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Solution of Differential Equation from the Transform Technique

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Abstract

In this article differential transform method is consider to solve second order differential equations. The analytical and numerical results of the equations have been obtained in terms of convergent series with easily comparable. Three examples are given to illustrate the efficiency of the present method. Differential transform technique may be considered as alternative and efficient for finding the approximate solutions of the boundary values problems.

Keywords: Differential transform method, Second order differential equation, Laplace transform.

Mathematical subject classification (2010): 65Mxx, 65M99.

Introduction

The Differential transform method has been successfully used by Zhou[6] to solve a linear and nonlinear initial value problems in electric circuit analysis. This method constructs an analytical solution in the form of a polynomial.. The differential transform method is an alterative method for finding the analytic solution of the differential equations. In this paper, we apply the differential transform method which is based on Taylor expansion to construct analytical approximate solutions of the initial value problem.

This paper is organized as follows: In Section 2, the differential transformation method is described. In Section 3, the method is implemented to three examples, and conclusion is given in Section 4.

Differential transformation method

Differential transformation of function $y(x)$ is defined as follows

$$Y(k) = \frac{1}{k!}\left[\frac{d^k y(x)}{dx^k}\right]_{x=0} \tag{1}$$

In (1), $y(x)$ is the original function and $Y(k)$ is the transformed function. Differential inverse transform of $Y(k)$ is defined as follows

$$y(x) = \sum_{k=0}^{\infty} x^k Y(k) \tag{2}$$

In fact, from (1) and (2), we obtain

$$y(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!}\left[\frac{d^k y(x)}{dx^k}\right] \tag{3}$$

Eq. (3) implies that the concept of differential transformation is derived from the Taylor series expansion. From the definitions (1) and (2), it is easy to obtain the following mathematical operations:

1. If $f(x) = g(x) \pm h(x)$ then $F(k) = G(k) \pm H(k)$.
2. If $f(x) = cg(x)$ then $F(k) = cG(k)$, where 'c' is constant.
3. If $f(x) = e^x$ then $F(k) = \frac{1}{k!}$
4. If $f(x) = \frac{d^n g(x)}{dx^n}$ then $F(k) = \frac{(k+n)!}{k!} G(k+n)$.
5. If $f(x) = g(x).h(x)$ then $F(k) = \sum_{r=0}^{\infty} G(k)F(k-r)$.
6. If $f(x) = x^n$ then $F(k) = \delta(k-n)$, where δ the Kronecker delta.

Numerical Examples

To demonstrate the method introduced in this study, three examples are solved here.

Example 1

Consider the equation

$$y^{11} + 5y^1 + 6y = 5e^t \tag{4}$$

the initial conditions is

$$y(0)=2 \text{ and } y^1(0)=1 \tag{5}$$

Taking the differential transform both sides of (4), we obtain

$$(k+2)(k+1)Y(k+2)+(k+1)Y(k+1)+6Y(k)=5\frac{1}{k!} \tag{6}$$

where $Y(k)$ is the differential transform.

From the initial condition given by Eq.(5) we have

$$Y(0)=2 \text{ and } Y(1)=1 \tag{7}$$

Taking Eq.(7) in Eq.(6) and by recursive method, we have

$$Y(2)=-6 \; Y(3)=\frac{59}{6} \; Y(4)=\frac{-109}{12} \; Y(5)=\frac{247}{40}$$

The form of the solution can be written as

$$y(x)=\sum_{k=0}^{\infty}x^kY(k)=2+x-6x^2+\frac{59}{6}x^3-\frac{109}{12}x^4+\frac{247}{40}x^5+........$$

$$y(x)=\frac{5}{12}e^x+\frac{16}{3}e^{-2x}-\frac{15}{4}e^{-3x},$$

which is exact solution of the equation (4)

Now applying laplace transform to the equation (4), we get the same solution.

Example 2

Consider the equation

$$y^{11}-2y^1+2y=0 \tag{8}$$

the initial conditions is

$$y(0)=1 \text{ and } y^1(0)=1 \tag{9}$$

Taking the differential transform both sides of (8), we obtain

$$(k+2)(k+1)Y(k+2)-2(k+1)Y(k+1)+2Y(k)=0 \tag{10}$$

where $Y(k)$ is the differential transform.

From the initial condition given by Eq.(9) we have

$$Y(0)=1 \text{ and } Y(1)=1 \tag{11}$$

Taking Eq.(11) in Eq.(10) and by recursive method, we have

$$Y(2)=0 \; Y(3)=\frac{-1}{3} \; Y(4)=\frac{-1}{6} \;\; Y(5)=\frac{-1}{30}$$

The form of the solution can be written as

$$y(x) = \sum_{k=0}^{\infty} x^k Y(k) = 1 + x - \frac{1}{2}x^3 - \frac{1}{6}x^4 - \frac{1}{30}x^5 + ...$$

$y(x) = e^x \cos x$, which is exact solution of the equation (8)

Now applying laplace transform to the equation (8) we get the same solution.

Example 3

Consider the equation

$$y^{11} + y^1 = x^2 + 2x \tag{12}$$

the initial conditions is

$$y(0) = 4 \text{ and } y^1(1) = -2 \tag{13}$$

Taking the differential transform both sides of (12), we obtain

$$(k+2)(k+1)Y(k+2) + (k+1)Y(k+1) = \delta(k-2) + 2\delta(k-1) \tag{14}$$

where $Y(k)$ is the differential transform.

From the initial condition given by Eq.(13) we have

$$Y(0) = 4 \text{ and } Y(1) = -2 \tag{15}$$

Taking Eq.(15) in Eq.(14) and by recursive method, we have

$$Y(2) = 1 \quad Y(3) = 0 \quad Y(4) = \frac{1}{12} \quad Y(5) = \frac{-1}{60}$$

The form of the solution can be written as

$$y(x) = \sum_{k=0}^{\infty} x^k Y(k) = 4 - 2x + x^2 + \frac{1}{12}x^4 - \frac{1}{60}x^5 + ...$$

$y(x) = 2 + 2e^{-x} + \frac{1}{3}x^3$, which is exact solution of the equation (12)

Now applying laplace transform to the equation (12), we get the same solution.

Conclusion

In this paper, we have shown that the differential transform method can be used successfully for finding the solution of second order differential equations and we compare the solution with other alternate method. It may be concluded that this technique is very powerful and efficient in finding solutions.

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