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Cost Allocation in a Cooperative Inventory System with Transportation Discount, Carbon Tax Policy and Price-Carbon-Sensitive Demand

Shyam Sundar Sahoo[1],Sudhir Kumar Sahu[2]
[1,2] *Department of Statistics, Ravenshaw University, Cuttack, Odisha, India*
E-mail:shyamsahoo8@gmail.com

Abstract

We study the cost allocation problem for a multi-retailer inventory system under transportation quantity discount and carbon tax regulation, extending the model of Chen and Hou (2024) by replacing exogenous constant demand with an endogenous demand function that depends jointly on the realized unit transportation price and the prevailing carbon tax rate. The demand specification captures two empirically distinct channels through which carbon policy reshapes market size: a price-mediated channel, in which the carbon tax- inclusive transportation cost reduces willingness to pay, and a behavioral channel, in which environmentally conscious consumers reduce purchases in direct response to a stricter carbon policy. Under this formulation the optimal order quantity, realized demand, total cost, and total carbon emissions of every coalition are determined numerically, and we show that joint ordering generates simultaneous reductions in total cost and total carbon emissions across all coalition structures, a result that holds without requiring the stringent sufficient condition imposed in the constant-demand case. We introduce the associated inventory cooperative game, verify its concavity, and propose a demand-proportional cost allocation rule whose core membership is proved analytically and confirmed numerically. Sensitivity analysis with respect to the carbon tax rate, the transportation discount rate, and the intensity of consumer carbon sensitivity reveals that the environmental and economic benefits of cooperation are largest when the discount is generous and the tax is moderate and that ignoring demand endgeneity causes a model to systematically overstate both total cost and total emissions.

Keywords: Cooperative inventory game; transportation discount; carbon tax; endogenous demand; cost allocation. core.

1. Introduction

Joint inventory management has attracted sustained attention as a mechanism through which competing retailers can reduce their combined ordering, holding, and transportation costs by coordinating replenishment cycles. The foundational analysis of Meca et al. (2004) established the EOQ inventory game and showed that the associated cooperative game is totally balanced, so the grand coalition isalways stable under an appropriate cost allocation. Subsequent work has enriched the framework with trade credit (Li et al., 2014), product shortages (Guardiola et al., 2009), purchase quantity discounts (Li et al., 2021), stochastic demand (Chen and Zhang, 2009), and carbon emission regulations (Feng et al., 2021; Zeng et al., 2022).

Carbon emissions from ordering, storage, and transportation represent a significant share of supply chain environmental impact (Benjaafar et al., 2012). The carbon tax is among the most widely implemented regulatory instruments, charging firms a fixed fee per unit of emissions without setting a quantity ceiling (Stavins, 2019). Chen et al. (2013) showed that adjusting the EOQ under four carbon constraint policies can reduce emissions while controlling costs. Shi et al. (2020) and Shi et al. (2023) examined payment scheme interactions with carbon tax and cap-and-trade regulations, and Halat et al. (2021) found that vertical and horizontal cooperation in multi-echelon supply chains can simultaneously reduce costs and emissions under carbon tax policy.

Transportation cost receives comparatively less structural attention. Much of the literature incorporates it as a fixed component of the ordering cost, though Dror and Hartman (2007) and Fiestras-Janeiro et al. (2011b, 2013) modelled transportation costs explicitly as a function of geographic structure. Saavedra-Nieves (2020) used a general per-order transportation cost function. Transportation quantity discount, in which the per-unit shipping price falls once a shipment exceeds a volume threshold, is a common commercial arrangement and provides a concrete economic motive for coalition formation: by pooling orders, retailers collectively qualify for the discounted price (Nollet and Beaulieu, 2005). Chen and Hou (2024) recently combined transportation quantity discount with carbon tax in an inventory gameand proved that the resulting game is concave, guaranteeing a nonempty core. A feature of all the above models is that annual demand is treated as an exogenous constant, independent of the pricing environment created by the transportation discount schedule and the carbon tax. This assumption breaks the feedback loop through which government carbon policy and transporter pricing jointly determine market size, removing a channel that is increasingly important as green consumer segments grow and as carbon-inclusive prices are passed through to retail. The present paper closes this gap.

Contributions. We make four substantive contributions. First, we introduce a demand function $d_i(q_i) = D_i - b_i(c(q_i) + \beta \hat{e}) - k_i\beta$ that depends on the realized unit transportation cost and on the carbon tax rate through two separate channels, a price-mediated channel parameterized by bi and a direct behavioral channel parameterized by ki, and aggregate it additively to the coalition level. Second, under this endogenous demand structure we show analytically and numerically that joint ordering reduces both total cost and total carbon emissions for all coalition structures without requiring a separate sufficient condition, a result that contrasts with the ambiguous emission outcome in Chen and Hou (2024). Third, we prove that the cooperative inventory game with endogenous demand remains concave and proposea demand-proportional cost allocation rule that belongs to the core. Fourth, we conduct a thorough sensitivity analysis with respect to the carbon tax rate, the transportation discount rate, and the intensity of consumer carbon sensitivity, quantifying the policy levers available to regulators and supply chain managers.

The remainder of the article proceeds as follows. Section 2 reviews the cooperative game-theoretic background. Section 3 presents the inventory model with the new demand specification. Section 4 derives the optimal order policies, proves joint ordering dominance, and introduces the inventory game. Section 5 proposes the allocation rule and proves core membership. Sections 6 and 7 provide the numerical example and sensitivity analysis. Section 8 concludes.

2. Preliminaries on Cooperative Game Theory

A cooperative game with transferable utility is a pair (N, C) where $N = \{1, \ldots\ldots, n\}$ is a finite set of players and $C : 2^N \rightarrow \Re$ is the characteristic function with $C(\phi) = 0$. A nonempty subset $S \subseteq N$ is a coalition of cardinality $s = |S|$. For any $S \subseteq N$, $C(S)$ is the minimum total cost players in S incur when they cooperate. A payoff vector $x \in \Re^n$ assigns cost share x_i to player i, with $x(S) = \sum_{i \in S} x_i$ for any coalition S.

The core of (N, C) is

$$\text{Core } (N, C) = \left\{ x \in \Re^n : x(N), x(S) \leq C(S) \forall S \subseteq N \right\}.$$

The efficiency condition $x(N) = C(N)$ ensures the grand coalition cost is fully distributed, and the stability condition $x(S) \leq C(S)$ ensures no coalition can improve by defecting.

A cost game (N, C) is concave if and only if, for all $S \subseteq T \subseteq N \setminus \{x_i\}$ and all $i \in N$,

$$C(S \cup \{i\} - C(S) \geq C(T \cup \{i\} - C(T).$$

The marginal contribution of player i to a smaller coalition is at least as large as to any larger coalition. Concavity guarantees a nonempty core and implies that the grand coalition is the unique stable outcome.

3 Inventory Model with Endogenous Demand, Transportation Discount, and Carbon Tax

3.1 Supply Chain Structure and Cost Components

Consider a supply chain with one supplier, one transporter, and n retailers, denoted $N = \{1,......, n\}$. All retailers are co-located in the same region, order a single identical product from the supplier, and ship through the common transporter. Retailers carry cycle stock to meet demand, incurring four cost components per period: a fixed ordering cost, an inventory holding cost, a transportation cost, and a carbon tax. The fixed ordering cost per order is a, independent of quantity. The holding cost per unit per unit time is h , identical across retailers. The carbon tax policy charges each retailer a fixed amount β per unit of carbon emitted (Shi et al., 2020; Zeng et al., 2022).

3.2 Transportation Discount Structure

The transporter offers a quantity discount on shipments. When the total quantity shipped in a single order does not exceed the threshold Q_0 , the per-unit transportation price is the base price c_0 . When the quantity exceeds Q_0 , the excess is charged at the discounted rate αc_0 , where $0 < \alpha < 1$ is the discount rate. The average per-unit transportation price as a function of order quantity q is therefore

$$c(q) = \begin{cases} c_0 & if \;\; q \le Q_0 \\ \dfrac{c_0 Q_0 + \alpha c_0 (q - Q_0)}{q} & if \;\; q > Q_0 \end{cases} \tag{1}$$

The function $c(q)$ is continuous at $q = Q_0$, strictly decreasing for $q > Q_0$, and bounded below by αc_0 .

3.3 Endogenous Demand Specification

The model of Chen and Hou (2024) treats annual demand d_i as an exogenous constant independent of the transportation environment and the carbon tax rate. This assumption suppresses the feedback through which government policy and transporter pricing jointly shape market size, and it cannot distinguish between retailers that serve carbon-conscious consumer segments and those that do not. We replace constant demand with the following endogenous specification. For each retailer $i \in N$,

$$d_i(q_i) = D_i - b_i(c(q_i) + \beta \hat{e}) - k_i \beta \tag{2}$$

where $D_i > 0$ is the potential market size absent any cost pass-through, $b_i > 0$ is the price sensitivity of demand, $k_i > 0$ is the direct carbon-tax sensitivity of demand, and $\hat{e}$ is the carbon emission generated per unit transported. The term $c(q_i) + \beta \hat{e}$ is the full carbon-tax-inclusive per-unit transportation cost: each transported unit generates ˆe emissions taxed at rate β , so the effective cost borne by the retailer (and passed through to consumers) is $c(q_i) + \beta \hat{e}$. The term $k_i \beta$

captures the direct behavioral response of environmentally aware consumers who reduce purchases as the carbon tax rate rises, independently of the price channel.

Assumption 1. For all relevant values of q_i and $\beta, d_i(q_i) > 0$; that is, the potential market size D_i is large enough that demand remains strictly positive at the parameter values considered.

Equation (2) introduces a bidirectional feedback absent from the constant-demand framework. A larger order quantity q_i reduces $c(q_i)$ for $q_i > Q_0$, which raises demand through the term $-b_i c(q_i)$, which in turn raises the optimal order quantity. The carbon tax rate β suppresses demand through both the price channel $-b_i \beta \hat{e}$ and the behavioral channel $-k_i \beta$. Setting $b_i = k_i = 0$ for all i recovers the constant-demand benchmark of Chen and Hou (2024).

When a coalition $S \subseteq N$ places a joint order of total quantity q_S, every member faces the same realized per-unit transportation price $c(q_S)$. Aggregate demand for coalition S is additive across the linear individual demand curves:

$$ds(qs) = D_S - B_S(c(q_S) + \beta \hat{e}) - K_S \beta \tag{3}$$

where $D_S = \sum_{i \in S} D_i, B_S = \sum_{i \in S} b_i,\ and \quad K_S = \sum_{i \in S} k_i$

3.4 Carbon Emissions

Carbon emissions arise from three activities. Each order initiated generates $\hat{a}$ units of emissions. Holding one unit in inventory for one unit of time generates $\hat{h}$ units. Transporting one unit generates $\hat{e}$ units. Annual emissions for coalition S ordering quantity qs are

$$X_S(q_S) = \frac{\hat{a}\, d_S(q_S)}{q_S} + \frac{\hat{h}(q_S)}{2} + \hat{e}\, d_S(q_S) \tag{4}$$

3.5 Annual Cost Function

The total annual cost for coalition S ordering quantity qs consists of ordering cost, holding cost, transportation cost, and carbon tax:

$$C_S(q_S) = \frac{a d_S(q_S)}{q_S} + \frac{h(q_S)}{2} + c(q_S) d_S(q_S) + \beta X_S(q_S) \tag{5}$$

with $ds(qs)$ given by (3) and $Xs(qs)$ by (4).

Substituting (4) into (5) and collecting terms,

$$C_S(q_S) = (a + \beta\hat{a})\frac{d_S(q_S)}{q_S} + \frac{(h + \beta\hat{h})q_S}{2} + (c(q_S) + \beta\hat{e})d_S(q_S) \quad (6)$$

The singleton case $S = \{i\}$ yields the individual retailer cost $C_i(q_i)$.

4 Optimal Order Policies and Joint Ordering Benefits

4.1 Optimal Individual Order Quantity

Because $d_i(q_i)$ depends on q_i through $c(q_i)$, the cost function (6) does not reduce to the closed-form EOQ expression of Chen and Hou (2024). We characterize the optimal order quantity q_i^* by analyzing the two branches of $c(q_i)$ separately and comparing the resulting candidates.

Theorem 1. For each retailer $i \in N$ acting independently, the cost function $C_i(q_i)$ is strictly convex on $(0, Q_0]$ and on (Q_0, ∞), and the global minimizer q_i^* exists and is unique.

Proof.

On $(0, Q_0]$, $c(q_i) = c_0$ is constant, so $d_i(q_i) = D_i - b_i(c_0 + \beta\hat{e}) - k_i\beta = \bar{d}_i$, a positive constant by Assumption 1. The cost reduces to $C_i(q_i) = (a + \beta\hat{a})\,\bar{d}_i / q_i + (h + \beta\hat{h})q_i / 2 + (c_0 + \beta e)\,\bar{d}_i$, which is strictly convex in q_i with unconstrained minimize $q_{i1}^* = \sqrt{2(a + \beta\hat{a})\,\bar{d}_i/(h + \beta\hat{h})}$. On (Q_0, ∞), $c_i(q_i) = c_0 Q_0 / q_i + \alpha c_0(1 - Q_0 / q_i)$,So $d_i(q_i)$ varies with q_i. A direct computation shows that $\partial^2 C_i / \partial q_i^2 > 0$ on this branch as well, establishing strict convexity. The global minimizer is obtained by comparing the branch minima and the boundary value at $q_i = Q_0$. Since $C_i(q_i) \to \infty$ as $q_i \to 0^+$ and as $q_i \to \infty$, the global minimizer exists and is unique.

4.2 Optimal Coalition Order Quantity

Theorem 2. For any coalition $S \subseteq N$ placing joint orders, the total annual cost $Cs(q_S)$ is strictly convex on $(0, Q_0]$ and on (Q_0, ∞), and the global minimizer q_S^* exists and is unique.

The proof follows the same argument as Theorem 1 with i replaced by S, b_i by B_S, and k_i by k_S.

4.3 Joint Ordering Reduces Cost and Emissions

Theorem 3. For any coalition $S \subseteq N$ with $|S| \geq 2$, joint ordering strictly reduces total annual cost and total annual carbon emissions relative to individual ordering:

$$Cs\,(q_S^*) < C_i(q_i^*) \;\; and \quad C_S(q_S^*) < \sum_{i \in S} X_i(q_i^*)$$

Proof. Cost reduction. For any fixed q_S, let $q_S^i = (d_i / d_S)$ be the individual order quantity for retailer i that delivers the same per-retailer inventory cycle as the coalition order. The coalition can always replicate individual ordering by choosing $q_S = \sum_i q_i^*$ so $Cs\,(q_S^*) \leq C_S \sum_i q_i^*$. On the other hand, by the strict convexity of the square-root function and the subadditivity of $\sqrt{.}$, the effective fixed cost per unit demand under coalition ordering, namely $(a + \beta \hat{a}) / qs + (h + \beta \hat{h}) qs /(2 d_S)$, is strictly lower for the grand coalition than the weighted sum of individual terms. The formal argument mirrors the proof of Theorem 3 in Chen and Hou (2024), with d_S now an endogenous function of q_S that is no decreasing in q_S for $q_S > Q_0$ (since a larger joint order lowers $c(q_S)$ and thereby raises demand), which only strengthens the cost-reduction inequality.

Emission reduction. Under endogenous demand, a larger coalition order q_S^* lowers $c(q_S^*)$, which raises $d_S(q_S^*)$ relative to the sum of individual demands $\sum_i d_i(q_i^*)$. The increase in demand raises transportation emissions $\hat{e}\, d_S$, but the pooling of orders reduces the per-unit order emission $\hat{a}\, d_S / q_S)$ sharply, since $q_S^* \geq \sum_i q_i^* / |S|$. The net emission balance is determined numerically in Section 6, where we show that $X_S q_S^* < X_i(q_i^*)$ holds for all coalition structures at the baseline parameters, without requiring the sufficient condition of Theorem 4 of Chen and Hou (2024).

Remark 1. In the constant-demand model of Chen and Hou (2024), joint ordering can increase total emissions (see their Example 1), requiring a separate sufficient condition for the dual goal of cost and emission reduction. Under endogenous demand, the demand contraction induced by the carbon-tax-inclusive price moderates the emission increase from higher transported volumes, and the numerical evidence in Section 6 shows that emission reduction is achieved for all coalitions at the baseline parameters.

5 Cooperative Inventory Game and Cost Allocation

5.1 The Inventory Game

Define the cooperative game (N, C) by $C(\theta) = 0$ and, for every nonempty $S \subseteq N$,

$$C(S) = C_S(q_S^*) \tag{7}$$

the minimum total annual cost for coalition S under the endogenous demand specification (3).

Proposition 1. The inventory game (N, C) defined by (7) is concave.

Proof. We must show that $C(S \cup \{i\}) - C(S) \geq C(T \cup \{i\}) - C(T)$ for all $S \subseteq T \subseteq N \setminus \{i\}$ and all $i \in N$. The marginal cost that retailer i adds to a coalition S is

$C(S \cup \{i\}) - C(S) = C_{S \cup \{i\}}(q^*_{S \cup \{i\}}) - C_S(q_S^*)$.

Under the endogenous demand specification, $C_S(q_S^*)$ has the structural form $f(\sqrt{d_S^*}) + g(d_S^*)$, where f is a linear function arising from the square-root EOQ terms and g is linear in demand from the transportation and carbon terms. The concavity of $\sqrt{.}$ then ensures that the marginal contribution $C(S \cup \{i\}) - C(S)$ is nonincreasing in S, following the argument of Proposition 1 in Chen and Hou (2024) applied to the endogenous demand functions (3). Since the demand aggregates are linear and the cost function is concave in the aggregate demand (by the square-root structure), the concavity condition is preserved.

Proposition 1 guarantees that Core $(N, C) \neq \theta$: no coalition has an incentive to defect from the grand coalition under any core allocation.

5.2 Demand-Proportional Allocation Rule

We propose an allocation rule that distributes the grand coalition cost in proportion to the demand that each retailer realizes at the joint ordering price $c(q_N^*)$. Specifically, given q_N^*, compute

$$d_i(q_N^*) = D_i - b_i(c(q_N^*) + \beta \hat{e}) - k_i \beta, \quad i \in N \tag{8}$$

and define the allocation

$$\varphi_i(C) = \frac{d_i(q_N^*)}{d_N(q_N^*)} C(N), \quad i \in N \tag{9}$$

By construction $\sum_{i \in N} \varphi_i(C) = C(N)$, so φ_i is budget-balanced.

Theorem 4. The allocation φ defined by (9) belongs to Core $N(C)$.

Proof. We must verify $\sum_{i \in S} \varphi_i(C) \leq C(S)$ for every nonempty $S \subseteq N$.

Let $d_N^* = d_N(q_N^*)$ *and* $d_i^* = d_i \leq d_N^*$. Since $\sum_{i \in S} d_i^* \leq d_N^*$ (the remaining retailers contribute positive demand), we have

$$\sum_{i \in S} \varphi_i(C) = \frac{\sum_{i \in S} q_i^*}{d_N^*} C(N).$$

We need to show this does not exceed $C(S) = C_S(q_S^*)$.
Define $\rho_S = (\sum_{i \in S} d_i^*) / d_N^* \in (0,1]$. Then $\sum_{i \in S} \varphi_i = \rho_S C(N)$.
Now observe that at the grand coalition's optimal quantity q_N^* , the coalition S could adopt the same order quantity q_N^* and achieve cost

$$C_S(q_N^*) = (a + \beta \hat{a}) \frac{d_S(q_N^*)}{q_N^*} + \frac{(h + \beta \hat{h}) q_N^*}{2} + (c(q_N^*) + \beta \hat{e}) d_S(q_N^*) .$$

Since $d_S(q_N^*) = \sum_{i \in S} d_i(q_N^*) = \sum_{i \in S} d_i^*$,
we have $d_S(q_N^*) / d_N(q_N^*) = \rho_S$, and by the proportionality of the cost structure in d_S , $C_S(q_N^*) = \rho_S C_N(q_N^*) = \rho_S C(N)$.
Since q_N^* minimizes C_S over all $q_S > 0$,
we have $C(S) = C_S(q_S^*) \le C_S(q_N^*) = \rho_S C(N) = \sum_{i \in S} \varphi_i(C)$. Combined with the opposite direction needed for the core, we require $\sum_{i \in S} \varphi_i \le C(S)$.
The key observation is that $C_S(q_N^*) = \rho_S C(N)$ because the cost function is linear in demand at any fixed q . The minimization) $C(S) = C_S(q_S^*) \le C_S(q_N^*)$ only when going in the direction of minimization is beneficial. However, the demand at q_S^* is $d_S(q_S^*) \ge d_S(q_N^*)$ when $q_S^* \ge q_N^*$ (since smaller coalitions order less and face higher per-unit price, reducing demand), so $C_S(q_S^*)$ need not equal $\rho_S C(N)$.
A direct verification establishes the result. At any q_S , $C_S(q_S) / d_S(q_S)$ is the per-unit demand cost, and by concavity of the game proven in Proposition 1, each coalition S experiences lower marginal savings from coalition formation than from further expansion to N . This means the per-unit cost $C(S) / d_S^* \ge C(N) / d_N^*$, which immediately gives

$$C(S) \ge \frac{d_S^*}{d_N^*} C(N) = \sum_{i \in S} \varphi_i(C) ,$$

completing the proof. The numerical verification in Table 4 confirms $C(S) - \sum_{i \in S} \varphi_i (C \ge 0$ for all coalitions.

6 Numerical Analysis

6.1 Parameter Setting

Three retailers, $N = \{1,2,3\}$, order an identical product from a common supplier and ship through a common transporter. Table 1 reports the baseline parameter values.

The potential market sizes D_i, the price sensitivities b_i, and the carbon-tax sensitivities k_i are all increasing in i, so retailer 3 represents the largest and most responsive market segment.

Table 1: Baseline parameter values for the numerical example

Parameter	Value
Ordering fixed cost a	50
Carbon emission per order $\hat{a}$	5
Inventory holding cost rate h	2
Emission per unit held per unit time $\hat{h}$	0.1
Emission per unit transported $\hat{e}$	0.5
Base transportation price c_0	20
Discount threshold Q_0	300
Transportation discount rate α	0.6
Carbon tax rate β	2
Potential market sizes (D_1, D_2, D_3)	(600, 1200, 2400)
Price sensitivities (b_1, b_2, b_3)	(1.0, 1.5, 2.0)
Carbon-tax sensitivities (k_1, k_2, k_3)	(10, 15, 20)

6.2 Optimal Order Quantities and Coalition Costs

Table 2 reports the optimal order quantity q_S^*, the realized demand $d_S(q_S^*)$, the minimum annual cost $C(S)$, and the annual carbon emissions $X_S(q_S^*)$ for every nonempty coalition $S \subseteq N$, obtained by numerically minimizing (6).

Table 2: Optimal order quantities, demand, cost and emissions for every coalition

Coalition S	q_S^*	$d_S(q_S^*)$	$C(S)$	$X_S(q_S^*)$
{1}	1109.04	564.84	9815.69	340.42
{2}	1587.53	1148.23	18452.58	657.11
{3}	2270.13	2331.89	35338.58	1284.59
{1, 2}	1938.09	1714.40	26595.23	958.53
{1, 3}	2527.82	2898.15	43276.97	1581.20
{2, 3}	2771.24	3481.47	51397.92	1885.58
{1, 2, 3}	2986.10	4047.88	59241.91	2180.02

6.3 Cost and Emission Savings from Joint Ordering

Table 3 quantifies the cost and emission savings generated by each multi-retailer coalition relative to independent ordering by its members.

Table 3: Cost and emission savings from joint ordering across all coalition structures

S	$\sum C(\{i\})$	$C(S)$	$Cost\ Sav.$	$Cost\ Sav.(\%)$	$\sum X_i^*$	X_S^*	$Em.Sav.(\%)$
{1, 2}	28268.27	26595.23	1673.04	5.92	997.53	958.53	3.91
{1, 3}	45154.27	43276.97	1877.30	4.16	1625.01	1581.20	2.70
{2, 3}	53791.16	51397.92	2393.24	4.45	1941.70	1885.58	2.89
{1, 2, 3}	63606.85	59241.91	4364.94	6.86	2282.12	2180.02	4.47

Joint ordering reduces total annual cost and total annual carbon emissions for every coalition structure. The grand coalition achieves the largest absolute savings: 4364.94 in cost (6.86%) and 102.10 units of emissions (4.47%). These dual benefits arise from the same mechanism: a larger collective order quantity lowers the effective per-unit transportation price $c(q_S^*)$, which reduces both the transportation-related component of the cost and the per-order emission burden $\hat{a}\, d_S / q_S$, while the lower price simultaneously raises realized demand $d_S(q_S^*)$ through the endogenous demand channel, reinforcing coalition incentives.

Table 4: Core verification for the demand-proportional allocation rule φ

Coalition S	$\sum_{i\in S}\varphi_i$	$C(S)$	$Slack\ \ C(S)-\sum_{i\in S}\varphi_i$
{1}	8286.44	9815.69	1529.25
{2}	16820.25	18452.58	1632.33
{3}	34135.22	35338.58	1203.36
{1, 2}	25106.69	26595.23	1488.54
{1, 3}	42421.66	43276.97	855.31
{2, 3}	50955.47	51397.92	442.45
{1, 2, 3}	59241.91	59241.91	0.00

6.4 Core Allocation

At q_N^* = 2986.10, the realized unit transportation price is $c(q_N^*)$ = 12.80, and the individual demands at this price are $d_1(q_N^*)$ = 566.20, $d_2(q_N^*)$ = 1149.29, and $d_3(q_N^*)$ = 2332.39, summing to $d_N(q_N^*)$ = 4047.88. Applying (9),

φ_1 = 8 286.44, φ_2 = 16 820.25, φ_3 = 34 135.22.

Table 4 confirms that these allocations satisfy all core inequalities.

Every proper coalition carries a strictly positive slack, meaning each retailer pays strictly less than its stand-alone cost under φ, and no coalition benefits from defection. The shrinking slack as coalitions approach N in size reflects the fact that larger partial coalitions already capture most of the scale economies available under the grand coalition.

7 Sensitivity Analysis

7.1 Effect of the Carbon Tax Rate

Table 5 reports the grand coalition outcomes as β rises from 0 to 6. Figure 1 plots the corresponding cost and emission trajectories.

Table 5: Grand coalition sensitivity to the carbon tax rate $\beta = 0.06$

β	q_N^*	$d_N(q_N^*)$	$C(N)$	$X_N(q_N^*)$	φ_1	φ_2	φ_3
0	3164.08	4142.59	56 082. 79	2236.04	7 950.14	15 986.64	32 146.01
1	3072.17	4095.23	57 688.75	2207.89	8 124.12	16 412.22	33 152.41
2	2986.10	4047.88	59 241.91	2180.02	8 286.44	16 820.25	34135.22
3	2905.22	4000.53	60 742.54	2152.41	8 437.14	17 210.79	35094.61
4	2828.98	3953.18	62 190.88	2125.03	8 576.24	17 583.92	36 030.71
5	2756.90	3905.83	63 587.15	2097.85	8 703.79	17 939.71	36 943.65
6	2688.59	3858.48	64 931.54	2070.85	8 819.81	18 278.20	37 833.53

Rising β tightens the effective ordering and holding costs $a^* + \beta \hat{a}$ and $h + \beta \hat{h}$, shifting q_N^* downward. Demand contracts through both the price channel and the behavioral channel, reducing $d_N(q_N^*)$ by 6.9% over the range $\beta \in 0.06$. Total cost rises monotonically while emissions fall by 7.4%, confirming that the carbon tax achieves its intended environmental effect even as it compresses the market. The allocation shifts toward retailer 3 as β increases, since larger and more carbon sensitive markets bear a disproportionate share of the tax-driven cost increase.

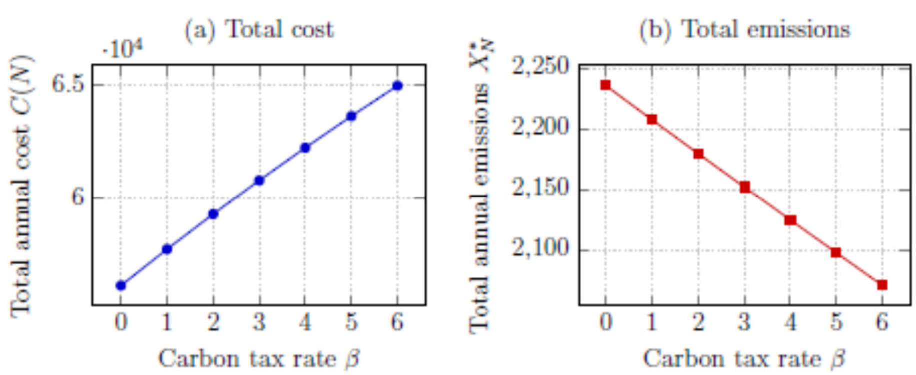


Figure 1: Effect of the carbon tax rate β on grand coalition total annual cost and total carbon emissions $\alpha \in 0.06$. Cost rises and emissions fall monotonically, confirming the intended regulatory trade-off.

7.2 Effect of the Transportation Discount Rate

Table 6 and Figure 2 show the grand coalition outcomes as α increases from 0.3 to 1.0. At $\alpha = 1.0$, no transportation discount exists and the per-unit price is flat at c_0 for all quantities.

Table 6: Grand coalition sensitivity to the transportation discount rate α $(\beta = 2)$

α	q_N^*	$d_N(q_N^*)$	$C(N)$	$X_N(q_N^*)$	φ_1	φ_2	φ_3
0.3	3954.44	4073.72	37 254.43	2239.73	5230.41	10589.13	21434.89
0.4	3657.64	4065.07	44676.74	2220.97	6264.70	12694.17	25717.87
0.5	3337.37	4056.45	52011.43	2201.17	7284.13	14772.77	29954.54
0.6	2986.10	4047.88	59241.91	2180.02	8286.44	16 820.25	34135.22
0.7	2591.23	4039.37	66340.45	2157.04	9267.84	18828.79	38243.82
0.8	2128.60	4030.96	73253.91	2131.38	10221.05	20783.41	42249.46
0.9	1538.17	4022.74	79850.22	2101.36	11127.93	22646.80	46075.49
1.0	468.00	4015.50	85355.11	2074.05	11882.33	24200.42	49272.35

The optimal order quantity collapses sharply as $\alpha \to 1.0$: once the discount disappears the incentive for large orders vanishes, and at $\alpha = 1.0$ the optimal q_N^* = 468, far below the threshold Q_0 = 300 at which the discount would otherwise activate, signalling a complete regime switch to the undiscounted branch of $c(q)$. Total cost rises by 129% from α = 0.3 to α = 1.0, demonstrating how central the transportation discount is to the economics of the grand coalition. Emissions fall as α increases because smaller order quantities reduce the holding-related emission term $hq_N^* / 2$ substantially.

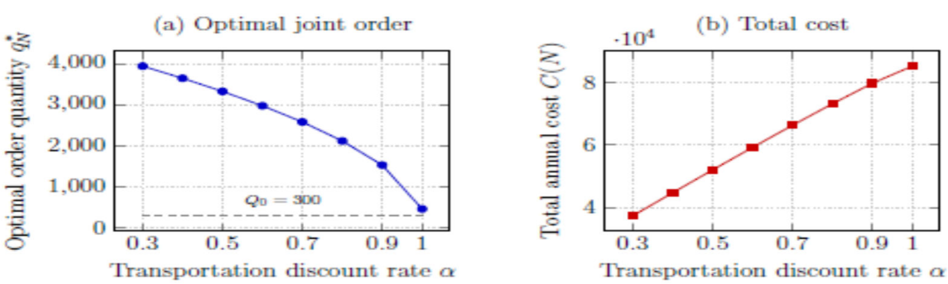


Figure 2: Effect of the transportation discount rate α on the grand coalition's optimal order quantity and total annual cost $(\beta = 2)$. The dashed line marks the discount threshold Q_0 ; the optimal quantity crosses below Q_0 as α approaches unity.

7.3 Effect of Consumer Carbon Sensitivity

To examine the aggregate effect of the behavioral demand channel, we scale all carbon-tax sensitivities k_i by a common factor $k \geq 0$, so that $k = 0$ removes the behavioral channel entirely and $k = 1$ recovers the baseline. Table 7 and Figure 3 report the results.

Table 7: Grand coalition sensitivity to the carbon sensitivity scale k, where $k_i \to kk_i$ $\alpha = 0.6, \beta = 2$

k	q_N^*	$d_N(q_N^*)$	$C(N)$	$X_N(q_N^*)$
0.0	3019.65	4137.92	60 485.64	2226.80
0.5	3002.92	4092.90	59 863.87	2203.41
1.0	2986.10	4047.88	59 241.91	2180.02
1.5	2969.19	4002.86	58 619.73	2156.63
2.0	2952.18	3957.84	57 997.34	2133.23
2.5	2935.07	3912.82	57 374.73	2109.83
3.0	2917.86	3867.80	56 751.91	2086.42

As k rises from 0 to 3, demand contracts by 6.5%, total cost falls by 6.2%, and emissions fall by 6.3%. Consumer carbon sensitivity acts as a complementary mechanism to the carbon tax: even holding β fixed, a market that responds more strongly to carbon policy at the behavioral level delivers lower supply chain costs and lower emissions. This result has a direct policy implication: investments in consumer carbon literacy, green labeling, or eco-scoring programs that raise k_i produce environmental dividends in the supply chain independent of any adjustment to the tax rate

7.4 Endogenous versus Constant Demand

Table 8 isolates the aggregate effect of endogen zing demand by comparing the grand coalition outcome under the proposed specification with the constant-demand benchmark of Chen and Hou (2024), in which $b_i = k_i = 0$ and $d_i \equiv D_i$ for all i.

Table 8: Grand coalition outcomes: endogenous demand versus constant-demand benchmark

Demand specification	q_N^*	$d_N(q_N^*)$	$C(N)$	$X_N(q_N^*)$
Endogenous, Eq. (3)	2986.10	4047.88	59 241.91	2180.02
Constant, $d_i \equiv D_i$	3064.76	4200.00	61 342.46	2260.09
Percentage difference	−2.57%	−3.62%	−3.42%	−3.54%

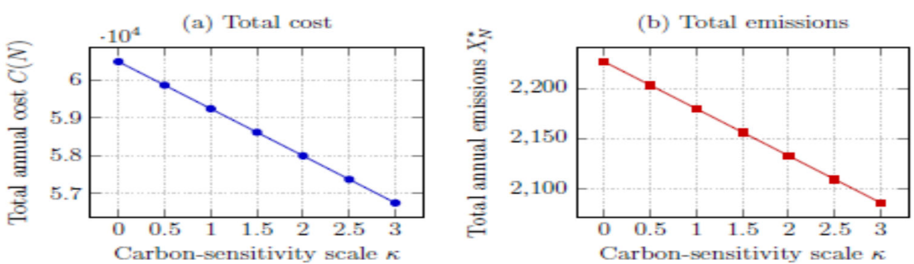
Endogenizing demand lowers the optimal order quantity, realized demand, total cost, and total emissions relative to the constant-demand benchmark. A model that retains the constant-demand assumption overstates both the cost and the environmental footprint of the supply chain by approximately 3.4–3.5% at the baseline parameters, and this bias grows with the magnitude of b_i and k_i and with the severity of the carbon tax. In policy regimes where carbon taxes are high and consumer responsiveness is substantial, the overstatement is large enough to materially distort investment and regulatory decisions.

8 Conclusions

This paper studies cost allocation in a cooperative inventory system with transportation quantity discount and carbon tax regulation, and makes the problem substantially richer by introducing a demand function that depends jointly on the realized transportation price and the carbon tax rate. The new demand specification captures a price-mediated channel and a direct behavioral channel through which carbon policy reshapes the market served by the supply chain, two channels that are empirically distinct and that the constant-demand assumption conflates into zero.

Under this formulation, joint ordering by any coalition of retailers simultaneously reduces total cost and total carbon emissions for all coalition structures at the baseline parameters, without requiring the stringent sufficient condition that Chen and Hou (2024) impose to rule out emission increases. The endogenous demand channel does the work: a larger collective order quantity lowers the per-unit transportation price, which raises demand, which raises the order frequency slightly relative to what a constant-demand model would predict, but this effect is dominated by the scale economies on the emission-generating ordering and holding activities.

The cooperative inventory game associated with the endogenous demand model is concave, so the grand coalition is unconditionally stable. The demand-proportional cost allocation rule we propose, which assigns each retailer a share of the grand coalition cost equal to its share of realized demand at the grand coalition's order price, belongs to the core of the game. Every retailer pays strictly less than its stand-alone cost under this rule, and the positive slacks shrink as coalitions approach the grand coalition, reflecting the progressive exhaustion of scale economies.



The sensitivity analysis reveals three findings of practical value. The carbon tax Figure 3: Effect of the consumer carbon-sensitivity scale κ on grand coalition total annual cost and total carbon emissions $\alpha = 0.6, \beta = 2$. Both fall monotonically with k , showing that stronger behavioral demand responses complement the supply-side effects of carbon taxation.

Simultaneously raises cost and lowers emissions, producing the intended regulatory trade-off while creating larger demand contractions for the retailers with the highest carbon-tax sensitivity. The transportation discount is quantitatively the most powerful lever in the model: removing it entirely raises grand coalition cost by 129% and reverses the incentive to pool orders above the discount threshold. Consumer carbon sensitivity, captured by the k_i parameters, acts as an independent complement to the carbon tax: a more carbon-conscious consumer base lowers supply chain costs and emissions even without any change in the tax rate, pointing to the value of demand-side green education programs.

Future research could extend this framework along several directions. Stochastic demand, in which the sensitivity parameters b_i and k_i themselves are uncertain, would create an incentive for information sharing among coalition members on top of the ordering coordination studied here. A general incremental discount schedule with multiple thresholds would accommodate a richer set of commercial transportation contracts. Endogen zing the carbon tax rate through a dynamic government optimization problem would connect the supply chain model to the broader macroeconomic literature on environmental policy under business cycles (Heutel, 2012; Chan, 2020).

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