defectives often measured in parts per million.

Unfortunately, traditional methods in some particular situations fail to find out a minute defect in the product. In order to overcome such problems Quality Interval Sampling (QIS) plan is introduced. This paper designs the parameters for the plan indexed with quality regions involving QIS

2. Selection of the Sampling Plan

Perry (1970) has developed a system of sampling inspection plan known as SkSP-2. This plan involves inspection of only some fraction f of the submitted lots when quality of the submitted product is good as demonstrated by the quality of the product. These plans are applicable to products produced or furnished in successive lots or batches.

Operating Procedure

A SkSP-2 plan is one that uses a given lot inspection plan by the method of attributes called the reference plan together with a procedure that calls for normally inspecting every lot, but for inspecting only a fraction of the lots when the quality is good. The plan includes specific rules based on the record of lot acceptance and rejection, for switching back and forth between normal inspection and skipping inspection

The operating procedure:

a. Start with normal inspection, using the reference plan

b. When i consecutive lots are accepted on normal inspection, switch to skipping inspection of inspecting a fraction f of the lots

c. When a lot is rejected on skipping inspection, switch to normal inspection.

d. Screen each rejected and correct to replace all defective units found.

Associated with SkSP-2 are, a given reference plan, and the parameters i and f. In general 0< f< 1 and i is a positive integer. When f=1, the plan degenerates to the original reference plan

Operating Characteristic function for SkSP-2 Plan

The OC function associated with SkSP-on Plan is of type B , based on probabilities sampling from an infinite universe or process. The conditions associated with sampling from an infinte universe re based on the notion of a process producing a theroetical continuous infinte product flow.

Perry (1970) has derived the OC function for SkSP-2 plan by two approaches namely

1. Power series approach and 2. Markov Chain approach

The measure of SkSP-2 for the probability of acceptance is

$$\text{Pa(f,i)} = \frac{(1-\text{f})\text{P i} + \text{fP}}{(1-\text{f})\text{Pi} + \text{f}}$$

Where P is the OC function for the reference sampling plan. It is noted that p(f,i) is a function of i clearing interval f sampling fraction and the reference plan.

3. Selection procedure

Designing of Quality Interval Skip-Lot Sampling Plan-2 Indexed Through Trigonometric Ratios

This section contains a new procedure for designing Quality Interval Skip-lot Sampling Plan-2 indexed through Trigonometric Ratios based on Probabilistic Quality Region (PQR) and Indifference Quality Region (IQR). Tables are constructed for the selection of parameters for this plan. Certain numerical illustrations are also provided for selection.

3.1 Selection of the sampling plan

This section provides a new procedure for designing Quality Interval Skip-lot Sampling Plan-2 indexed through Trigonometric Ratios. Also Considering the ability of the declination angles of the tangent at the inflection point on the OC curve for discrimination of the Skip-lot Sampling Plan

Here, $\tan\theta_1 = \frac{0.95 - P_a(p_*)}{d_1}$ ………… (1)

Where $\tan\theta_1$ is the tangent angle for OC Curve with quality levels p_1, p_* and d_1, Quality Decision Region (QDR). From equation (1) one can find the parameters (f, i) for a specific $Pa(p_*)$ and d_1. It is noted that both θ_1 and d_1 are uniquely determines the parameters for Skip-lot Sampling Plan-2.

Similarly, $\tan\theta_2 = \frac{\text{Pa}(P_*) - 0.10}{d_3}$ ………..(2)

Where LQR (d_3) = d_2-d_1 and tan θ_2 is the tangent angle for OC Curve with quality levels p_* and p_2. The Probabilistic Quality Region was represented by d_2. From equation (2) one can find the parameters (f , i) for a particular $P_a(p_*)$ and d_3.
It is noted that both θ_2 and d_3 uniquely determine the parameters for Skip-lot Sampling Plan.

And, $$\tan \theta_3 = \frac{\text{Pa}(P_*)}{d_2} \quad \ldots\ldots\ldots\ldots (3)$$

From equation (3), one can find (f,i) for a particular $P_a(p_*)$ and d_2. It is noted that both θ_3 and d_2 are uniquely determines the parameters for Skip-lot Sampling Plan-2.Figure 3.2.1 it is noted that ΔABC represents the approximate area inscribed by the quality levels p_1 and p_*., ΔCDE represents the approximate area inscribed by the quality levels p_* and p_2. and the ΔBFG represents the approximate area inscribed by the quality levels p_1 and p_2. θ_1 is inscribed triangle for OC Curve with quality levels p_1 and p_*. Then θ_2 represent the inscribed triangle for OC Curve with quality levels p_* and p_2. Further θ_3 and θ_4 are the inscribed triangles for OC Curve with quality levels p_1 and p_2.

3.2 Designing plans for given QDR and PQR

From the table 1 one can find out the sampling plan for specified $P_a(p^*)$ and p*. Prefixing d_1, np_1, $P_a(p^*)$, tan θ_1,and p*, one can find out i and f using the relation $\tan \theta_1 = \frac{0.95\text{-Pa}(P_*)}{d_1}$.
Then one can easily read off corresponding i and f from the table.

Designing plans for given LQR and MAPD

From the table 1 one can find out i the sampling plan for specified $P_a(p^*)$ and p*. Prefixing d_3, np_2, $P_a(p^*)$, tan θ_2, and p* we can find out i, f from the relation tan $\theta_2 = \frac{\text{Pa}(P_*)\text{- }0.10}{d_3}$ Then one can easily read off corresponding i, f using the table

Designing plans for given PQR and MAPD

From the table 1, one can find out the sampling plan for specified $P_a(p^*)$ and p*.Prefixing d_2, np_1, Pa(p*), tan θ_3,and p* one can find out i,f from the relation , tan $\theta_3 = \frac{\text{Pa}(P_*)}{d_2}$ Then one can easily read off corresponding i,f from the table.

Construction of Tables

- ✓ Quality decision Range denoted as d_1 = (p_*- p_1) is derived from probability of acceptance

$$P_a(p_1 < p < p_*) = \frac{\text{f P}+(1-\text{f})[\text{P}]^i}{\text{f }+(1-\text{f}))[\text{P}]^i} \text{ for } p_1 < p < p_* \quad \ldots\ldots\ldots 4$$

- ✓ Probabilistic Quality Range denoted as d_2 =(p_2- p_1) is derived using the probability of acceptance

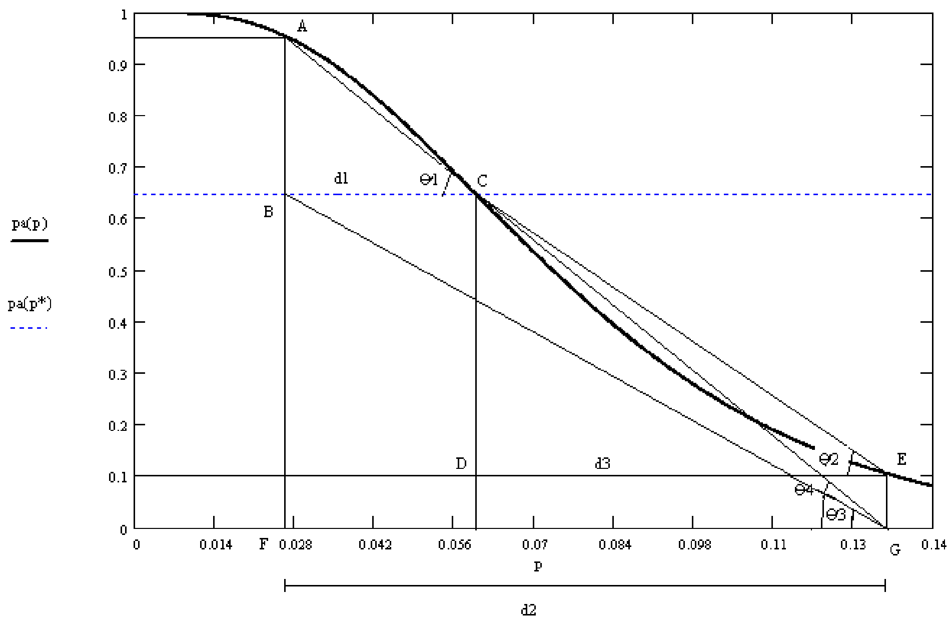
$P_a(p_1<p<p_2)= \frac{f P+(1-f)[P]^i}{f+(1-f))[P]^i}$ for $p_1 < p < p_2$ ……........5

- Limiting Quality Range denoted as is derived through probability of acceptance

$P_a(p_*<p<p_2)= \frac{f P+(1-f)[P]^i}{f+(1-f))[P]^i}$ for $p_* < p < p_2$ ……….6

- Indifference Quality Range denoted as $d_0 =(p_0- p_1)$ is derived through probability of acceptance
- $P_a(p_1<p<p_0)= \frac{f P+(1-f)[P]^i}{f+(1-f))[P]^i}$ for $p_1< p < p_0$ ……7
- From figure 3.2.1
- $tan\,\theta_1 = \frac{0.95-P_a(p_*)}{d_1}$ the declination angle
- $\theta_1 = tan^{-1}(\frac{0.95-P_a(p_*)}{d_1})$
- Similarly tan $\theta_2 = \frac{Pa(P_*)-0.10}{d_3}$ and the declination angle is
- $\theta_2 = tan^{-1}(\frac{Pa(P_*)-0.10}{d_3})$
- And tan $\theta_3 = \frac{Pa(P_*)}{d_2}$, the declination angle is
- $\theta_3 = tan^{-1}(\frac{Pa(P_*)}{d_2})$

Figure 3.2.1: Area of OC Curve for the skip lot sampling plan-2 with quality regions incurred with certain quality levels



- For different values of i then $P_a(p^*)$ is determined through equation 4. Substituting the appropriate values in equation 5,6 and 7 for fixed $L(P_*)$,d_1,d_2,d_3 and hence angle $\theta_1, \theta_2, \theta_3$ and i,f are obtained.

Table 1 shows the values of i and corresponding ranges d_1= nQDR and d_2=nPQR, Table 1 is given for the values of L(p*) for selected values of i and f.

- Table – 2 provides the area of triangle ABC, CDE and BFG for a fixed n for different values of f and i. Table – 3 provides the operating ratio R_1 R_2 R_3 for different values of f and i.

Example 1

- For given sample size n=100 and to attain an area of 0.0802. Find the acceptance to be taken for attain a better OC curve. Using table 6 we can easily read off, for area ABC=0.0802 the corresponding acceptance number f=1/3 and i= 6.

Example 2

- For given sample size n=100 and to attain an area of 0.2272. Find the acceptance to be taken for attain a better OC curve. Using table 6 we can easily read off, for area CDE=0.2272 the corresponding acceptance number f=1/2 and i=4.

Example 3

- For given sample size n=100 and to attain an area of 0.6401. Find the acceptance to be taken for attain a better OC curve. Using table 6 we can easily read off, for area BFG=0.6401 the corresponding acceptance number f=1/4 and i=4 .

Significance of Tangent Angle Plans

- From the table given below it is very clear that, for fixed MAPD (p_*) and an increasing value of Quality regions (that is for various QDR, PQR, LQR), the values of angles θ_1, θ_2 , θ_3 decreases.
- It can also be verified from the table that for a fixed MAPD (p_*) and varying quality regions a better and optimum sampling plan can be obtained
- This implies that tangent angles and Quality Regions have strong power of discrimination and the resultant sampling plan is unique. When angle θ_3 is minimized, the OC curve comes closer to the ideal OC curve.
- Therefore, designing of sampling plan through tangent angle gives more protection to both producer and consumer. These are the advantages of tangent angle sampling plan which can be seen towards designing compared to other available designing procedures.

- **Conclusion**
- Acceptance Sampling is a method that involves making decisions to accept or reject lots or processes based on the evaluation of samples. This paper primarily focuses on the development and selection of performance measures for Quality Interval Sampling (QIS) inspection plans categorized by Quality Regions. It also includes parameter conversions for ease of use. Additionally, tables have been created that are specifically designed for practical, ready-to-use application in industrial shop-floor environments.

The scope of the paper is comprehensive since special purpose plan are discussed with suitable illustrations are well covered. The design parameters with number of groups and the acceptance number are determined by satisfying both the producers and consumers risk at incoming and outgoing quality levels. Certain comparisons are also made for the performance of sampling procedures, these techniques are proved better than the existing ones, which are already attempted and notices through numerical illustrations and the operating characteristic curves for the easy understanding and application for the floor engineers.

Table 1: Parametric values for Skip-lot Sampling Plan-2

i	L(p*)	np_1	np_2	nd_1	nd_2
0	0.48	0.335	3.867	0.625	3.512
1	0.435	0.187	2.47	0.335	2.263
2	0.366	0.142	2.305	0.231	2.143
3	0.311	0.119	2.283	0.172	2.144
4	0.268	0.104	2.283	0.134	2.159
5	0.235	0.094	2.283	0.107	2.169
6	0.208	0.086	2.283	0.087	2.177
7	0.187	0.08	2.283	0.071	2.183
8	0.169	0.074	2.283	0.059	2.189
9	0.154	0.07	2.283	0.048	2.193
10	0.141	0.067	2.283	0.039	2.196
∞	-	0.031	2.283	-	2.232

Table 2: The area of Triangle ABC, CDE and BFG for a fixed n

i	L(p*)	area ABC	area of CDE	area BFG
0	0.48	0.1469	0.5485	0.8429
1	0.435	0.0863	0.3229	0.4922
2	0.366	0.0675	0.2543	0.3922
3	0.311	0.055	0.208	0.3334

4	0.268	0.0457	0.1701	0.2893
5	0.235	0.0383	0.1392	0.2549
6	0.208	0.0323	0.1129	0.2264
7	0.187	0.0271	0.0919	0.2041
8	0.169	0.023	0.0735	0.185
9	0.154	0.0191	0.0579	0.1689
10	0.141	0.0158	0.0442	0.1548
∞	-			-

Table 3: The ratio of area of Triangle ABC and CDE, ABC and BFG, CDE and BFG

i	area ABC	area CDE	area BFG	R_1	R_2	R_3
0	0.1469	0.5485	0.8429	3.7347	5.7388	1.5366
1	0.0863	0.3229	0.4922	3.7437	5.7059	1.5241
2	0.0675	0.2543	0.3922	3.7762	5.814	1.5422
3	0.055	0.208	0.3334	3.7858	6.0667	1.6025
4	0.0457	0.1701	0.2893	3.7226	6.3314	1.7008
5	0.0383	0.1392	0.2549	3.6386	6.6625	1.8311
6	0.0323	0.1129	0.2264	3.4966	7.0145	2.0061
7	0.0271	0.0919	0.2041	3.3918	7.5355	2.2217
8	0.023	0.0735	0.185	3.1895	8.0284	2.5171
9	0.0191	0.0579	0.1689	3.0316	8.839	2.9157
10	0.0158	0.0442	0.1548	2.8031	9.8138	3.5012

∞	0	0	-	-	-	-

Table 4: Parametric values for Skip-lot Sampling Plan-2

f	i	np_1	np_2	nd_1	nd_2	L(p*)
1	-	0.0493	2.3006	0.1663	2.2493	0.2011
	4	0.0693	2.301	0.1463	2.2298	0.202
	6	0.0669	2.3006	0.1149	2.2317	0.1799
2/3	8	0.0648	2.3006	0.0894	2.2338	0.1568
	10	0.0631	2.3006	0.0703	2.2355	0.1378
	12	0.0616	2.3006	0.0583	2.237	0.1226
	4	0.0869	2.3015	0.2499	2.2126	0.3315
	6	0.0817	2.3006	0.173	2.2169	0.2638
1/2	8	0.0775	2.3006	0.1278	2.2211	0.2181
	10	0.0742	2.3006	0.0982	2.2244	0.1857
	12	0.0714	2.3006	0.0772	2.2272	0.165
	4	0.1169	2.3024	0.3459	2.1834	0.4864
1/3	6	0.106	2.3006	0.2245	2.1927	0.364
	8	0.0978	2.3006	0.1611	2.2008	0.2917
	10	0.0915	2.3006	0.1218	2.2071	0.2437
	12	0.0864	2.3006	0.0952	2.2122	0.2093
	4	0.1422	2.3033	0.3979	2.1591	0.5929
1/4	6	0.1256	2.3006	0.2518	2.173	0.4333
	8	0.1138	2.3006	0.1784	2.1848	0.343
	10	0.105	2.3006	0.1341	2.1936	0.2843
	12	0.098	2.3006	0.1044	2.2006	0.2429

	4	0.164	2.3041	0.4328	2.1381	0.6767
1/5	6	0.1421	2.3006	0.2698	2.1565	0.4882
	8	0.1272	2.3006	0.1898	2.1714	0.3937
	10	0.1161	2.3006	0.1421	2.1825	0.3168
	12	0.1075	2.3006	0.1105	2.1911	0.2698

Table 5: The area of Triangle ABC, CDE and BFG for a fixed n

f	i	L(p*)	area ABC	area of CDE	area BFG
1	-	0.2011	0.0623	0.1053	0.2262
	4	0.202	0.0547	0.1063	0.2252
	6	0.1799	0.0442	0.0846	0.2007
2/3	8	0.1568	0.0355	0.0609	0.1751
	10	0.1378	0.0285	0.0409	0.154
	12	0.1226	0.0241	0.0246	0.1371
	4	0.3315	0.0773	0.2272	0.3667
	6	0.2638	0.0594	0.1674	0.2924
1/2	8	0.2181	0.0468	0.1236	0.2422
	10	0.1857	0.0375	0.0911	0.2065
	12	0.165	0.0303	0.0699	0.1837
	4	0.4864	0.0802	0.355	0.531
1/3	6	0.364	0.0658	0.2598	0.3991
	8	0.2917	0.053	0.1955	0.321
	10	0.2437	0.043	0.1498	0.2689

	12	0.2093	0.0353	0.1157	0.2315
	4	0.5929	0.071	0.434	0.6401
1/4	6	0.4333	0.0651	0.3202	0.4708
	8	0.343	0.0541	0.2438	0.3747
	10	0.2843	0.0446	0.1898	0.3118
	12	0.2429	0.0369	0.1498	0.2673
	4	0.6767	0.0591	0.4917	0.7234
1/5	6	0.4882	0.0623	0.3662	0.5264
	8	0.3937	0.0528	0.291	0.4274
	10	0.3168	0.045	0.2212	0.3457
	12	0.2698	0.0376	0.1766	0.2956

Table 6: The ratio of area of Triangle ABC and CDE, ABC and BFG, CDE and BFG

f	i	area ABC	area CDE	area BFG	R_1	R_2	R_3
1	-	0.062271	0.105296	0.226167	1.690925	3.632	2.1479
	4	0.054716	0.106259	0.22521	1.941993	4.116	2.1195
	6	0.044242	0.084566	0.200741	1.911435	4.5373	2.3738
2/3	8	0.035456	0.060901	0.17513	1.717647	4.9394	2.8757
	10	0.028549	0.040922	0.154026	1.433414	5.3952	3.7639
	12	0.024119	0.024619	0.137128	1.020756	5.6855	5.5699

	4	0.077282	0.227183	0.366738	2.939673	4.7455	1.6143
	6	0.059356	0.167395	0.292409	2.820179	4.9263	1.7468
1/2	8	0.046768	0.123609	0.242211	2.64301	5.1789	1.9595
	10	0.037527	0.091108	0.206536	2.427781	5.5036	2.2669
	12	0.030301	0.069875	0.183744	2.30603	6.064	2.6296
	4	0.08018	0.355005	0.531003	4.427621	6.6227	1.4958
1/3	6	0.065779	0.259802	0.399071	3.949655	6.0669	1.5361
	8	0.053026	0.195505	0.320987	3.686965	6.0534	1.6418
	10	0.043014	0.149829	0.268935	3.483283	6.2523	1.7949
	12	0.035257	0.115694	0.231507	3.281419	6.5662	2.001
	4	0.071045	0.434048	0.640065	6.109472	9.0093	1.4746
¼	6	0.065053	0.320168	0.47078	4.921684	7.2369	1.4704
	8	0.054144	0.243778	0.374693	4.50236	6.9203	1.537
	10	0.044635	0.189783	0.31182	4.251868	6.986	1.643
	12	0.036911	0.149773	0.267263	4.057734	7.2408	1.7844
	4	0.059142	0.491723	0.723426	8.314265	12.232	1.4712
1/5	6	0.062297	0.366208	0.526402	5.878446	8.4499	1.4374
	8	0.052793	0.290998	0.42744	5.512069	8.0965	1.4689
	10	0.044989	0.221179	0.345708	4.916314	7.6843	1.563
	12	0.037581	0.176643	0.295579	4.700319	7.8651	1.6733

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