International Journal of Computational Science and Mathematics.
ISSN 0974-3189 Volume 3, Number 1 (2011), pp. 35-46

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An Inventory Model for Product Life Cycle with Defective Items and Shortages Allowed

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Abstract

To manage inventory, the classical economic order quantity (EOQ) equation can be used to decide how much to order. The EOQ is the order quantity that theoretically minimizes the total of the cost of ordering and holding inventory and it assumes that the demand is constant, does not vary over time. This paper deals with an inventory model for product life cycle with and without defective items and shortages allowed for a single product manufacturing /purchasing system. The defective rate is considered as a variable of known proportions. The objective is to minimize the total net inventory cost and to find the optimal quantity in product life cycle. The relevant model is built, solved and some main results about the uniqueness of this solution with the use of rigorous mathematical methods are obtained. This seems to be the first time where such a inventory model for product life cycle is mathematically treated and numerically verified.

Keywords: Inventory, Cycle time, Shortages, Defective Items, Demand and Production.

Introduction

A product life cycle is the life span of a product which the period begins with the initial product specification and ends with the withdrawal from the market of both the product and its support. A new product is first developed and then introduced to the market. Once the introduction is successful, a growth period follows with wider awareness of the product and increasing sales. The product enters maturity when sales stop growing and demand stabilizes. Eventually, sales may decline until the product is finally with drawn from the market or redeveloped. A product’s life cycle can be divided into several stages characterized by the revenue generated by the product. The life cycle concept may apply to a brand or to a category of product. Its duration may be as short as a few months for a fad item or a century or more for product categories such as the gasoline powered automobile. During the introductory stage the firm is likely to incur additional costs ie advertising cost associated with the initial distribution of the product. These higher costs coupled with a low sales volume usually make the introduction stage a period of negative profits. During the introduction stage, the primary goal is to establish a market and build primary demand for the product class. The growth stage is a period of rapid revenue growth. Sales increase as more customers become aware of the product and its benefits and additional market segments are targeted. Once the product has been proven a success and customers begin asking for it, sales will increase further as more retailers become interested in carrying it. The marketing team may expand the distribution at this point. The maturity stage is the most profitable. While sales continue to increase into this stage, they do so at a slower pace. Because brand awareness is strong, advertising expenditures will be reduced. Eventually sales begin to decline as the market becomes saturated, the product becomes technologically obsolete or customer tastes change. If the product has developed brand loyalty, the profitability may be maintained longer. Unit costs may increase with the declining production volumes and eventually no more profit can be made.

The production of inventory is dependent on the type of inventory system that the organization is adopting. To facilitate solving the problem of inventory, it is required to build models, which explain the inventory fluctuation. In some cases such as retailer, wholesaler/distributor, where items are purchased externally, if the problem of inventory exists, then there are two main questions, which generally arise and face any organization. These are how many to order and when to order. Having too much inventory reduces both purchases and/or ordering costs but it may tie up capital, which may lead to unnecessary holding cost and possibility of deteriorating items. Whereas having too little inventory reduces the holding cost but it can result in lost of customers, which may affect the reliability of the organization, which minimizes its total inventory cost.

Assumptions and Notations

a. Assumptions: The assumption and notations of an inventory model for product life cycle are as follows:

1. The demand rate is known, constant and continuous.
2. Items are produced and added to the inventory.
3. Shortages are allowed.
4. The item is a single product, it does not interact with any other inventory items.
5. The production rate is always greater than or equal to the sum of the demand rate.

b. Notations: This section defines the notations used in the inventory models

P – Production rate in units per unit time

D – Demand rate in units per unit time
Q_1 – on hand inventory level
Q*-Optimal size of production run
C_p – Production /Purchase Cost per unit
W – rate of defective items from end customers in units per unit time (W = Dy)
Y – proportion of defective items after distribution to end customers (y is between 0 to 1)
x – proportion of defective items from regular production (x is between 0 to 1)
C_d – unit scrap cost per item of imperfect quality.
C_g – Cost of customer return (cost of disposal, shipment and penalty)
C_h -Holding cost per unit/year
C_0 – Setup cost / ordering cost
t – unit time in one cycle
t_i - unit time in periods i (i = 1, 2, 3, ………..)
TC - Total cost
TOC* - Minimum total cost per cycle $\left(= \frac{TotalCost}{TimeInOneCycle} \right)$

Problem Formulation

The objective of this research is to develop mathematical models to minimize the expected total cost of inventory. Initially, the manufacturer must define all costs (such as the cost of production, holding cost, setup cost) production characteristics and all capabilities of the production process. These have to be accurate because these variables will directly affect the production quantity and total cost. This paper deals with a finite production inventory model for a single product imperfect manufacturing system. The defect rate is considered as a variable of known proportions. The mathematical models for optimal production lot size in this research can be classed as follows:

1. Model 1 : EOQ model for product life cycle with shortages.
2. Model 2 : EOQ model for product life cycle with shortages and defective items.

Model 1 : An Economic Order Quantity Model for Product Life Cycle with Shortages: The system operates as follows: It starts at time t_0 at a demand rate D. Then production starts where the inventory level increases at a rate P-D in order to satisfy the demand until time t_1. At this point, the production ceases and the inventory level reaches its maximum. Then the demand in the constant rate of D until time t_2. Thereafter, the inventory level declines continuously at a rate D and becomes zero at time $t_1 + t_2 + t_3$ (end of the cycle). The process is repeated. The variation of the underlying inventory system for one cycle is shown in the following figure. During production period t_1, inventory is increasing at the rate of P and simultaneously decreasing at the rate of D. Thus inventory accumulates at the rate of P-D units. Therefore, the maximum level shall be equal to $(P-D)t_1$. Time t_1 needed to build up Q_1 units of items . From the diagram,

$$t_1 = \frac{Q_1}{P-D};\ t_2 = \frac{Q_1}{D};\ t_3 = \frac{Q_1}{D};\ t_4 = \frac{x}{D};\ t_5 = \frac{x}{D} \text{ and } t = \frac{Q}{D}$$

Production Cycle:

$$t = t_1 + t_2 + t_3 + t_4 + t_5$$

$$= \frac{Q_1}{P-D} + \frac{Q_1}{D} + \frac{Q_1}{D} + \frac{x}{D} + \frac{x}{P-D} = \frac{Q_1(2P-D)}{D(P-D)} + \frac{xP}{D(P-D)}$$

$$\frac{Q}{D} = \frac{Q_1(2P-D)}{D(P-D)} + \frac{xP}{D(P-D)}$$

therefore,

$$Q_1 = \frac{Q(P-D) - xP}{(2P-D)}$$

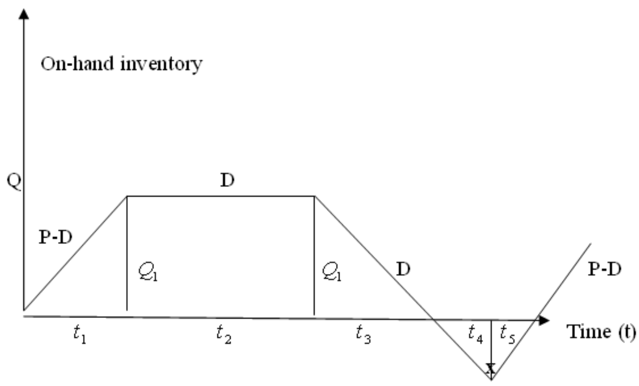

Figure 1: On-hand inventory of non- defective items for Model 1.

Preposition: 1 The optimal solution for the inventory policy is given by

$$Q = \sqrt{\frac{2DC_0(2P-D)^2\left[C_hP(3P-2D) + C_S(2P-D)^2\right]^2}{C_h(P-D)(3P-2D)\begin{bmatrix}\left(C_hP(3P-2D) + C_S(2P-D)^2\right)^2 \\ -C_SP(2P-D)^2(3P-2D) - P^2C_h^2(3P-2D)^2\end{bmatrix}}}$$

Proof: The total cost components consist of Purchase/production cost, setup cost and holding cost and shortages cost.

a. Purchase/Production Cost = $C_P Q$
b. Ordering/Setup cost = C_0
c. Holding Cost: The holding cost is derived as follows:

$$HC = C_h\left(\frac{Q_1 t_1}{2} + Q_1 t_2 + \frac{Q_1 t_3}{2}\right) = \frac{C_h Q_1}{2}\left(\frac{Q_1}{P-D} + \frac{2Q_1}{D} + \frac{Q_1}{D}\right)$$

$$= \frac{C_h {Q_1}^2}{2}\left(\frac{D + 2(P-D) + P - D}{D(P-D)}\right) = \frac{C_h}{2}\left(\frac{Q(P-D) - xP}{2P-D}\right)^2\left(\frac{3P-2D}{D(P-D)}\right)$$

$$= \frac{C_h Q^2 (P-D)(3P-2D)}{2D(2P-D)^2} + \frac{C_h x^2 P^2 (3P-2D)}{2D(P-D)(2P-D)^2} - \frac{xPQC_h(3P-2D)}{D(2P-D)^2}$$

d. Shortages Cost : = $C_S\left(\frac{1}{2}xt_4 + \frac{1}{2}xt_5\right) = \frac{C_S P x^2}{2D(P-D)}$

e. Total Cost: The total cost is as follows:

$$TC = C_p Q + C_0 + \frac{C_h Q^2 (P-D)(3P-2D)}{2D(2P-D)^2} + \frac{C_h x^2 P^2 (3P-2D)}{2D(P-D)(2P-D)^2} - \frac{xPQC_h(3P-2D)}{D(2P-D)^2}$$

$$+ \frac{C_S P x^2}{2D(P-D)}$$

The total cost per unit time is given below:

$$TC = DC_p + \frac{DC_0}{Q} + \frac{C_h Q(P-D)(3P-2D)}{2(2P-D)^2} + \frac{C_h x^2 P^2 (3P-2D)}{2Q(P-D)(2P-D)^2}$$

$$- \frac{xPC_h(3P-2D)}{(2P-D)^2} + \frac{C_S P x^2}{2D(P-D)}$$

Partial differentiate w.r.t. to Q,

$$\frac{\partial}{\partial Q} = 0 \Rightarrow -\frac{DC_0}{Q^2} + \frac{C_h (P-D)(3P-2D)}{2(2P-D)^2} - \frac{C_h x^2 P^2 (3P-2D)}{2Q^2 (P-D)(2P-D)^2} - \frac{PC_S x^2}{2Q^2 (P-D)} = 0$$

$$\frac{\partial^2}{\partial Q^2}(TC) = \frac{2DC_0}{Q^3} + \frac{C_h x^2 P^2 (3P-2D)}{Q^3 (2P-D)^2 (P-D)} + \frac{C_S P x^2}{Q^3 (P-D)} > 0$$

$$\frac{1}{Q^2}\left(DC_0 + \frac{C_h x^2 P^2 (3P-2D)}{2(P-D)(2P-D)^2} + \frac{PC_S x^2}{2(P-D)}\right) = \frac{C_h (3P-2D)(P-D)}{2(2P-D)^2}$$

$$Q^2 = \frac{2DC_o (2P-D)^2}{C_h (P-D)(3P-2D)} + \frac{x^2 P^2}{(P-D)^2} + \frac{C_S P x^2 (2P-D)^2}{C_h (P-D)^2 (3P-2D)} \qquad \text{(A)}$$

Partially differentiate with respect to x,

$$\frac{\partial}{\partial x}(TC) = \frac{-C_h P(3P-2D)}{(2P-D)^2} + \frac{P^2 x C_h (3P-2D)}{Q(2P-D)^2(P-D)} + \frac{PC_S x}{Q(P-D)} = 0$$

$$\frac{\partial^2}{\partial x^2}(TC) = \frac{C_h P^2 (3P-2D)}{Q(2P-D)^2(P-D)} + \frac{C_S P}{Q(P-D)} > 0$$

$$x\left(\frac{C_h P^2(3P-2D)}{Q(P-D)(2P-D)^2} + \frac{PC_S}{Q(P-D)}\right) = \frac{PC_h(3P-2D)}{(2P-D)^2}$$

$$x = \frac{QC_h(3P-2D)(P-D)}{C_h P(3P-2D) + C_S(2P-D)^2}$$ substitute in the equation (A)

$$Q^2 = \left(\begin{array}{l} \dfrac{2DC_o(2P-D)^2}{C_h(P-D)(3P-2D)} + \dfrac{Q^2P^2C_h^2(3P-2D)^2}{\left[C_h P(3P-2D)+C_S(2P-D)^2\right]^2} \\ + \dfrac{C_S P Q^2 C_h (2P-D)^2(3P-2D)}{\left[C_h P(3P-2D)+C_S(2P-D)^2\right]^2} \end{array}\right)$$

$$Q = \sqrt{\frac{2DC_0(2P-D)^2\left[C_h P(3P-2D)+C_S(2P-D)^2\right]^2}{C_h(P-D)(3P-2D)\left[\begin{array}{l}\left(C_h P(3P-2D)+C_S(2P-D)^2\right)^2 \\ -C_S P(2P-D)^2(3P-2D) - P^2C_h^2(3P-2D)^2\end{array}\right]}}$$

Illustrative Example, consider the following parameters

P=1500 units/year, D = 1,000 units/year, C_0 = Rs.150,

C_h = 10 per units/year, C_P = Rs.100, C_S = 30

Solution: Q = 321.50 units, x = 25.52; Total cost = 100,934.73

Model 2 : An Economic Order Quantity Model for Product Life Cycle with Shortages and Defective Items: The proposed inventory system operates as follows: The cycle starts at time t = 0 and the inventory accumulates at a rate P-D-W upto time t_1 where production stops. After that , the inventory level starts to decrease due to demand and defective items at a rate D + W upto time t_2 and t_3. The process is repeated. The variation of the underlying inventory system for on cycle is shown in the following figure. The production rate of imperfect items:

W = D(x+y) (external)

The production rate of good items is always greater than or equal to the sum of the demand rate and the rate which defective items are produced . So, we must have:

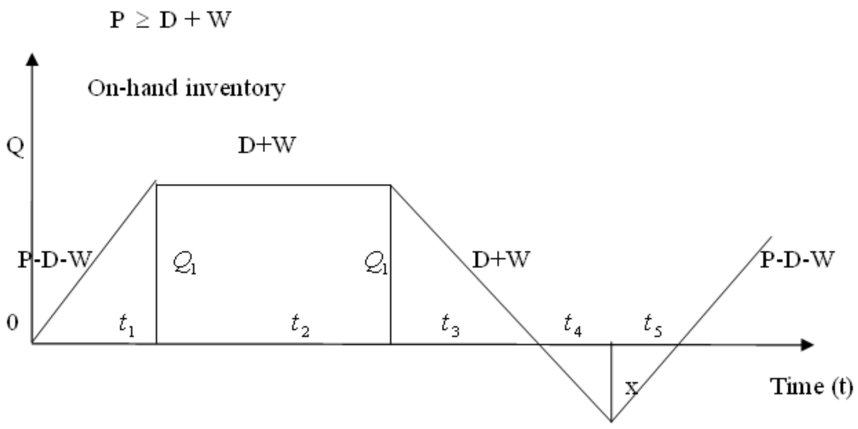

Figure 2: On-hand inventory of defective items for Model 2.

During production period t_1, the maximum level shall be equal to $(P-D-W)t_1$. From the above figure, Time t_1 needed to build up Q_1 units of items. From the figure,

$$t_1=\frac{Q_1}{P-D-W};\ t_2=\frac{Q_1}{D+W};\ t_3=\frac{Q_1}{D+W};\ t_4=\frac{x}{D+W};\quad t_5=\frac{x}{P-D-W};\ \text{t}$$

$$=\frac{Q}{D+W}$$

Production Cycle: t = $t_1+t_2+t_3+t_4+t_5$

$$=\frac{Q_1}{P-D-W}+\frac{Q_1}{D+W}+\frac{Q_1}{D+W}+\frac{x}{D+W}+\frac{x}{P-D-W}$$

i.e. $\dfrac{Q}{D+W}=\dfrac{Q_1(2P-D-W)}{P-D-W}+\dfrac{xP}{P-D-W}$

Therefore, $Q_1=\dfrac{Q(P-D-W)-xP}{2P-D-W}$

Preposition 2: The optimal solution for the inventory policy is given by

$$Q=\sqrt{\frac{2C_0(D+W)(2P-D-W)^2\left[PC_h(3P-2D-2W)+C_S(2P-D-W)^2\right]^2}{C_h(3P-2D-2W)(P-D-W)\begin{pmatrix}\left[PC_h(3P-2D-2W)+C_S(2P-D-W)^2\right]^2\\-P^2C_h^2(3P-2D-2W)^2\\-C_SPC_h(2P-D-W)^2(3P-2D-2W)\end{pmatrix}}}$$

Proof: The total costs per cycle appropriate to this model are as follows:

a. Purchase/Production Cost = $C_P Q$
b. Ordering/Setup cost = C_0
c. Holding Cost: The holding costs should include that of all produced items, defective and non-defective.

$$HC = C_h\left(\frac{Q_1t_1}{2}+Q_1t_2+\frac{Q_1t_3}{2}\right) = \frac{C_hQ_1}{2}\left(\frac{Q_1}{P-D-W}+2\left(\frac{Q_1}{D+W}\right)+\frac{Q_1}{D+W}\right)$$

$$= \frac{C_h}{2}\left(\frac{3P-2D-2W}{(D+W)(P-D-W)}\right)\left(\frac{Q(P-D-W)-xP}{2P-D-W}\right)^2$$

$$= \frac{C_hQ^2(P-D-W)(3P-2D-2W)}{2(D+W)(2P-D-W)^2}+\frac{x^2P^2C_h(3P-2D-2W)}{2(D+W)(2P-D-W)^2(P-D-W)}$$

$$-\frac{QPxC_h(3P-2D-2W)}{(D+W)(2P-D-W)^2}$$

d. Shortage Cost = $\frac{C_S}{2}x\left(\frac{x}{D+W}+\frac{x}{P-D-W}\right)=\frac{C_Sx^2P}{2(D+W)(P-D-W)}$

e. Defective Cost : Cost per defect passed forward customers (scrap and penalty costs)

$= C_d$ Q(x+y) + C_g Q(x+y)

Total Cost: The total cost (TC) would be:

TC = Purchase Cost + Ordering cost + holding cost + Shortages cost + Cost of defective items

$$TC=C_pQ+C_0+\frac{C_hQ^2(3P-2D-2W)(P-D-W)}{2(D+W)(2P-D-W)^2}$$

$$+\frac{x^2P^2C_h(3P-2D-2W)}{2(D+W)(2P-D-W)^2(P-D-W)} - \frac{QPxC_h(3P-2D-2W)}{(D+W)(2P-D-W)^2}$$

$$+\frac{Px^2C_S}{2(D+W)(P-D-W)} + C_d\,Q(X+Y) + C_g\,Q(X+Y)$$

Total Cost per unit time

$$TC=C_p(D+W)+\frac{C_0(D+W)}{Q}+\frac{C_hQ(3P-2D-2W)(P-D-W)}{2(2P-D-W)^2}$$

$$+\frac{x^2P^2C_h(3P-2D-2W)}{2Q(2P-D-W)^2(P-D-W)} - \frac{PxC_h(3P-2D-2W)}{(2P-D-W)^2}+\frac{x^2C_SP}{2Q(P-D-W)}$$

$+\ C_d$ (D+W)(X+Y) + C_g (D+W)(X+Y)

Partially differentiate w.r.t. Q

$$\frac{\partial}{\partial Q}(TC)=0 \ \Rightarrow -\frac{C_0(D+W)}{Q^2}+\frac{C_h(3P-2D-2W)(P-D-W)}{2(2P-D-W)^2}$$

$$-\frac{x^2P^2C_h(3P-2D-2W)}{2Q^2(2P-D-W)^2(P-D-W)} - \frac{x^2C_SP}{2Q^2(P-D-W)}=0$$

$$\frac{\partial^2}{\partial Q^2}(TC)=\frac{2(D+W)C_0}{Q^3}+\frac{x^2P^2C_h(3P-2D-2W)}{Q^3(2P-D-W)^2(P-D-W)}+\frac{C_Sx^2P}{Q^3(P-D-W)}>0$$

$$\frac{1}{Q^2}\left(C_0(D+W)+\frac{x^2P^2C_h(3P-2D-2W)}{2(2P-D-W)^2(P-D-W)}+\frac{C_Sx^2P}{2(P-D-W)}\right)$$

$$=\frac{C_h(3P-2D-2W)(P-D-W)}{2(2P-D-W)^2}$$

$$Q^2=\frac{2C_0(D+W)(2P-D-W)^2}{C_h(3P-2D-2W)(P-D-W)}+\frac{x^2P^2}{(P-D-W)^2}+$$
$$\frac{x^2C_SP(2P-D-W)^2}{C_h(3P-2D-2W)(P-D-W)^2}$$

Partially differentiate w.r.t. x

$$\frac{\partial}{\partial x}(TC)=0\Rightarrow \frac{xP^2C_h(3P-2D-2W)}{Q(2P-D-W)^2(P-D-W)}-\frac{PC_h(3P-2D-2W)}{(2P-D-W)^2}+\frac{xC_SP}{Q(P-D-W)}$$
$$=0$$

$$\frac{\partial^2}{\partial x^2}(TC)=\frac{P^2C_h(3P-2D-2W)}{Q(2P-D-W)^2(P-D-W)}+\frac{C_SP}{Q(P-D-W)}>0$$

$$x\left(\frac{P^2C_h(3P-2D-2W)}{Q(2P-D-W)^2(P-D-W)}+\frac{C_SP}{Q(P-D-W)}\right)=\frac{PC_h(3P-2D-2W)}{(2P-D-W)^2}$$

$$x=\frac{QC_h(3P-2D-2W)(P-D-W)}{PC_h(3P-2D-2W)+C_S(2P-D-W)^2}$$

$$Q^2=\frac{2C_0(D+W)(2P-D-W)^2}{C_h(3P-2D-2W)(P-D-W)}+\frac{C_h^2Q^2P^2(3P-2D-2W)^2}{\left[C_hP(3P-2D-2W)+C_S(2P-D-W)^2\right]^2}$$

$$+ \frac{C_h Q^2 C_S P(2P-D-W)^2(3P-2D-2W)}{\left[C_h P(3P-2D-2W)+C_S(2P-D-W)^2\right]^2}$$

$$Q = \sqrt{\frac{2C_0(D+W)(2P-D-W)^2\left[PC_h(3P-2D-2W)+C_S(2P-D-W)^2\right]^2}{C_h(3P-2D-2W)(P-D-W)\begin{pmatrix}\left[PC_h(3P-2D-2W)+C_S(2P-D-W)^2\right]^2\\ -P^2C_h^2(3P-2D-2W)^2\\ -C_S PC_h(2P-D-W)^2(3P-2D-2W)\end{pmatrix}}}$$

Illustrative Example, consider the following parameters

D = 1,000 units/year, P = 1,500 units/year, C_0 = Rs.50, x = 0.05 , $C_S = 30$; y = 0.01 to 0.1, $C_h = 10$ per units/year, $C_P = 100$, C_d = 15, y = 0.05, C_g = 150

Table 1: Variation of Q, W and T.

Rate of Defective	Q	x	Q_1	t_1	t_2	t_3	t_4	t_5	t
0.01	360.08	28.05	67.52	0.1378	0.0669	0.0669	0.0278	0.0572	0.3566
0.02	365.34	27.92	67.42	0.1405	0.0661	0.0661	0.0269	0.0582	0.3578
0.03	370.73	27.78	67.30	0.1432	0.0653	0.0653	0.0270	0.0591	0.3599
0.04	376.27	27.64	67.15	0.1460	0.0646	0.0646	0.0266	0.0601	0.3619
0.05	381.98	27.49	67.00	0.1489	0.0638	0.0638	0.0262	0.0611	0.3638
0.06	387.85	27.33	66.84	0.1519	0.0631	0.0631	0.0258	0.0621	0.3660
0.07	393.89	27.16	66.65	0.1550	0.0623	0.0623	0.0254	0.0632	0.3682
0.08	400.13	27.00	66.44	0.1582	0.0615	0.0615	0.0250	0.0643	0.3705
0.09	406.56	26.81	66.22	0.1615	0.0608	0.0608	0.0246	0.0654	0.3731
1.00	413.21	26.62	65.98	0.1650	0.0600	0.0600	0.0242	0.0666	0.3758

From table 1, a study of rate of defective items with lot size Q, manufacturing time t_1, constant sales time t_2, decrease sales time t_3 and cycle time. We conclude from the above table, when the rate of defective items W increases then the Q, t_1, t_2 , t_3 and t also increases but x and Q_1 decreases.

Table 2: Variation of Rate of Defective and inventory and Total Cost.

Rate of Defective	Purchase Cost	Ordering Cost	Holding cost	Defective cost	Shortage cost	Total Cost
0.01	101000	420.74	420.74	6969.00	100.34	108910.82
0.02	102000	418.79	418.79	8211.00	100.02	111148.60
0.03	103000	416.75	416.75	9476.00	99.65	113409.15
0.04	104000	414.60	414.60	10764.00	99.31	115692.51

0.05	105000	412.32	412.32	12075.00	98.92	117998.56
0.06	106000	409.95	409.95	13409.00	98.48	120327.38
0.07	107000	407.47	407.47	14766.00	98.00	122678.94
0.08	108000	404.87	404.87	16146.00	97.60	125053.34
0.09	109000	402.15	402.15	17549.00	97.02	127450.32
1.00	110000	399.31	399.31	18975.00	96.47	129870.09

From table 2, a study of rate of defective items with inventory cost and total inventory cost. We conclude from the above table, when the rate of defective items increases then purchase cost, defective cost and total cost also increases but ordering cost and holding cost decreases.

Conclusion

In this paper, an EOQ model for product life cycle is considered in which each of the demand, production and as well as all cost parameters are known. Shortages are not allowed. The objective is to minimize the overall total relevant inventory cost. An exact mathematical model and a solution procedure is established. An illustrative example also explained. This seems to be the first time where such a inventory model for product life cycle is mathematically treated and numerically verified and it is conclude that when the rate of defective items increases then Optimal order Quantity (Q), t_1, t_2 , t_3 t, purchase cost, defective cost and total cost also increases but the ordering cost and holding cost decreases.

References

[1] R.W.Grubbstrom, Material Requirements Planning and Manufacturing Resource planning, International Encyclopedia of business and Management Rout ledge, London, 1996.

[2] R.W. Grubbstrom and A .Erdem, The EOQ with backlogging derived without derivatives, International Journal of Production Economics, Vol.59, pp529-530, 1999.

[3] L.E. Cardenas-Barron, The Economic Production quantity (EPQ) with shortage derived algebraically, International Journal of production Economics, Vol.70, pp 289-292, 2001.

[4] Chun-Hsiung Lan and Yen-chick Yu, A note on the improved Algebraic method for the EPQ Model with Stochastic Lead time, Information and Management Sciences, Vol.18, No.1, pp.91-96, 2007.

[5] Jon June and Eyler R. Coates, An Economic Order quantity model for Time varying demand, The International Journal of Modern Engineering, Vol.1, Number 2, Spring, 2001.

[6] Singh Sarbjit, Sharma Jitendra, Singh shivraj, Profit Maximizing probabilitics Inventory model under the Effect of Permission delay, Proceedings of the International Multi Conference of Engineers and Computer Scientists 2010 Vol, III, IMECS 2010, March 17-19, 2010, Hongkong.

[7] Zaid T. Balkhi and Ali S. Haj Bakry, A general and dynamic production lot size inventory model, International Journal of Mathematical Models and Methods in Applied Sciences, issue 3, volume 3, 2009, p. 187 – 193.

[8] C. Sugapriya, K. Jeyaraman, An EOQ model for non – instaneous deteriorating items in which holding cost varies with time. Electronic Journal of Applied Statistical Analysis, Vol. 1, pp 19- 27, 2008.