International Journal of Computational Science and Mathematics.
ISSN 0974-3189 Volume 3, Number 1 (2011), pp. 109-110

http://www.irphouse.com

A Note on a Common Fixed Point Theorem in Cone Metric Spaces of Huang, Zhu and Wen

K.P.R. Sastry[1], Ch. Srinivasarao[2], K. Sujatha[3] and G. Praveena[4]

[1]8-28-8/1, Tamil Street, Chinna Waltair, Visakhapatnam-530 017, India.
E-mail: kprsastry@hotmail.com
[2]Department of Mathematics, Mrs. A.V.N. College, Visakhapatnam-530 001, India.
E-mail: drcsr41@yahoo.com
[3]Department of Mathematics, St. Joseph's College for Women,
Visakhapatnam-530 004, India
E-mail: kambhampati.sujatha@yahoo.com
[5]Department of Mathematics, Mrs. A.V.N. College, Visakhapatnam-530 001, India
E-mail: praveenagorapalli29@gmail.com

Abstract

In this paper, we obtain the result of Xianjiu Huang, Zhu, Wen [2] in a simple way as a corollary from (L.G. Haung and X. Zhang [1], Theorem 1)

Mathematics Subject Classification: 54E40; 54E35; 54H25

Keywords: Cone metric spaces; Common fixed point; Sequences of maps.

Introduction

The object of this note is to obtain the result of (Xianjiu Huang, Zhu, Wen [2]) from (L.G. Haung and X. Zhang [1], Theorem 1) in a simple way as corollary.

For relevant definitions and other material we refer to (Xianjiu Huang, Zhu, Wen [2]).

Main Results

L.G. Haung and X. Zhang [1] proved the following theorem.

Theorem 2.1 (L.G. Haung and X. Zhang [1], Theorem 1): Let (X, d) be a complete cone metric space and P a normal cone with normal constant K. Suppose the mapping S: $X \to X$ satisfies

$$d(Sx,Sy) \le k\, d(x, y) \quad (2.1.1)$$

for some $0 < k < 1$ and for every $x, y \in$ X. Then S has unique fixed point in X.

Now we obtain Theorem 2.1 of [2] as a corollary of the above theorem in a simple way.

Corollary 2.2 ([2], Theorem 2.1): Let (X, d) be a complete cone metric space and P a normal cone with normal constant K. Suppose the sequence $\{T_n\}$ of self mappings on X

satisfies, for some positive integer m,

$$d({T_i}^m x, {T_j}^m y) \le a_{i,j}\, d(x, y) \quad (2.2.1)$$

for all i,j = 1,2,3….., $x, y \in X$,

where $a_{i,j}$ and k are constants with $0 \le a_{i,j} < \mathrm{k} < 1$.Then the sequence

$\{ {T_i}^m \}$ has a unique common fixed point in X.

Proof: By hypothesis, $d({T_i}^m x, {T_j}^m y) \le a_{i,j}\, d(x, y)$
$\le \mathrm{k}\, d(x,y)$ for all $x, y \in X$.

Hence, by taking $x = y$ we get ${T_i}^m x = {T_j}^m x$ for all i, j $=$ 1,2,3 ...
Thus ${T_1}^m = {T_2}^m = {T_3}^m =$...
Hence, by Theorem 2.2,
$\{ {T_1}^m \}$ has unique fixed point, say, y^*.
Then ${T_1}^m(T_1 y^*) = T_1({T_1}^m y^*) = T_1 y^*$
so that $T_1 y^*$ is a fixed point of ${T_1}^m$
By condition (2.2.1) follows that
$T_1 y^*$ is a fixed point of ${T_1}^m$.
Hence, by the uniqueness of fixed point of ${T_1}^m$ follows that

$$T_1 y^* = y^*$$

Thus y^* is a fixed point of $T_1 = T_2 = T_3 =$...
If z^* is a fixed point of T_1, then z^* is also a fixed point of ${T_1}^m$ so that $z^* = y^*$
Thus y^* is the unique fixed point of $T_1 = T_2 = T_3 =$...

References

[1] L.G. Huang, X. Zhang, Cone metric spaces and fixed point theorems of contractive mappings, J. Math. Anal. Appl. 332, 2007, 1468-1476.

[2] Xianjiu Huang, Chuanxi Zhu and Xi Wen, A common fixed point theorem in cone metric spaces, Int.jour. of math. Analysis, vol.4, 2010, no.15,721 – 726.